A Color Tree of Shapes with Illustrations on Filtering, Simplification, and Segmentation

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Context

About morphological tree representations:

- versatile and efficient → many apps;
- (very) easy to compute/manipulate [5, 2, 4],
- implicit multiscale analysis,
- some of them feature (very) desirable properties:
    - contrast change invariance,
    - self-duality. . .

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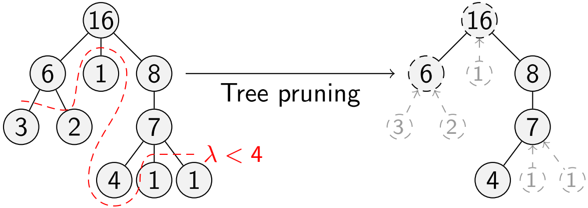
  - contrast change invariance,
  - self-duality...

Not convinced? Let's see...

Grain filters [3](1/2)

Method overview

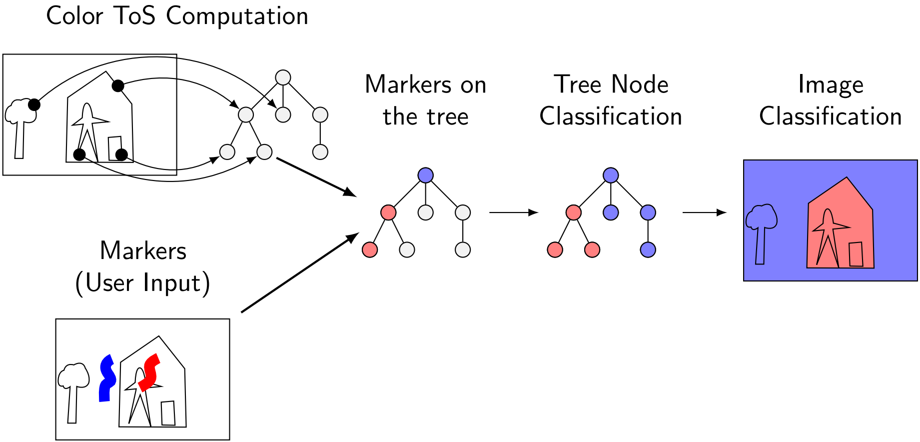


1. Compute the size attribute over the tree.
2. Threshold and collapse.

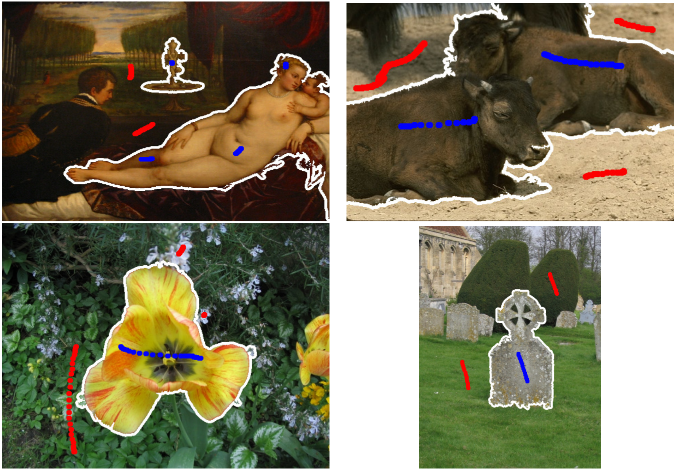
Grain filters (2/2): Document layout extraction

Interactive segmentation (1/2)

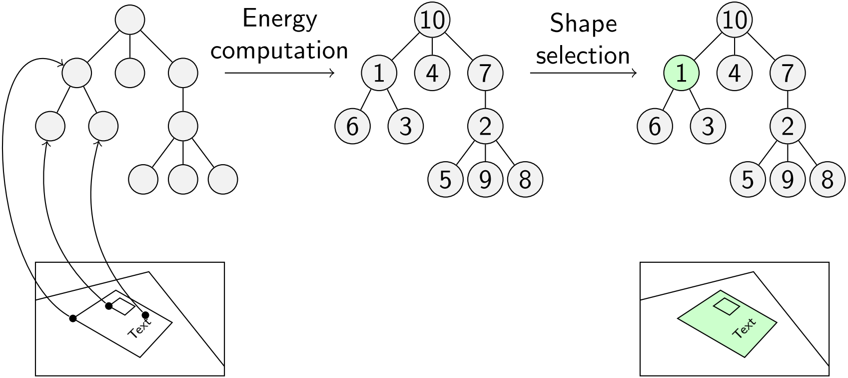
Method overview



Interactive segmentation (2/2)

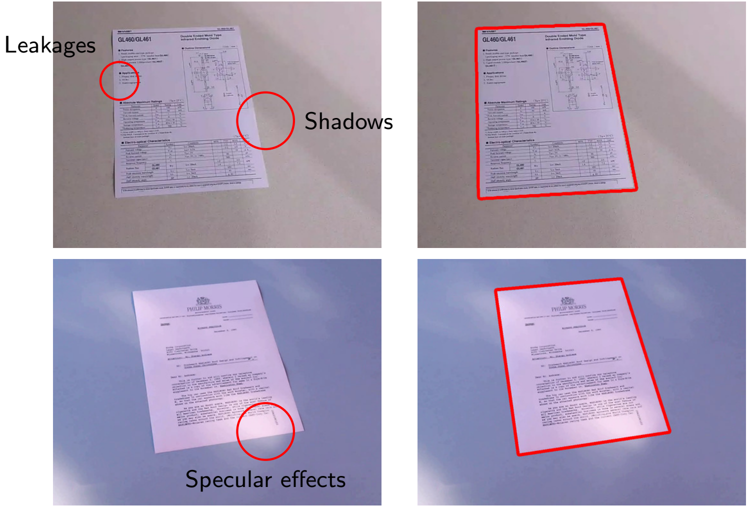


Document detection in videos (ICDAR SmartDoc'15)



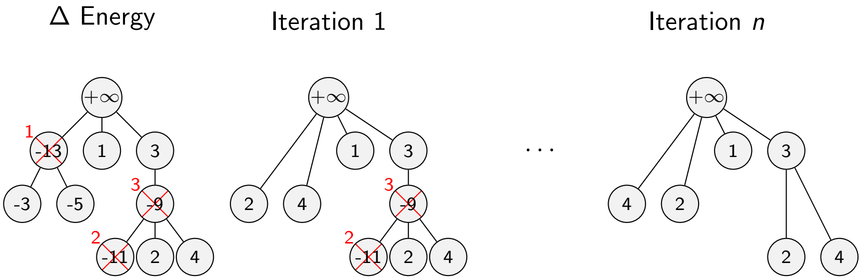
1. Valuate an energy adpated to the object to detect.
2. Retrieve the shape with the lowest energy.

Document detection in videos (ICDAR SmartDoc'15)



Natural Image Simplification[7]

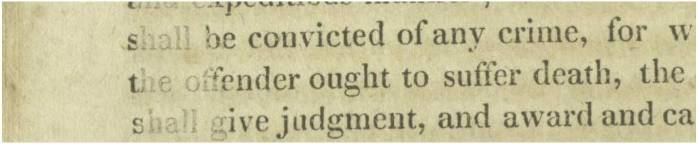
Principle. Mumford-Shah energy optimization constrained to the tree.



Natural Image Simplification[7]

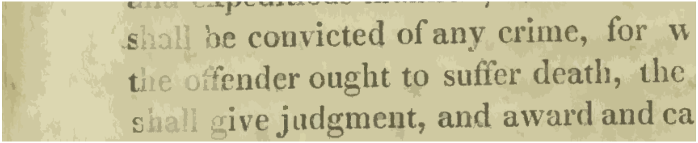
Image simplification: the simplified images have less than 100 nodes (original: ∼80k nodes)

Document Image Simplification[7]



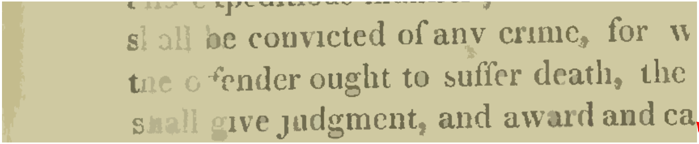
(a) Original (113k nodes).

nodes ÷100



(b) Strong simplification (1000 nodes).

nodes ÷1000



(c) Drastic simplification (285 nodes).

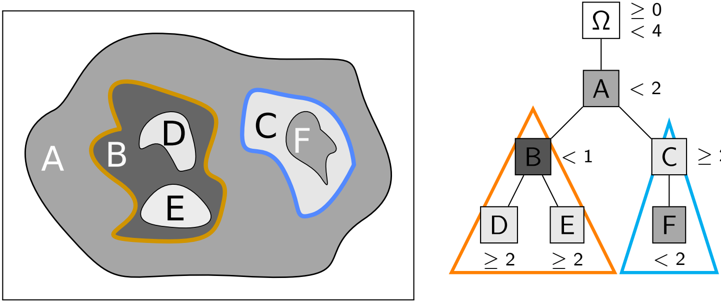
These applications use a single image representation:

The Color Tree of Shapes

Outline

What is the Tree of Shapes? (1/2)

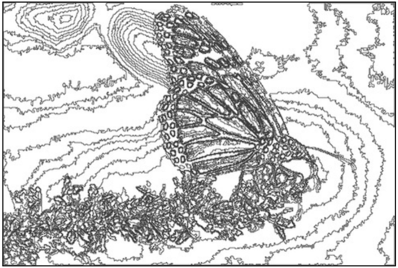
As the fusion of the min- and max- trees



The Tree of shapes (ToS) of u, formed by cavity-filled connected components of the min- and max- trees (self-dual representation)

What is the Tree of Shapes? (2/2)

As the inclusion tree of the level lines



u and its level lines (every 5 levels)

- The ToS also encodes the inclusion of the image level lines,
- They are the contours of shapes.

Properties of the ToS

We have:

- Invariance by **contrast change**:
  $T(g(u)) = T(u)$ for any increasing function g

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- A way to get self-dual connected operators:

  $\rightarrow$ they do not shift object boundaries

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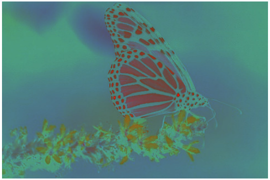
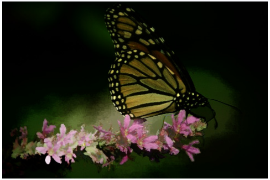
→ Yet, the ToS requires a **total** order on colors (does one make sense?)

Outline

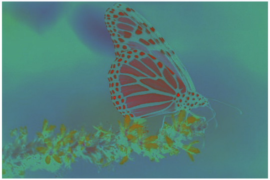


Independant Marginal contrast change & inversion.



Local contrast change

What do these images have in common?



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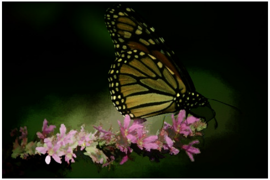

Local contrast change

What do these images have in common?

They share an exact same representation:
the Color Tree of Shapes

General Overview

What do we want?

- Given $\mathcal{M} = \{\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_n\}$, where $(\mathcal{S}_i, \subseteq)$ is a tree, we note $\mathcal{S} = \bigcup \mathcal{S}_i$ the primary shape set.
- We aim at defining a new set of shapes **S** such that:

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(Q) A "well-formed" tree: $\#nodes \simeq \#pixels$ and not degenerated.

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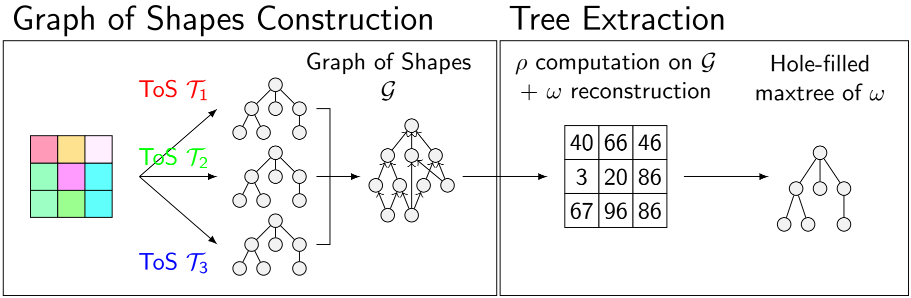
Scheme of the method



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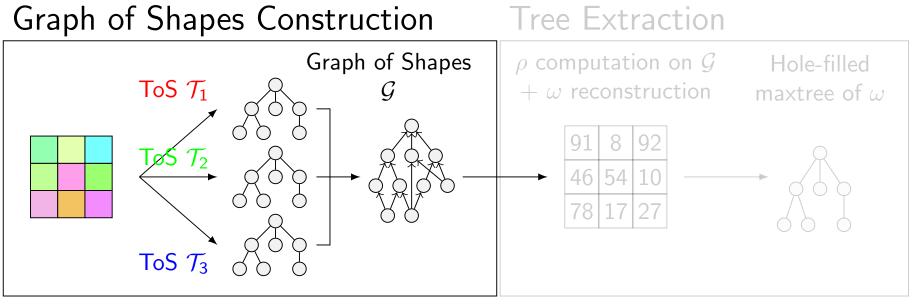

1. Get the primary shape set $\mathcal{S}$ from the marginal ToS.
2. Compute the Graph of Shapes $\mathcal{G} = (\mathcal{S}, \subseteq)$

General Overview

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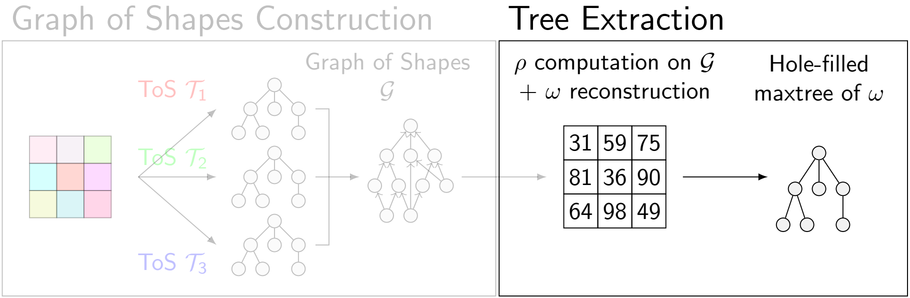
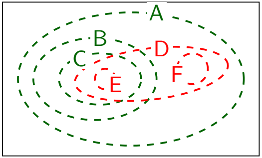
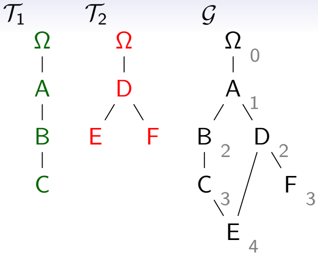


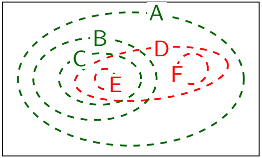
1. Compute the depth attribute ρ over $\mathcal{G}$,
2. Reconstruct the attribute map ω (in the image space),
3. Compute the cavity-filled maxtree of ω



(a) Input u



(b) Graph of Shapes + ρ



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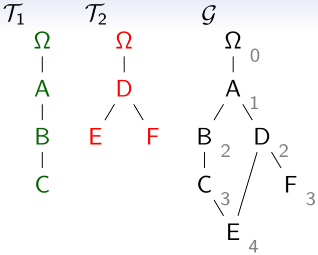

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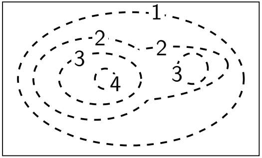
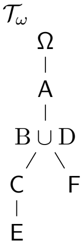


(c) ω map



(d) Cavity-filled Maxtree $\mathcal{T}_\omega$

Justification

There is no magic!

In gray level:
The ToS of u is related to the maxtree of the depth map (cf. paper).

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The ToS of u is related to the maxtree of the depth map (cf. paper).

Furthermore...

It fulfills the properties. (Proofs in an upcoming paper)

You can get effective results. (you've already seen that!)

Outline

Comparing on image simplification with classical approaches

Original Ours Gray-level Marginal Total preorder [6]

Original | Ours | Gray-level | Marginal | Total preorder [6]

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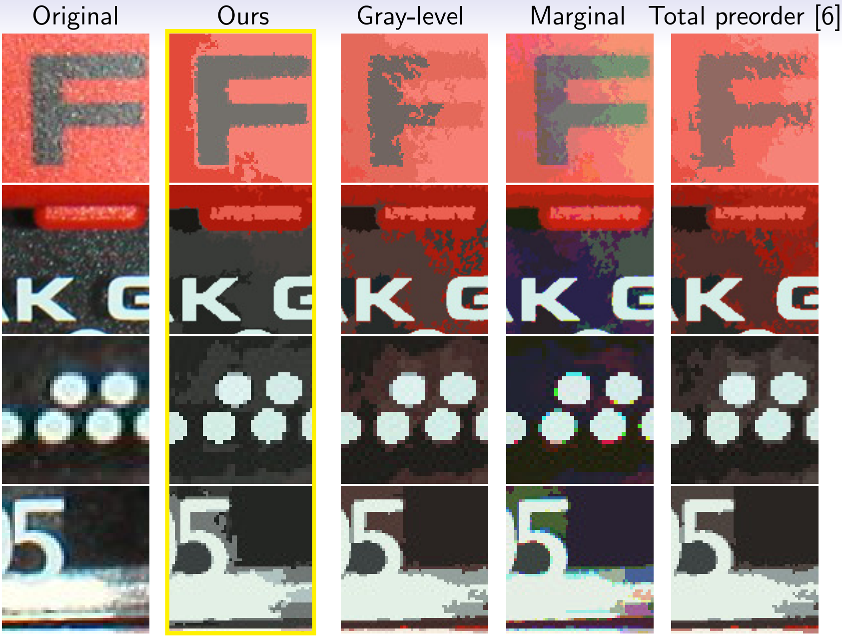
Original	Ours	Gray-level	Marginal	Total preorder [6]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Conclusion (1/2)

Key Idea. A method where the ordering is not based on colors (values), but on inclusion of shapes (components).

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What has been done?

1. A proposal for a Color Tree of Shapes
2. An a-posteriori validation: get convincing results for simplification, segmentation...

Conclusion (2/2)

Perspectives: Use it!

Reproductible research:

`http://publications.lrde.epita.fr/carlinet.15.itip`
→ Source code, binaries, and extra results.

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By the way... It's quite fast (2s on a 512×512 pixels image).

Plant a tree!

Questions?

[1] E. Aptoula and S. Lefèvre.
A comparative study on multivariate mathematical morphology.
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IEEE Transactions on Image Processing, 23(9):3885–3895, September 2014.

[3] V. Caselles and P. Monasse.
Grain filters.
Journal of Mathematic Imaging and Vision, 17(3):249–270, November 2002.

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A first parallel algorithm to compute the morphological tree of shapes of *n*D images.
In *Proc. of IEEE Intl. Conf. on Image Processing (ICIP)*, pages 2933–2937, Paris, France, 2014.

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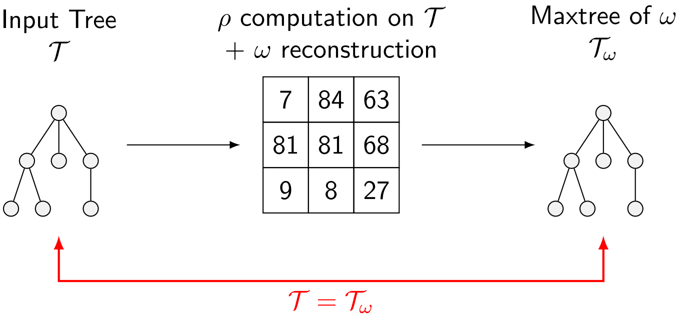
[6] O. Lézoray and A. Elmoataz.
Nonlocal and multivariate mathematical morphology.
In *Proc. of IEEE Intl. Conf. on Image Processing (ICIP)*, pages 129–132, Orlando, USA, 2012.

[7] Y. Xu, T. Géraud, and L. Najman.
Salient level lines selection using the Mumford-Shah functional.
In *Proc. of IEEE Intl. Conf. on Image Processing (ICIP)*, pages 1227–1231, Merlbourne, Australia, 2013.

Rationale (1/2)

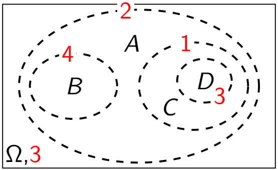
Idea 1. $\mathcal{T}$+ dec. attribute ρ + restitution ω_ρ + Maxtree $\mathcal{T}_{\omega_\rho} = \mathcal{T}$



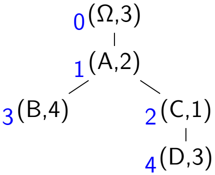
Rationale (2/2)

Idea 1. $\mathcal{T}$+ dec. attribute ρ + restitution ω_ρ + Maxtree $\mathcal{T}_{\omega_\rho} = \mathcal{T}$

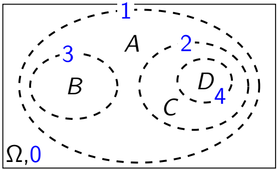
Idea 2. u level lines = ω_{TV} level lines (TV from the border).



(a) u and its level lines.



(b) The ToS of u and the valuation of ρ_{TV} (blue).



(c) The level lines of ω_{TV}.

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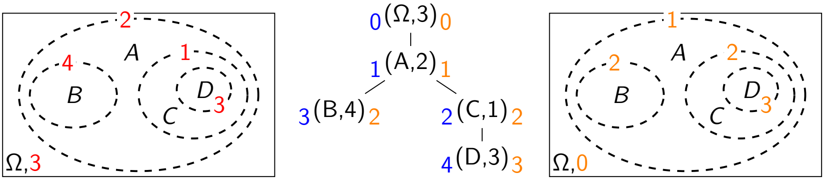
$= \omega_{CV}$ level lines (Counted variations).



(a) u and its level lines. (b) The ToS of u and ρ_{CV} (orange). (c) The level lines of ω_{CV}.

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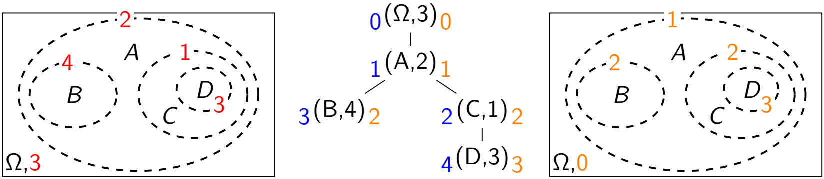
$\rightarrow$ ToS of u = Maxtree of ω_{CV}



(a) u and its level lines. (b) The ToS of u and ρ_{CV} (orange). (c) The level lines of ω_{CV}.

Conclusion. Use the depth attribute on $\mathcal{G}$ and reconstruct.
$\omega_{CV}(x)$ stands for:

> *The number of marginal level lines (that are nested) along the path from the border to the deepest shape that contains x.*

Differences with “Shape” component-graphs

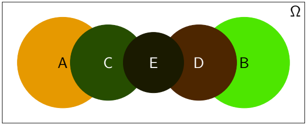
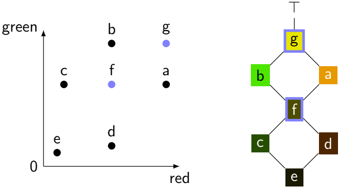


Image u



Lattice of the values

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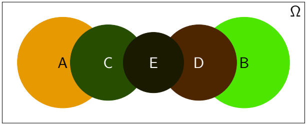

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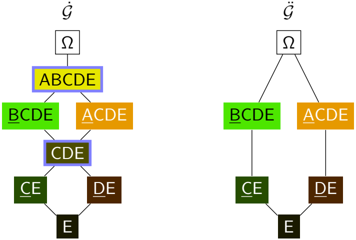
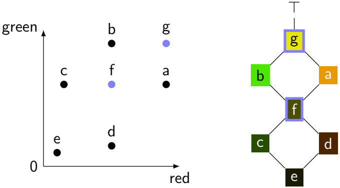


“Shape” Component graphs



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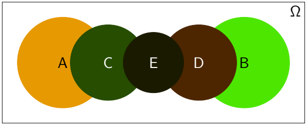

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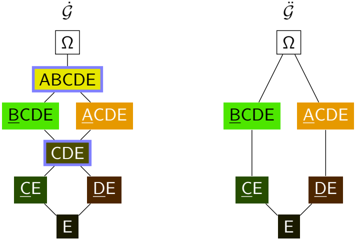

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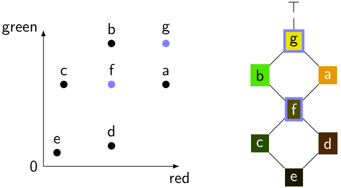

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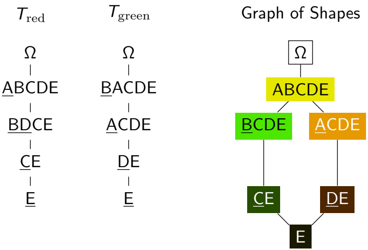
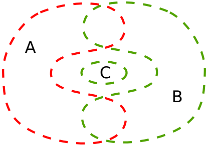
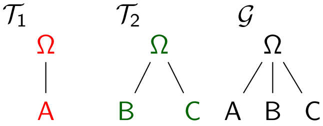


The graph of shapes

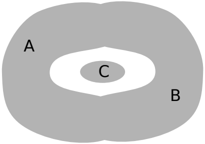
On the need of the saturation



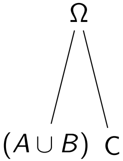
(a) Original.



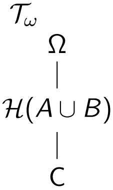
(b) Marginal ToS and GoS.



(c) ω map.

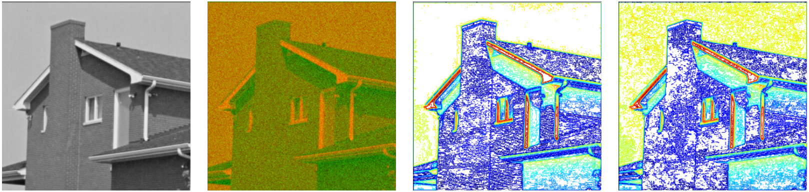


(d) Maxtree of ω (w/o cavity filling).



(e) Final Color ToS (with cavity filling).

Effect of noise



(a) House

(b) House (red channel) + Gaussian Noise ($\sigma = 20$, green channel)

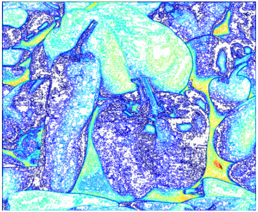
(c) Level lines of the tos of (a). Level lines: 24k, avg. depth: 37, max. depth: 124.

(d) Level lines of the ctos of (b). Level lines: 48k, avg. depth: 48, max. depth: 127.

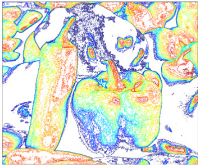
Effect of the dynamic

(a) Peppers (only red/green)

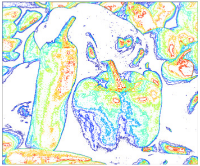
(b) Peppers (only red/green) with green sub-quant. to 10 levels



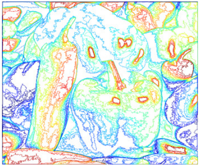
(c) Level lines of the red channel of (a) and (b)



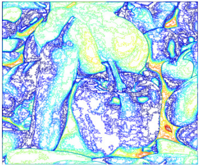
(d) Level lines of the green channel (a)



(e) Level lines of the green channel (b)



(f) Level lines of the ctos of (a)



(g) Level lines of the ctos of (b)