Effective Component Tree Computation with Application to Pattern Recognition in Astronomical Imaging

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International Conference on Image Processing (ICIP)
September 18, 2007

Effective Component Tree Computation with Application to Pattern Recognition in Astronomical Imaging

Context

- Goal: apply connected filters from mathematical morphology to astronomical images.
- Features of processed astronomical images
  - Huge sizes (order of magnitude: 100 MB – 1.5 GB)
  - Pixels encoded as floating-point values
  - High dynamic ranges

$\Rightarrow$ New tools needed to write these filters, in particular a component tree algorithm

- Joint-work between IAP and LRDE, in the context of the EFIGI project (Extraction of Idealized Patterns of Galaxies in Imaging)

http://www.efigi.org

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Component Tree

- Convenient and versatile representation of an image
- Parenthood relationship between nodes maps component (spatial) inclusion
- Applications
  - Classification
  - Image Filtering
  - Segmentation
  - Registration
  - Compression

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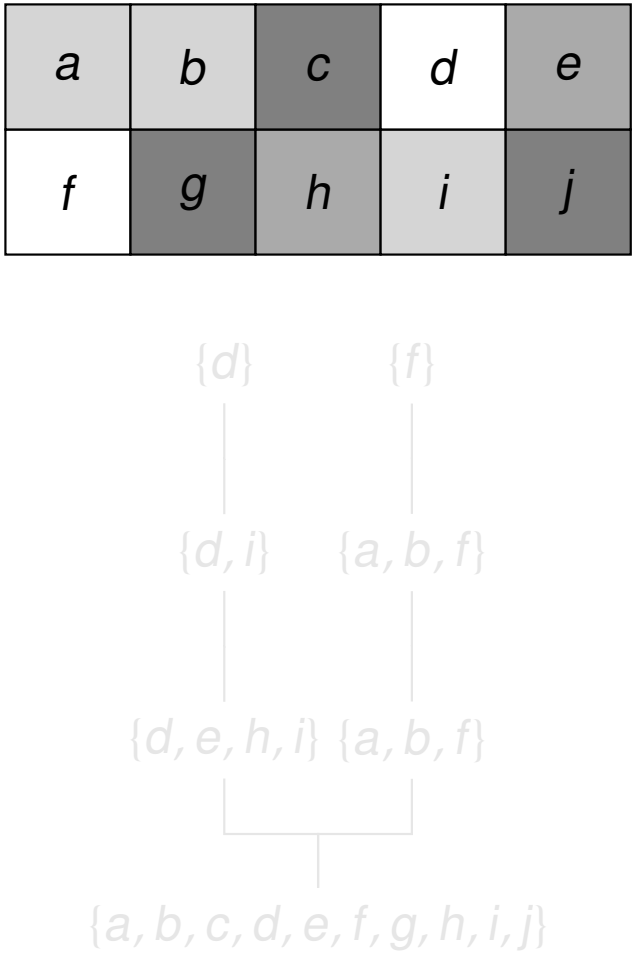

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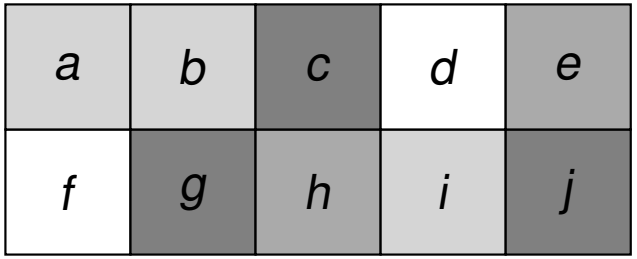

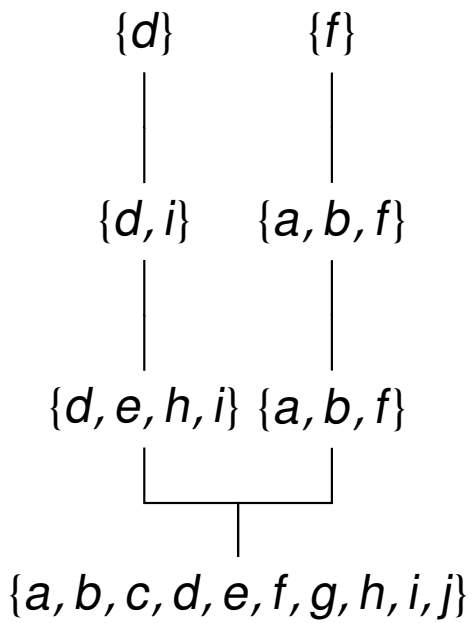

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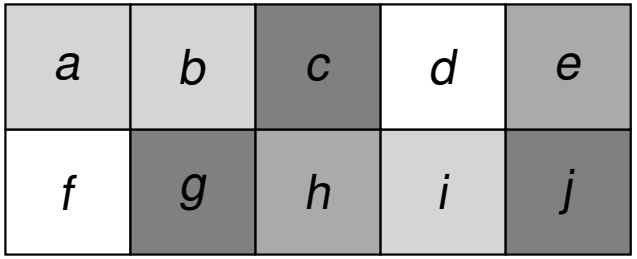

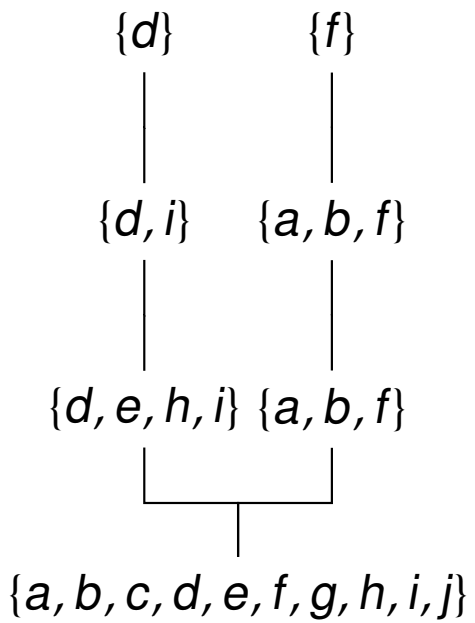

Connected Filters

- Properties
  - Rely on attributes of components (no structuring element)
  - Simplify the images
  - Do not create nor shift contours
- Relationship with the component tree
  - A connected filter can be expressed as a transformation of the component tree that does not add any branch
- Recent filters (about ten years of existence)

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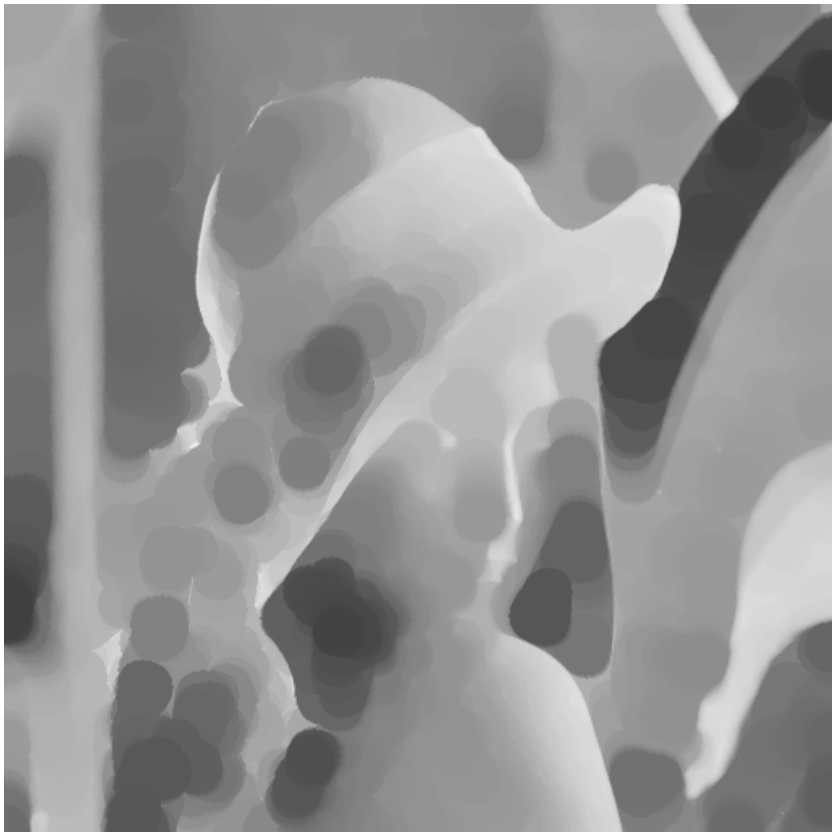
Example 1/4

Morphological Opening Using a Structural Element (disc, radius = 15 pixels)

Example 2/4

Morphological Closing Using a Structural Element (disc, radius = 15 pixels)



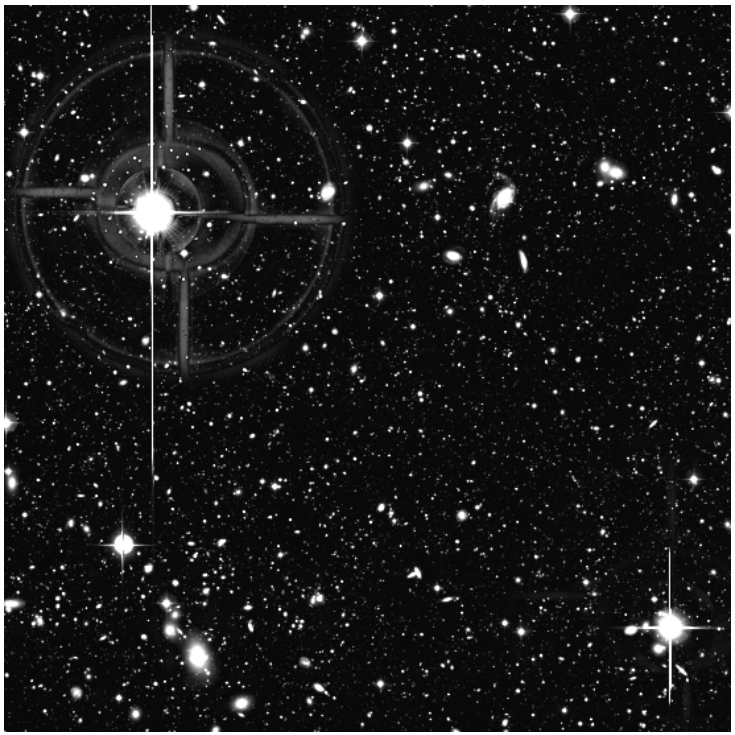
Example 3/4

Morphological Area (attribute) opening (area $\approx \pi 15^2$ pixels)

Example 4/4

Morphological Area (attribute) closing (area $\approx \pi 15^2$ pixels)

Acquisition and Nature of Data



- The observed image is convolved by a Point-Spread Function
- 32-bit, floating-point values
- Most pixels correspond to the (noisy) sky background (dark areas)
- Brighter pixels: objects (stars, galaxies) and optical effects (halos, etc.).

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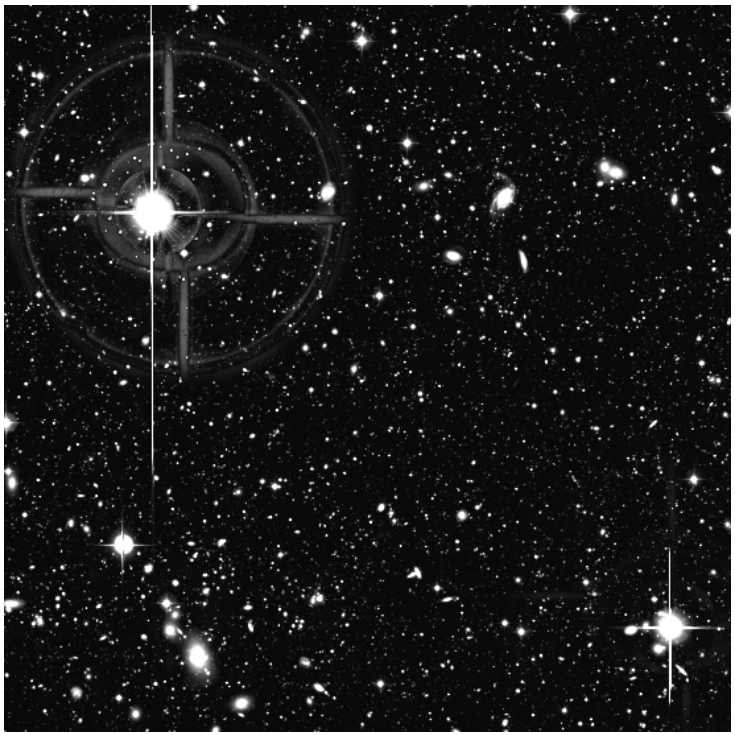

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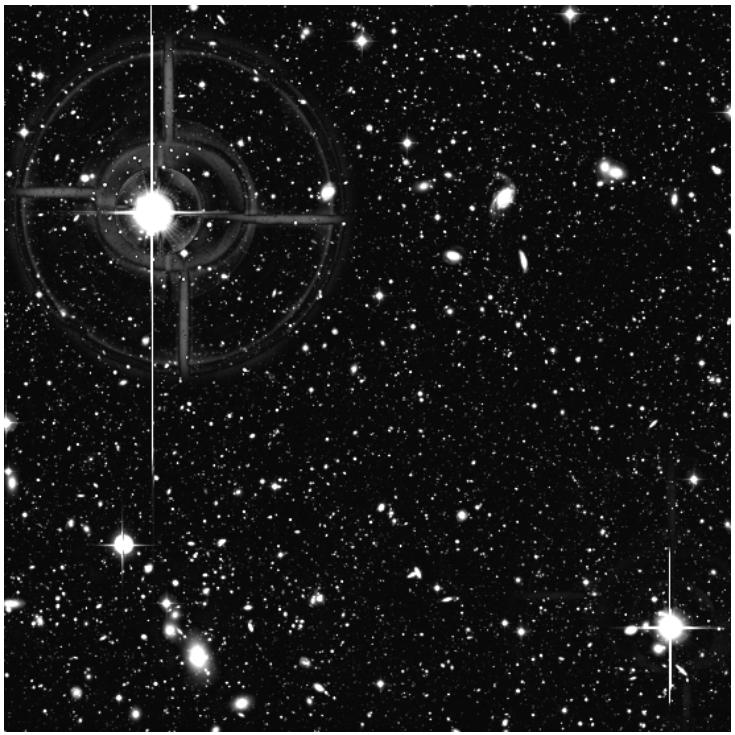

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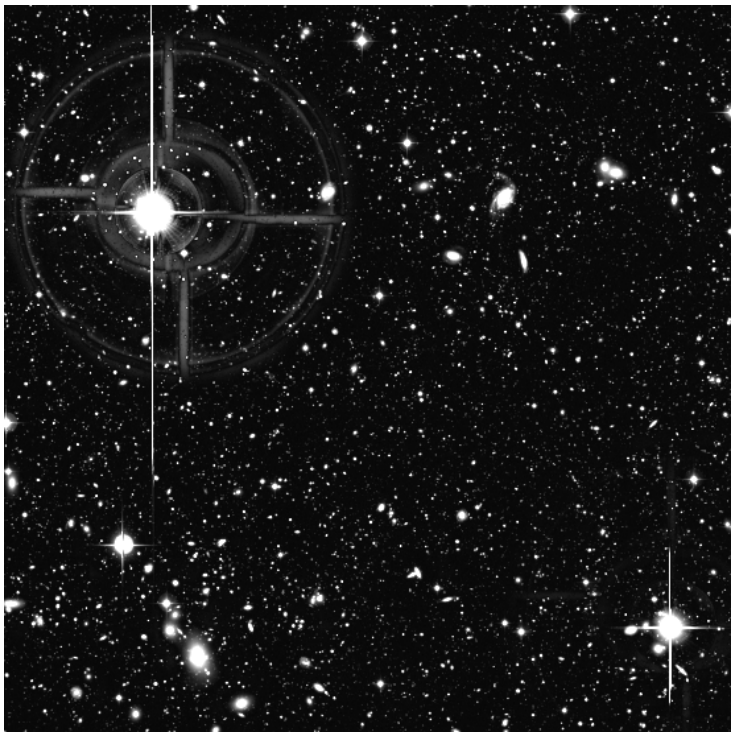

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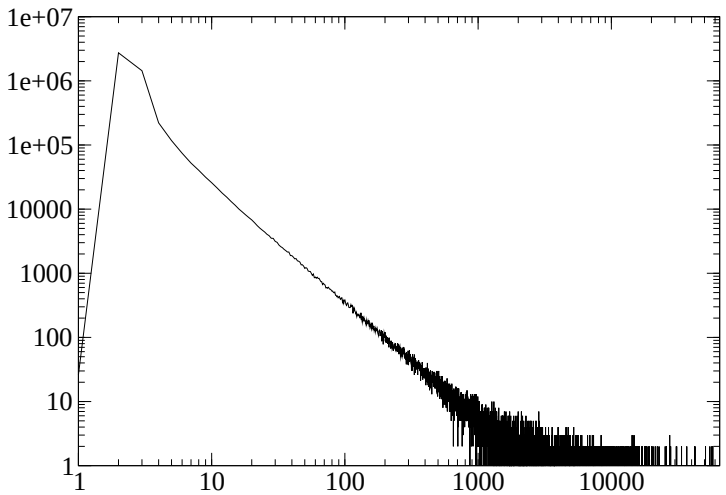
Quantization

Example

Histogram
(*log-log* scale)



- 16-bit, linear quantization
- Most pixels between 0 and 255
- The slope that appears on this plot is due to the presence of blur.

⇒ Need for an optimal quantization; or

⇒ direct processing of floating-point values with no quantization

Quantization

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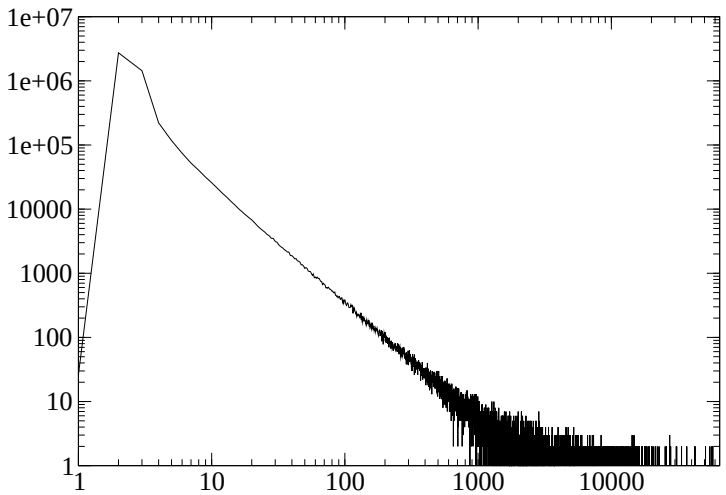

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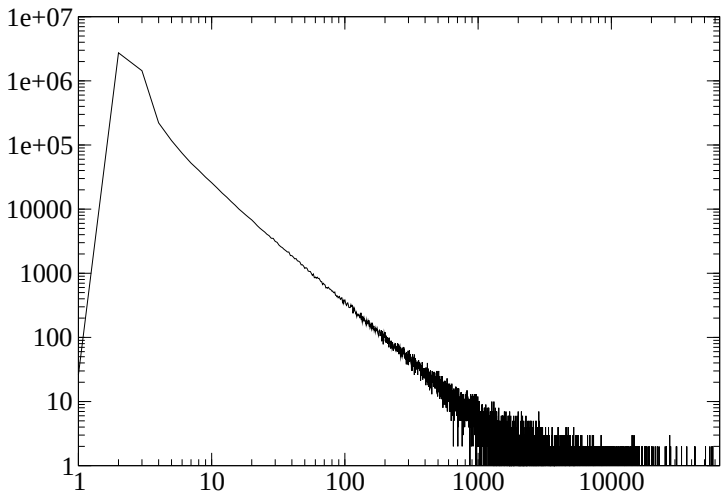

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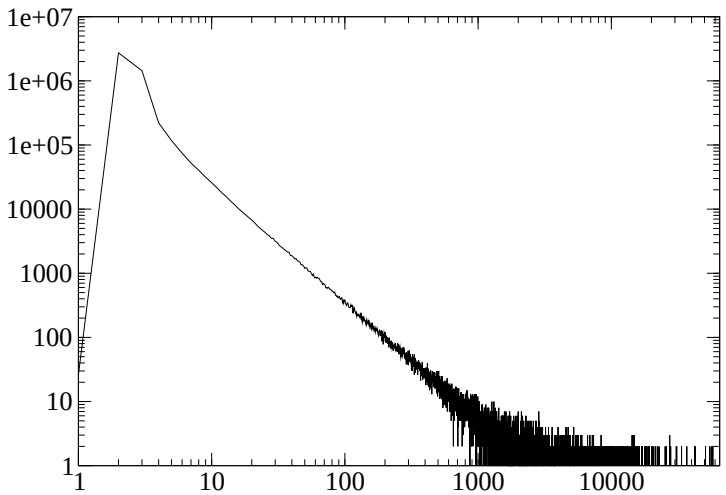

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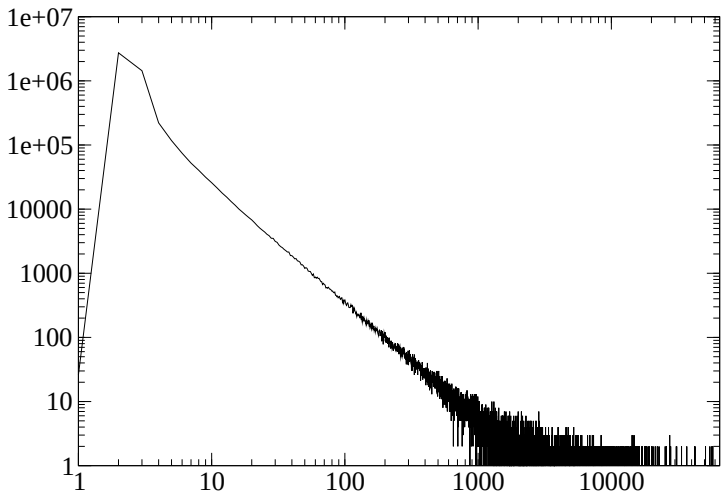

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Overview

- Based upon a variant of the Union-Find algorithm [Tarjan, 1975]
- Three-step strategy
  1. Ordering of the sites of the image f following a relationship $\mathcal{R}$ such as

$$p \mathcal{R} q \Leftrightarrow \begin{cases} f(p) > f(q), \text{ or} \\ f(p) = f(q) \text{ and } p \text{ is before } q \text{ in the classical video scan order} \end{cases}$$

  2. Actual computation of the component tree
  3. Canonization (compression)

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Component tree and *max-tree*

3	3	1	4	2
4	1	2	3	1

f

C	D	H	A	F
B	I	G	E	J

R

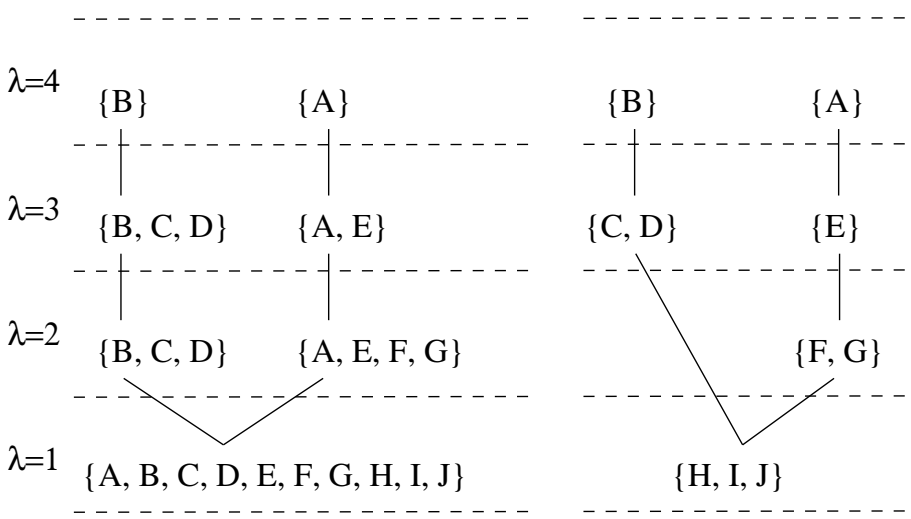

Image Level Sets

3	3	1	4	2
4	1	2	3	1

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3	3	1	4	2
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f

Level 4 ($\lambda = 4$)

C	D	H	A	F
B	I	G	E	J

R

$\lambda = 4$

Component tree

Max-tree

Image Level Sets

3	3	1	4	2
4	1	2	3	1

f

			A	

Level 4 ($\lambda = 4$)

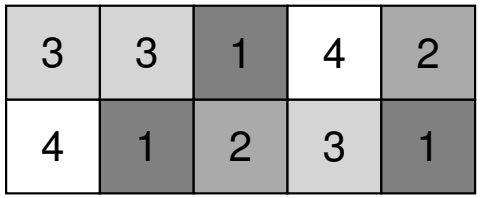
C	D	H	A	F
B	I	G	E	J

R

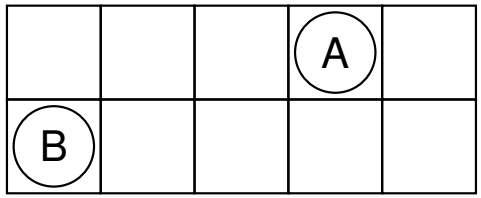
$\lambda = 4$ $\{A\}$ $\{A\}$

Component tree Max-tree

Image Level Sets



f



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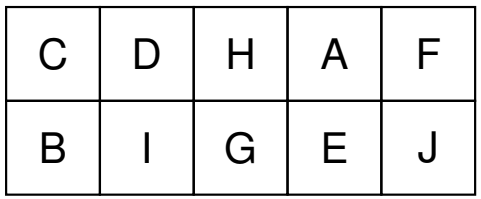

R

$\lambda = 4$ $\{B\}$ $\{A\}$ $\{B\}$ $\{A\}$

Component tree Max-tree

Image Level Sets

3	3	1	4	2
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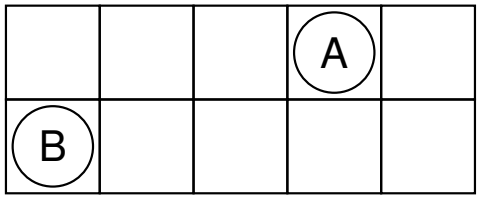
f



Level 4 ($\lambda = 4$)

Level 3 ($\lambda = 3$)

C	D	H	A	F
B	I	G	E	J

R

$\lambda = 4$ $\{B\}$ $\{A\}$ $\{B\}$ $\{A\}$

$\lambda = 3$ $\{B\}$ $\{A\}$

Component tree

Max-tree

Image Level Sets

3	3	1	4	2
4	1	2	3	1

f

			A	
B				

Level 4 ($\lambda = 4$)

C			A	
B				

Level 3 ($\lambda = 3$)

C	D	H	A	F
B	I	G	E	J

R

$\lambda = 4$ $\{B\}$ $\{A\}$ $\{B\}$ $\{A\}$

$\lambda = 3$ $\{B, C\}$ $\{A\}$ $\{C\}$

Component tree Max-tree

Image Level Sets

3	3	1	4	2
4	1	2	3	1

f

			A	
B				

Level 4 ($\lambda = 4$)

C	D		A	
B				

Level 3 ($\lambda = 3$)

C	D	H	A	F
B	I	G	E	J

R

$\lambda = 4$ $\{B\}$ $\{A\}$ $\{B\}$ $\{A\}$

$\lambda = 3$ $\{B, C, D\}$ $\{A\}$ $\{C, D\}$

Component tree Max-tree

Image Level Sets

3	3	1	4	2
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f

			A	
B				

Level 4 ($\lambda = 4$)

C	D		A	
B			E	

Level 3 ($\lambda = 3$)

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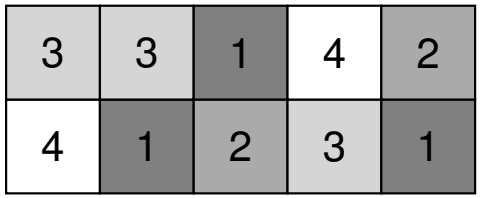

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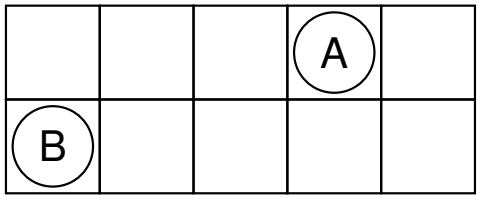

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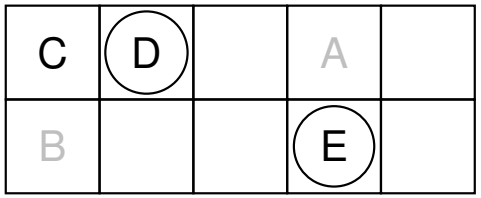

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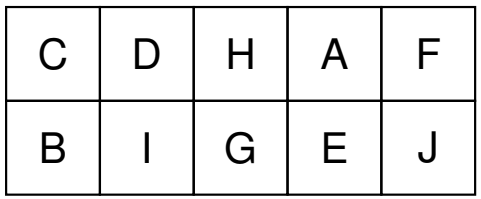
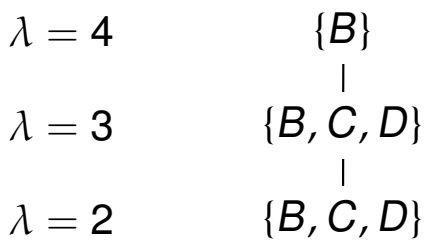
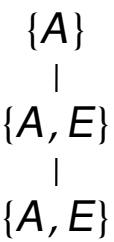


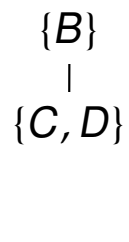
R

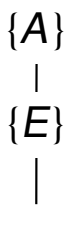
Level 2 ($\lambda = 2$)





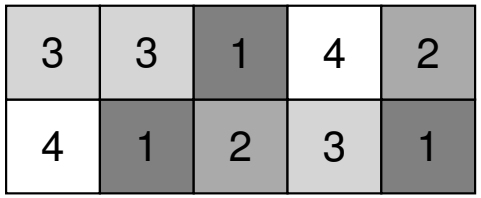
Component tree



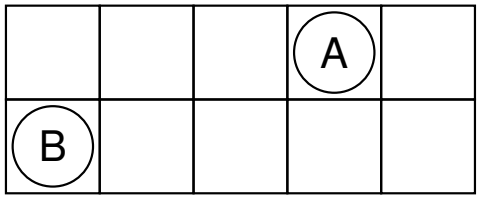


Max-tree

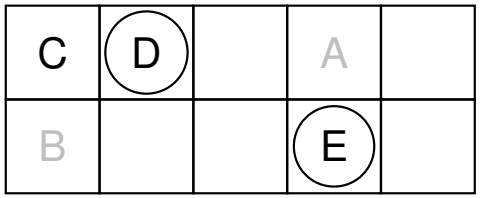
Image Level Sets



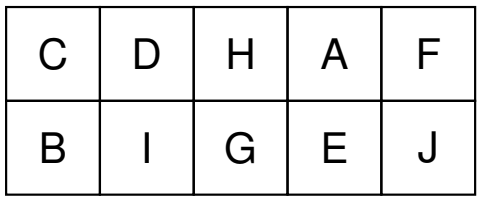
f



Level 4 ($\lambda = 4$)

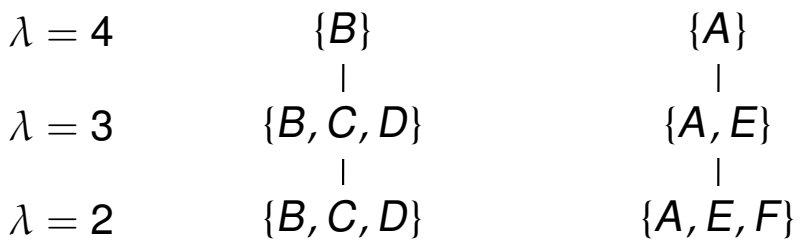


Level 3 ($\lambda = 3$)

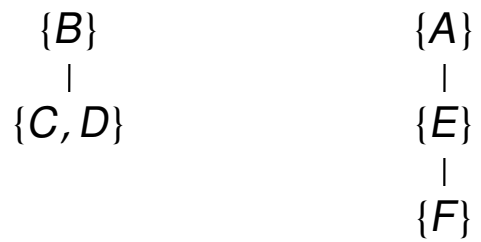


R

Level 2 ($\lambda = 2$)

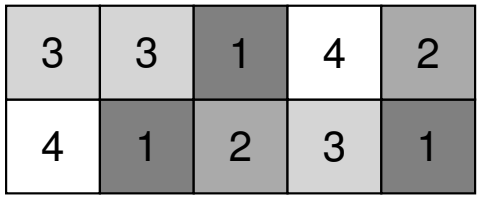


Component tree



Max-tree

Image Level Sets



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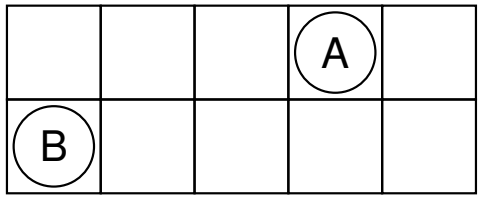
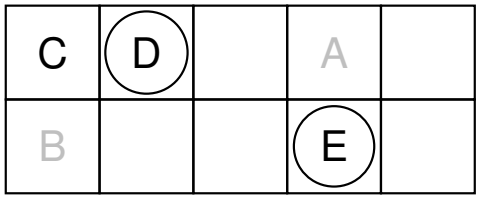
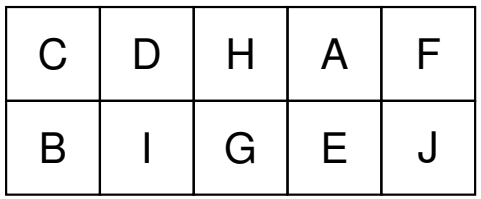


Level 4 ($\lambda = 4$)



Level 3 ($\lambda = 3$)



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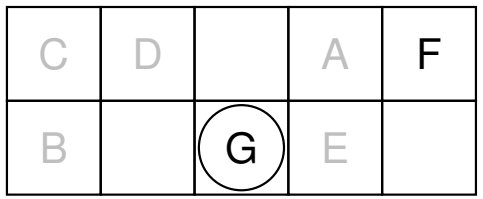
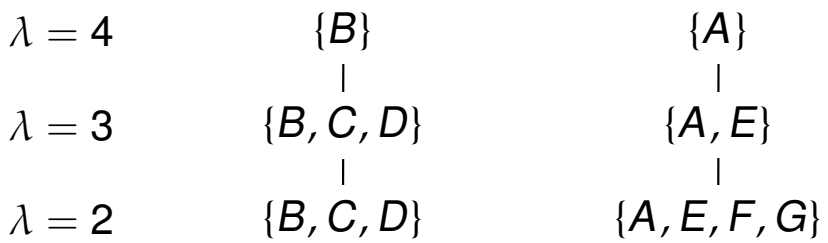


Level 2 ($\lambda = 2$)



Component tree

Max-tree

Image Level Sets

3	3	1	4	2
4	1	2	3	1

f

			A	
B				

Level 4 ($\lambda = 4$)

C	D		A	
B			E	

Level 3 ($\lambda = 3$)

C	D	H	A	F
B	I	G	E	J

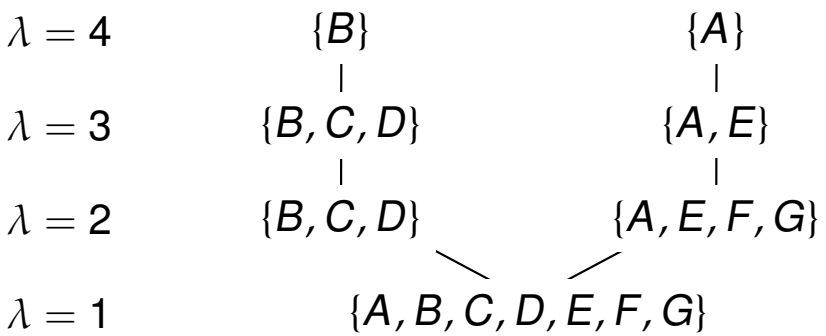
R

C	D		A	F
B		G	E	

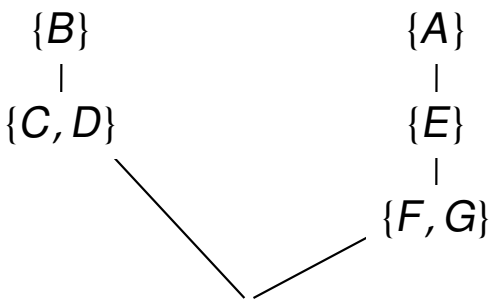
Level 2 ($\lambda = 2$)

C	D		A	F
B		G	E	

Level 1 ($\lambda = 1$)

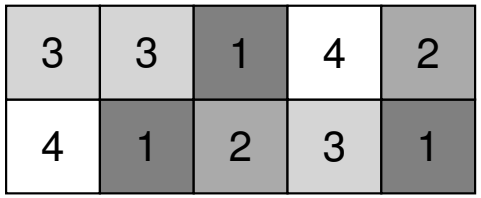


Component tree

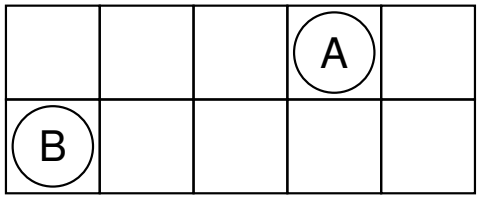


Max-tree

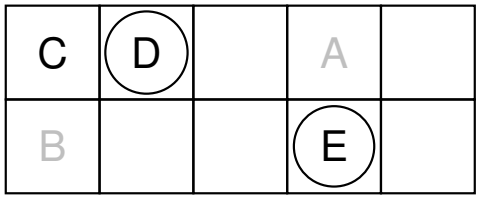
Image Level Sets



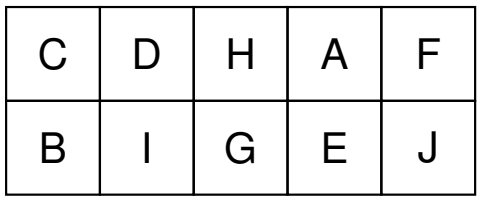
f



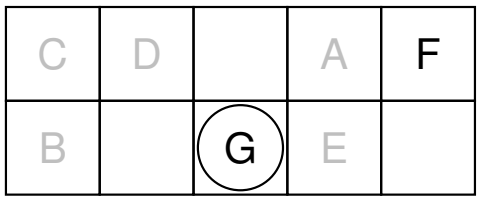
Level 4 ($\lambda = 4$)



Level 3 ($\lambda = 3$)

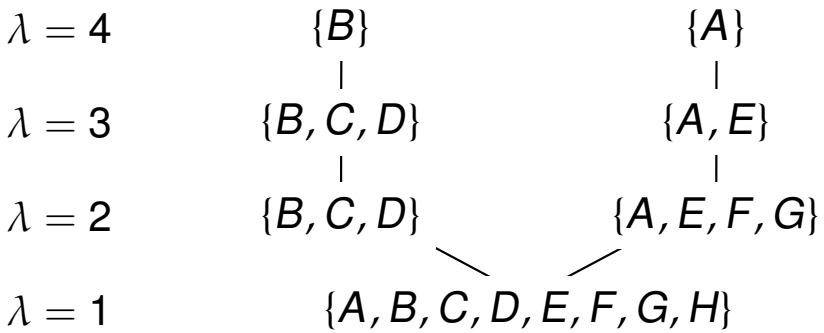


R

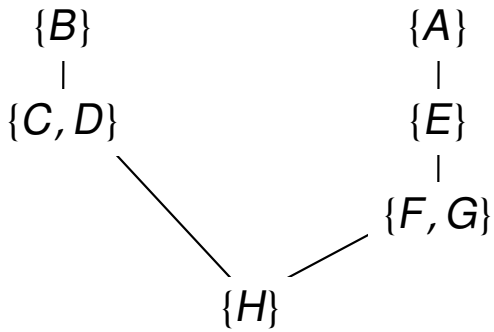


Level 2 ($\lambda = 2$)

Level 1 ($\lambda = 1$)

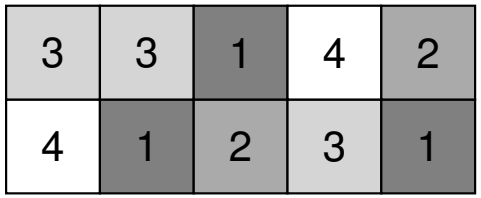


Component tree

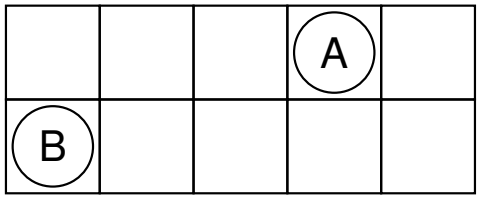


Max-tree

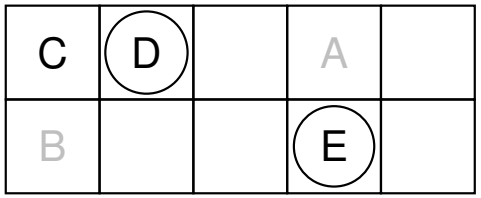
Image Level Sets



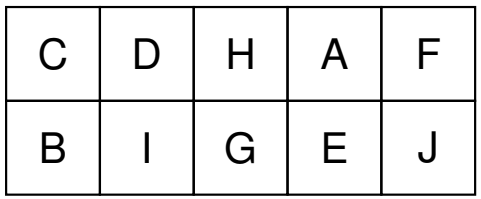
f



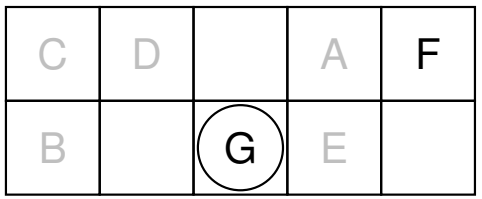
Level 4 ($\lambda = 4$)



Level 3 ($\lambda = 3$)

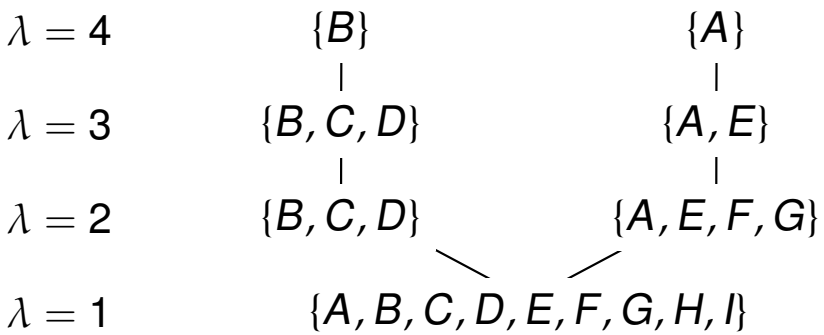


R

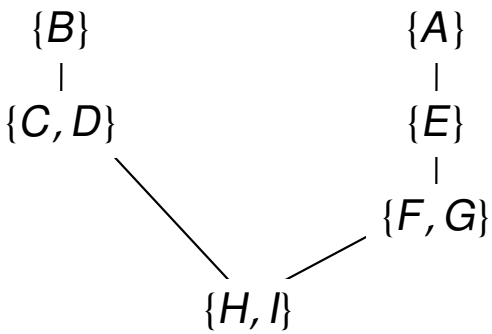


Level 2 ($\lambda = 2$)

Level 1 ($\lambda = 1$)

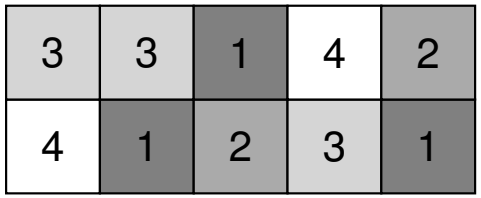


Component tree

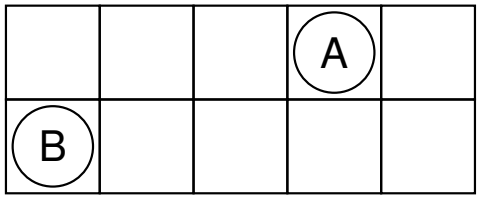


Max-tree

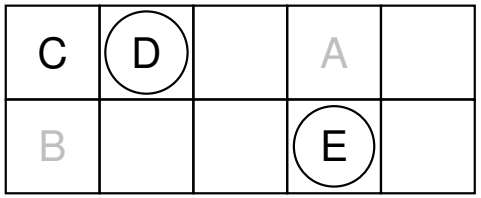
Image Level Sets



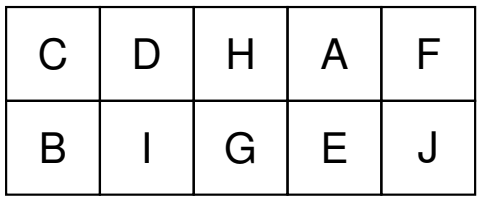
f



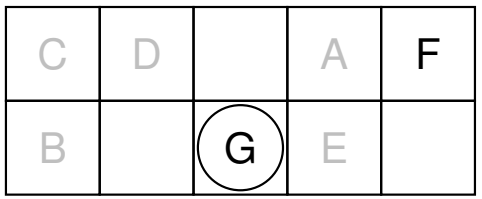
Level 4 ($\lambda = 4$)



Level 3 ($\lambda = 3$)

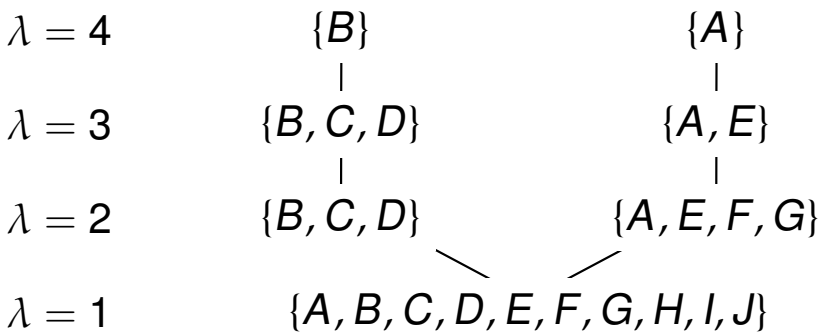


R

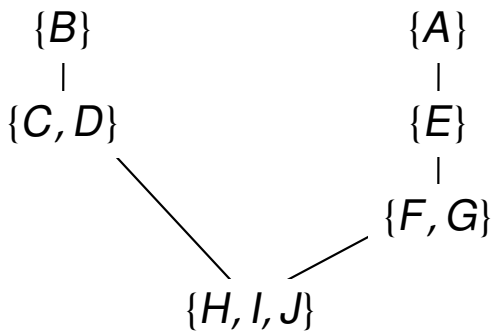


Level 2 ($\lambda = 2$)

Level 1 ($\lambda = 1$)



Component tree



Max-tree

Density of the max-tree, canonization

3	3	1	4	2
4	1	2	3	1

f

C	D	H	A	F
B	I	G	E	J

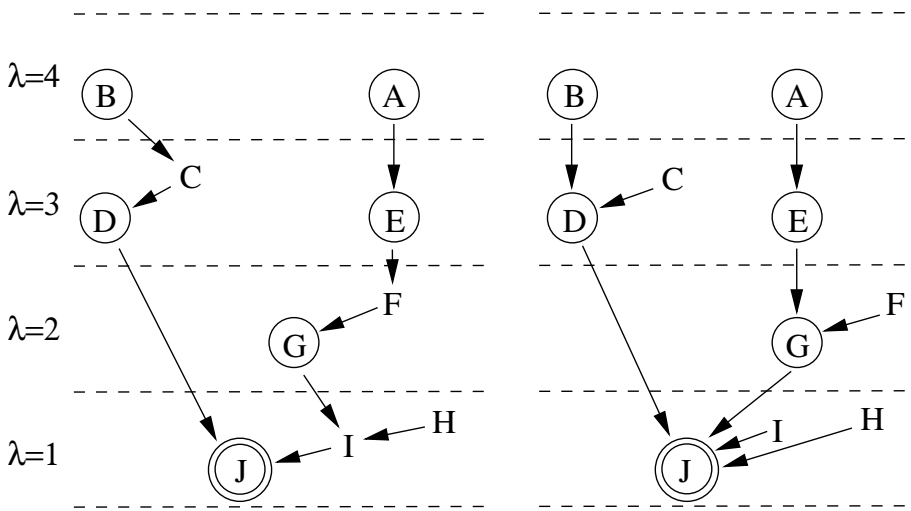
R

D	J	I	E	G
C	J	I	F	J

parent

D	J	J	E	G
D	J	J	G	J

CANONIZE-TREE(*parent*)



Computation

COMPUTE-TREE

FIND-ROOT(x)

```
1  if zpar(x) = x then return x
2    else { zpar(x) ← FIND-ROOT(zpar(x)) ; return zpar(x) }
```

COMPUTE-TREE(f)

```
1  for each p, zpar(p) ← undef
2  R ← REVERSE-SORT(f)   // maps R into an array
3  for each p ∈ R in direct order
4    parent(p) ← p ; zpar(p) ← p
5    for each n ∈ N(p) such as zpar(n) ≠ undef
6      r ← FIND-ROOT(n)
7      if r ≠ p then { parent(r) ← p ; zpar(r) ← p }
8  DEALLOCATE(zpar)
9  return pair(R, parent)   // a "correct" function
```

Canonization

CANONIZE-TREE

CANONIZE-TREE($parent, f$)

1 **for** each $p \in R$ in reverse order
2 $\quad q \leftarrow parent(p)$
3 $\quad$ **if** $f(parent(q)) = f(q)$ **then** $parent(p) \leftarrow parent(q)$
4 **return** *parent* // a "canonized" function

Example

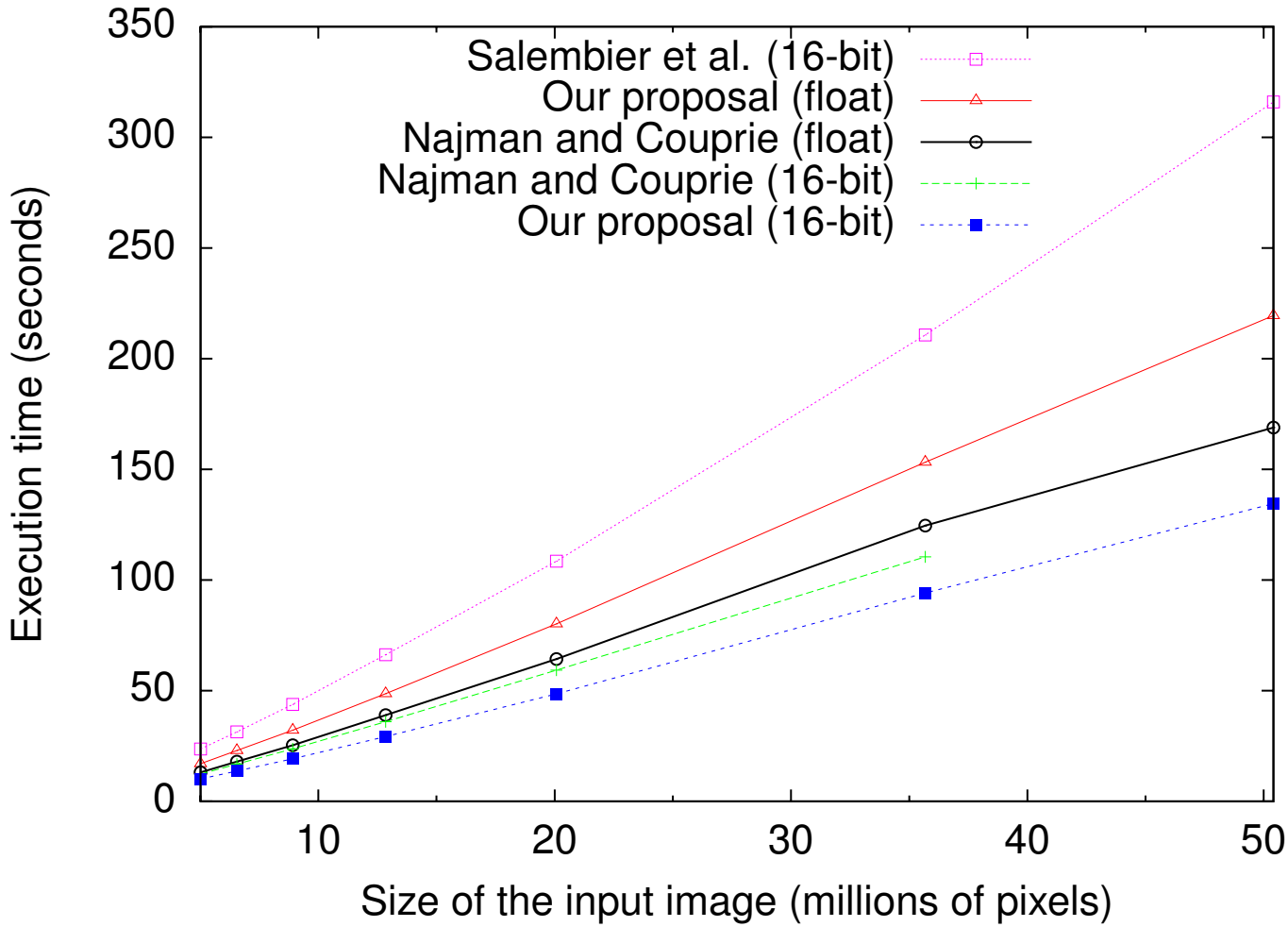
computation of the area of the components

COMPUTE-AREA($f, R, parent$)

1 **for** each $p \in R$, $area(p) \leftarrow 1$ // initialization
2 **for** each $p \in R$ in direct order
3 $area(parent(p)) \leftarrow area(parent(p)) + area(p)$ // update

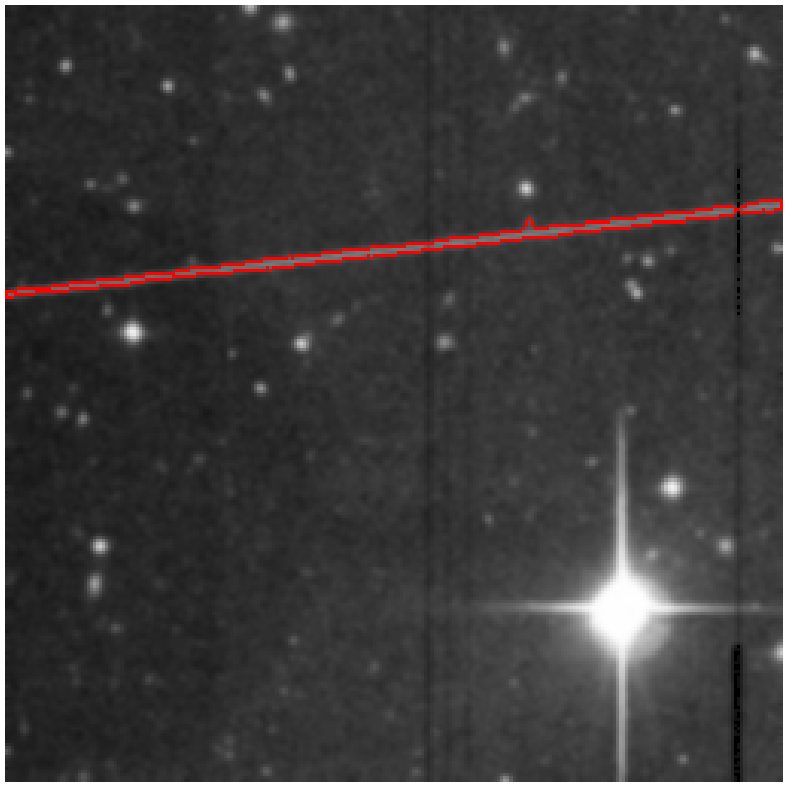
- Simple process
- Computation conducted in an iterative way
- Linear complexity

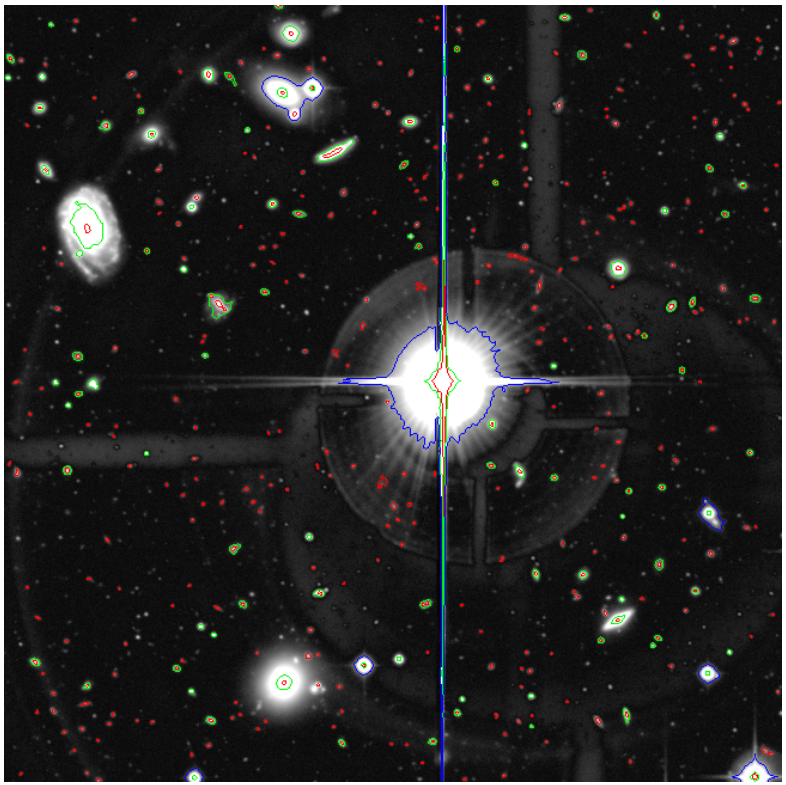
Comparison of Execution Times



(3 Ghz Bi-Xeon Processor with 2×1 MB of cache memory and 4 GB of RAM, running GNU/Linux.)

Applications





Conclusions

- Algorithm effective for images with high quantization and with no quantization
- About as efficient as the fastest known algorithm, and needs twice less memory
- Serves as a basis to build efficient connected filters for astronomical images
- On-going work on the selection of attributes and node labeling strategies

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Effective Component Tree Computation with Application to Pattern Recognition in Astronomical Imaging

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