A Type System for Weighted Automata and Rational Expressions

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http://vaucanson.lrde.epita.fr

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Vaucanson

General-purpose platform for weighted automata and transducers.

- Genericity
  - Acceptors and transducers
  - Rational expressions
  - Weights: Boolean, Usual, Tropical...
  - Labels: letter, word, ε...
  - Letters: `char`s, `int`s...
- Performance
  - C++ templated library
  - No dynamic polymorphism (`virtual`)
  - Efficient algorithms and data structures
- Flexibility
  - A dynamic API on top of the templated library
  - A dynamic typing system for automata, rational expressions, etc.
  - An interactive GUI on top of IPython

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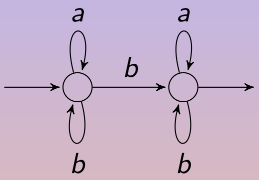
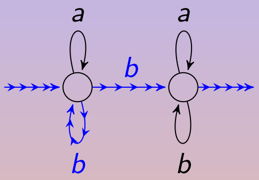
Weighted Automata

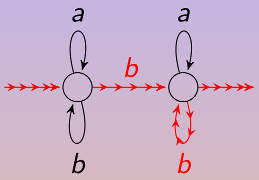


Weighted Automata



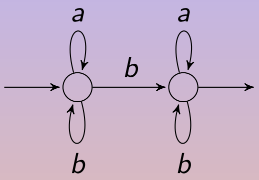
$\top \wedge \top$

Weighted Automata



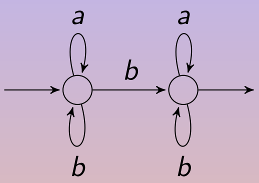
$\top \wedge \top$ $\top \wedge \top$

Weighted Automata

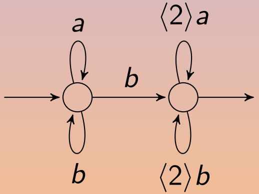


$\top \wedge \top \vee \top \wedge \top = \top$

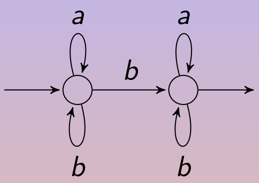
$bb \to \top$



$\top \wedge \top \vee \top \wedge \top \;=\; \top$ $\qquad bb \to \top$

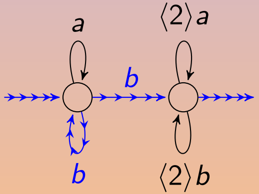


Weighted Automata



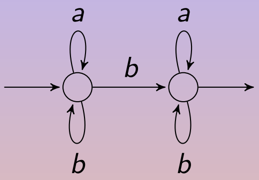
$\top \wedge \top \vee \top \wedge \top \; = \; \top$

$bb \to \top$



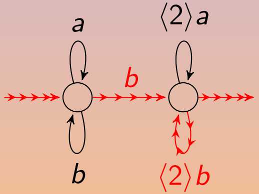
1×1

Weighted Automata



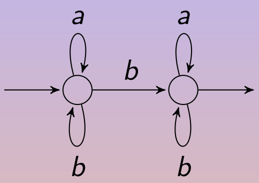
$\top \wedge \top \vee \top \wedge \top \; = \; \top$

$bb \rightarrow \top$

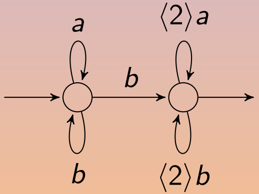


$1 \times 1 \; + \; 2 \times 1$

Weighted Automata



$\top \wedge \top \vee \top \wedge \top \;=\; \top \qquad bb \to \top$

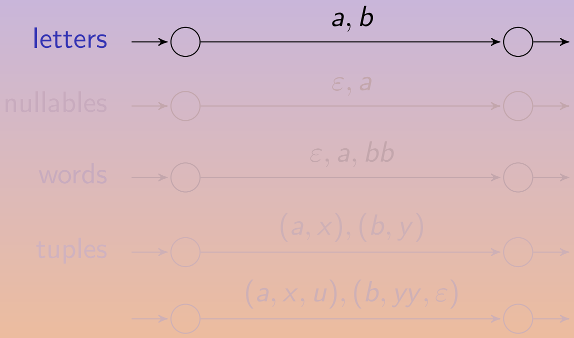


$1 \times 1 + 2 \times 1 \;=\; 3 \qquad bb \to 3$

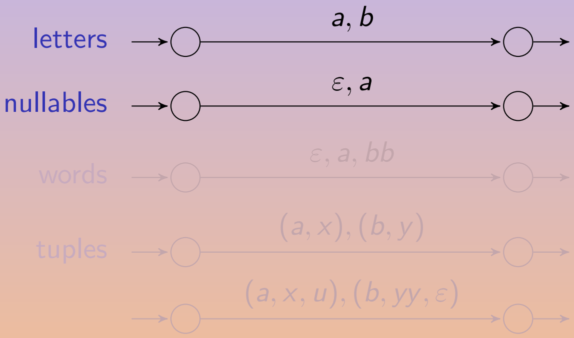
Many Different Kinds of Weights

- $\langle \mathbb{B}, \vee, \wedge \rangle$
- $\langle \mathbb{Z}, +, \times \rangle$
- $\langle \mathbb{Q}, +, \times \rangle$
- $\langle \mathbb{R}, +, \times \rangle$
- $\langle \mathbb{Z}, \mathsf{min}, + \rangle$
- Rational expressions
- Tuples
- ...

Many Different Kinds of Labels



Many Different Kinds of Labels



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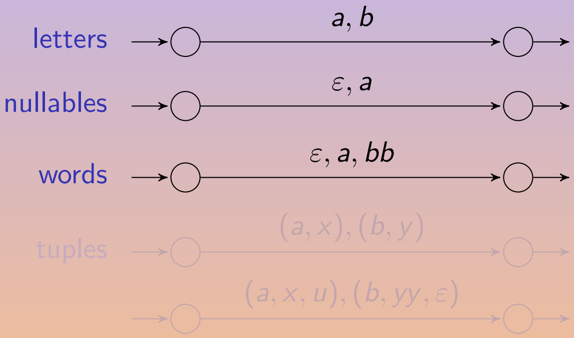

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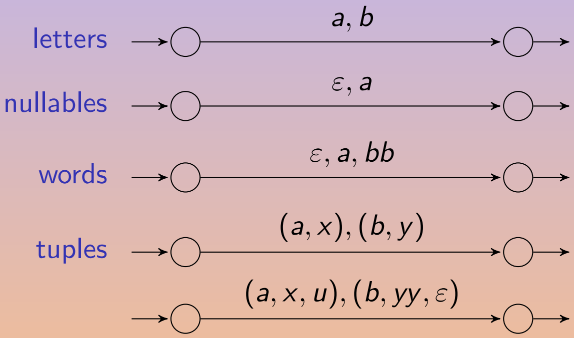
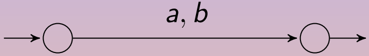


Contexts

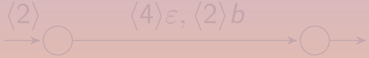
$$\textit{context} ::= \textit{labelset} \rightarrow \textit{weightset}$$

$\{a, b\} \rightarrow \mathbb{B}$



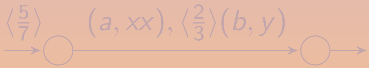
$(\varepsilon + a + b)^*$

$\{a, b, c\}^? \rightarrow \mathbb{Z}$



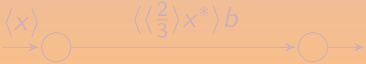
$\langle 2 \rangle (\langle -2 \rangle a + b)^*$

$\{a, b, c\} \times \{x, y, z\}^* \rightarrow \mathbb{Q}$



$\langle \frac{1}{2} \rangle (a, x) + \langle \frac{1}{3} \rangle (b, y)^*$

$\{a, b\} \rightarrow \mathrm{RatE}[\{x, y\} \rightarrow \mathbb{Q}]$



$\left(\langle x \rangle a + \langle \langle \frac{1}{3} \rangle (x + y)^* \rangle b \right)^*$

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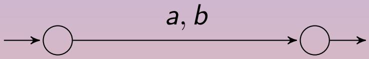

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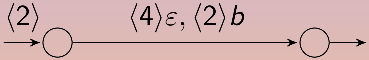

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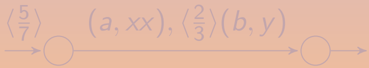

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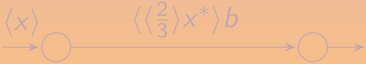

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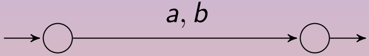

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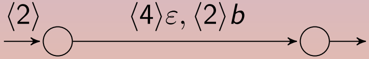

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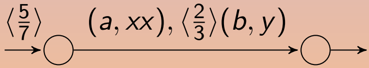

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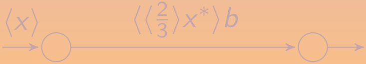

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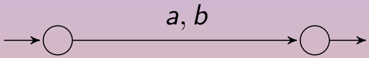

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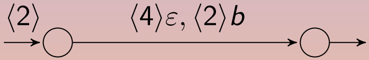

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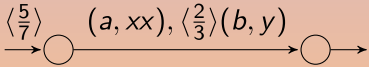
$\{a, b, c\} \times \{x, y, z\}^* \to \mathbb{Q}$



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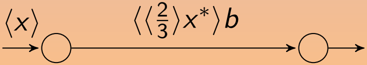

$$\big(\langle x \rangle a + \langle \langle \tfrac{1}{3} \rangle (x + y)^* \rangle b \big)^*$$

A Calculus on Types

Subtyping

$$A <: B$$

- *values of type A are values of type B*
- roughly $A \subseteq B$

Labelsets and Weightsets Subtypes

$$\{\varepsilon\} <: A^? \quad A <: A^? \quad A^? <: A^*$$

$$A <: B \quad A^? <: B^? \quad A^* <: B^*$$

$$\mathbb{B} <: \mathbb{N} <: \mathbb{Z} <: \mathbb{Q} <: \mathbb{R}$$

$$\mathbb{B} <: \mathbb{Z}_{\mathsf{min}}$$

A, B alphabets such that $A \subseteq B$

Labelsets and Weightsets Subtypes

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A, B alphabets such that $A \subseteq B$

$$L <: \mathrm{RatE}[L \to W] \qquad W <: \mathrm{RatE}[L \to W]$$

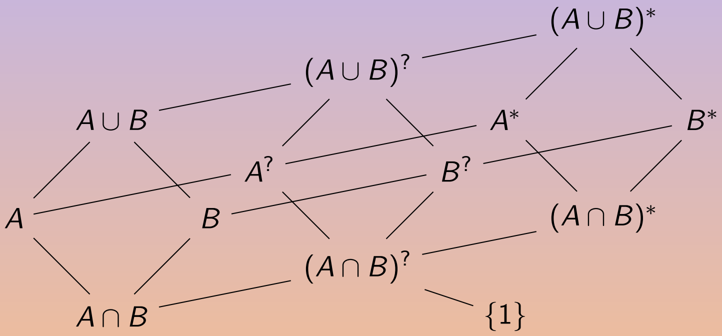
Subtype On Contexts, Expressions and Automata

$$\frac{L_1 <: L_2 \qquad W_1 <: W_2}{(L_1 \to W_1) <: (L_2 \to W_2)}$$

$$\frac{C_1 <: C_2}{\mathrm{RatE}[C_1] <: \mathrm{RatE}[C_2]}$$

$$\frac{C_1 <: C_2}{\mathrm{Aut}[C_1] <: \mathrm{Aut}[C_2]}$$

Labelset Subtypes



Meet and Join

$V_1 \vee V_2$ (join)
The least upper bound between V_1 and V_2.

$V_1 \wedge V_2$ (meet)
The greatest lower bound between V_1 and V_2.

$$\begin{array}{rcccl} \mathbb{Q} & \vee & \mathrm{RatE}[\{x,y,z\} \to \mathbb{B}] & = & \mathrm{RatE}[\{x,y,z\} \to \mathbb{Q}] \\ \{a,b,c\} & \wedge & \{a,b,d\} & = & \{a,b\} \end{array}$$

Union of Automata

$$\frac{\mathcal{A}_1 : L_1 \to W_1 \qquad \mathcal{A}_2 : L_2 \to W_2}{\mathcal{A}_1 \cup \mathcal{A}_2 : L_1 \vee L_2 \to W_1 \vee W_2}$$

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$$\{a, b, c\} \vee \{a, b, d\} = \{a, b, c, d\}$$

$$\mathbb{Q} \vee \mathrm{RatE}[\{x, y, z\} \to \mathbb{B}] = \mathrm{RatE}[\{x, y, z\} \to \mathbb{Q}]$$

$$\frac{\mathcal{A}_1 : \{a, b, c\} \to \mathbb{Q} \qquad \mathcal{A}_2 : \{a, b, d\} \to \mathrm{RatE}[\{x, y, z\} \to \mathbb{B}]}{\mathcal{A}_1 \cup \mathcal{A}_2 : \{a, b, c, d\} \to \mathrm{RatE}[\{x, y, z\} \to \mathbb{Q}]}$$

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Synchronized Product of Automata

$$\frac{\mathcal{A}_1 : A_1 \to W_1 \qquad \mathcal{A}_2 : A_2 \to W_2}{\mathcal{A}_1 \,\&\, \mathcal{A}_2 :}$$

Synchronized Product of Automata

$$\frac{\mathcal{A}_1 : A_1 \to W_1 \qquad \mathcal{A}_2 : A_2 \to W_2}{\mathcal{A}_1 \mathbin{\&} \mathcal{A}_2 : A_1 \wedge A_2 \to W_1 \vee W_2}$$

Synchronized Product of Automata

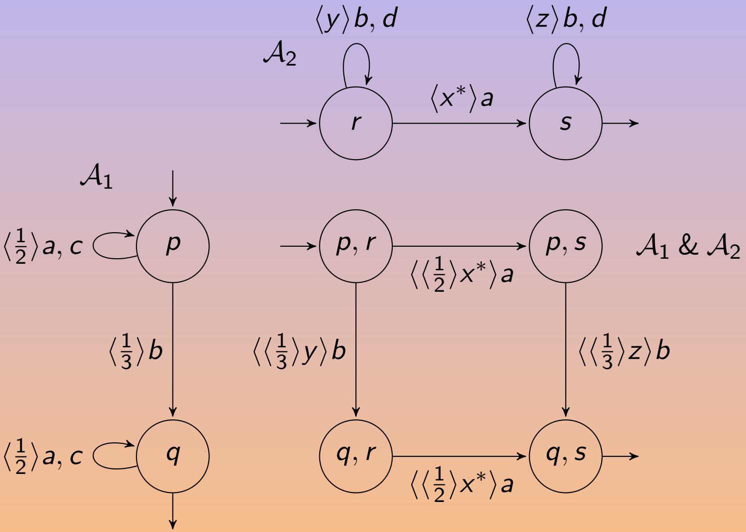
$$\frac{\mathcal{A}_1 : A_1 \to W_1 \qquad \mathcal{A}_2 : A_2 \to W_2}{\mathcal{A}_1 \,\&\, \mathcal{A}_2 : A_1 \wedge A_2 \to W_1 \vee W_2}$$

$$\{a, b, c\} \wedge \{a, b, d\} = \{a, b\}$$

$$\mathbb{Q} \vee \mathsf{RatE}[\{x, y, z\} \to \mathbb{B}] = \mathsf{RatE}[\{x, y, z\} \to \mathbb{Q}]$$

$$\frac{\mathcal{A}_1 : \{a, b, c\} \to \mathbb{Q} \qquad \mathcal{A}_2 : \{a, b, d\} \to \mathsf{RatE}[\{x, y, z\} \to \mathbb{B}]}{\mathcal{A}_1 \,\&\, \mathcal{A}_2 : \{a, b\} \to \mathsf{RatE}[\{x, y, z\} \to \mathbb{Q}]}$$

Synchronized Product in Vaucanson



Use in Vaucanson

Use of Types

- (Low-level) objects can synthetize a type identifier
- Dynamic objects wrap low-level objects
- The dynamic library dispatches on this type identifier
- The dispatching routine computes resulting types
- The type identifier can be used to generate code

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Operations on Rational Expressions

$$\frac{\mathsf{E}_1 : L_1 \to W_1 \qquad \mathsf{E}_2 : L_2 \to W_2}{\mathsf{E}_1 \odot \mathsf{E}_2 : L_1 \vee L_2 \to W_1 \vee W_2} \odot \in \{\cdot, +, \}$$

$$\frac{w_1 : W_1 \qquad \mathsf{E}_2 : L_2 \to W_2}{w_1 \cdot \mathsf{E}_2 : L_2 \to W_1 \vee W_2} \qquad \frac{\mathsf{E}_1 : L_1 \to W_1 \qquad w_2 : W_2}{\mathsf{E}_1 \cdot w_2 : L_1 \to W_1 \vee W_2}$$

Operations on Automata

$$\frac{\mathcal{A}_1 : L_1 \to W_1 \qquad \mathcal{A}_2 : L_2 \to W_2}{\mathcal{A}_1 \odot \mathcal{A}_2 : L_1 \vee L_2 \to W_1 \vee W_2} \odot \in \{\cdot, +\}$$

$$\frac{\mathcal{A}_1 : A_1 \to W_1 \qquad \mathcal{A}_2 : A_2 \to W_2}{\mathcal{A}_1 \odot \mathcal{A}_2 : A_1 \vee A_2 \to W_1 \vee W_2} \odot \in \{\between, \uparrow\}$$

$$\frac{w_1 : W_1 \qquad \mathcal{A}_2 : L_2 \to W_2}{w_1 \cdot \mathcal{A}_2 : L_2 \to W_1 \vee W_2} \qquad \frac{\mathcal{A}_1 : L_1 \to W_1 \qquad w_2 : W_2}{\mathcal{A}_1 \cdot w_2 : L_1 \to W_1 \vee W_2}$$

Conclusion

State
- Acceptors, transducers
- Efficient static API
- Flexible dynamic API with runtime compilation
- User(/student) friendly IPython interface

Transition
- Better transducers support
- Improved type-checking errors
- Richer expressions
- Metadata on states
- ...

Conclusion

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Questions?