New Algorithms for Multivalued Component Trees

Nicolas Passat[1], Romain Perrin[2], Jimmy Francky Randrianasoa[2,3], Camille Kurtz[4], Benoît Naegel[2]

[1] Université de Reims Champagne Ardenne, CRESTIC, Reims, France
[2] Université de Strasbourg, CNRS, ICube, Strasbourg, France
[3] EPITA Research Laboratory (LRE), Paris, France
[4] Université Paris Cité, LIPADE, Paris, France

Motivation

The component tree (CT) [3] can model grey-level images for various image processing / analysis purposes (filtering, segmentation, registration, retrieval...). Its generalized version, the multivalued component tree (MCT) [1] can model images with hierarchically organized values. We provide new tools to handle MCTs:

- a new algorithm for the construction of MCTs;
- two strategies for building hierarchical orders on values, required to further build MCTs.

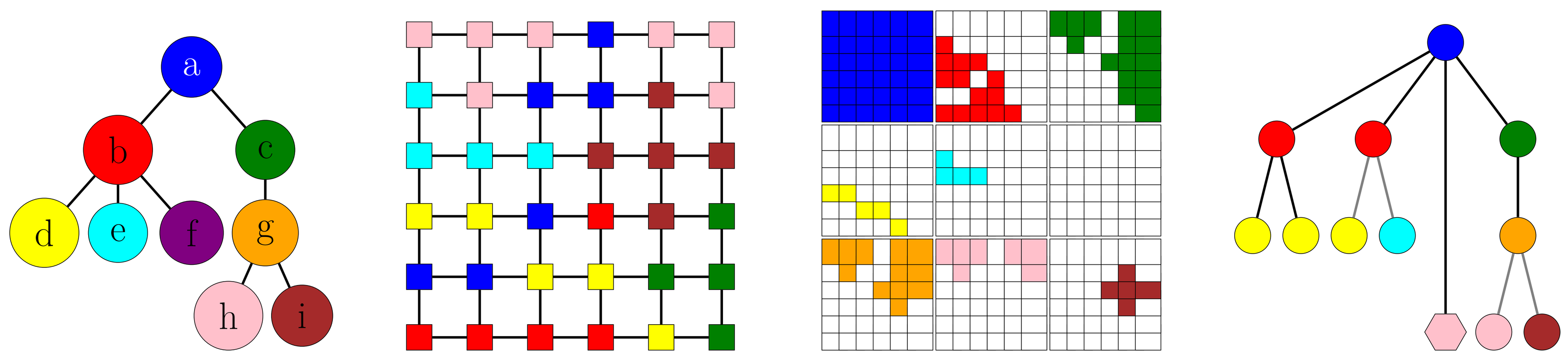
Multivalued Component Tree

Let $\mathfrak{G} = (\Omega, \frown)$ be a non-directed graph. Let $\mathcal{C}[X]$ be the set of the connected components of $X \subseteq \Omega$.
Let $\mathbb{V}$ be a finite set and $\leqslant$ a hierarchical order on $\mathbb{V}$, i.e. an order (1) which admits a minimum (resp. a maximum) and (2) such that for any $v \in \mathbb{V}$, the subset of the elements lower (resp. greater) than v is totally ordered by $\leqslant$.
Let us consider an image $\mathcal{F} : \Omega \to \mathbb{V}$. The threshold set of $\mathcal{F}$ at value $v \in \mathbb{V}$ is defined by $\Lambda_v(\mathcal{F}) = \{\mathbf{x} \in \Omega \mid v \leqslant \mathcal{F}(\mathbf{x})\}$. We define the set of nodes of the MCT as

$$\Theta = \bigcup_{v \in \mathbb{V}} \mathcal{C}[\Lambda_v(\mathcal{F})] \tag{1}$$

with $\mathbb{I}(X) = \{v \in \mathbb{V} \mid X \in \mathcal{C}[\Lambda_v(\mathcal{F})]\}$, $\omega(X) = \bigvee^{\leqslant} \mathbb{I}(X)$ and $\tau(X) = |\mathbb{I}(X)|$ for any node $X \in \Theta$.
The inclusion relation $\subseteq$ is a hierarchical order on Θ. Let $\lhd$ be the reflexive-transitive reduction of $\subseteq$.
The Hasse diagram $\mathfrak{T} = (\Theta, \lhd)$ of $(\Theta, \subseteq)$ is the multivalued component tree of the image $\mathcal{F}$. For any node $X \in \Theta$, we set $\rho(X) = X \setminus \bigcup_{Y \lhd X} Y = \{\mathbf{x} \in \Omega \mid \mathcal{F}(\mathbf{x}) = \omega(X)\}$.
The MCT $\mathfrak{T}$ is an image model of the image $\mathcal{F}$:

$$\forall \mathbf{x} \in \Omega, \mathcal{F}(\mathbf{x}) = \bigvee_{X \in \Theta}^{\leqslant} \mathbf{1}_{(X,\omega(X))}(\mathbf{x}) \tag{2}$$



(a) $(\mathbb{V}, \leqslant)$ (b) $\mathcal{F} : \Omega \to \mathbb{V}$ (c) $\Lambda_v(\mathcal{F})$ for $v \in \mathbb{V}$ (d) $\mathfrak{T} = (\Theta, \lhd)$

Building the Multivalued Component Tree

This construction algorithm derives from the CT construction of [3].

- `nodes`: stores the nodes of the multivalued component tree.
- `points`: stores the processed points of the image.
- `status`: stores the status of each point of the image.
- `nb_nodes` and `index`: store the number of nodes already fully built and the index of the node currently built at each value of $\mathbb{V}$.
- `progress`: indicates if there exists a node at value v, currently under construction or to be built, which is an ancestor of the node at value u currently being defined.

Algorithm 1: Build the multivalued component tree

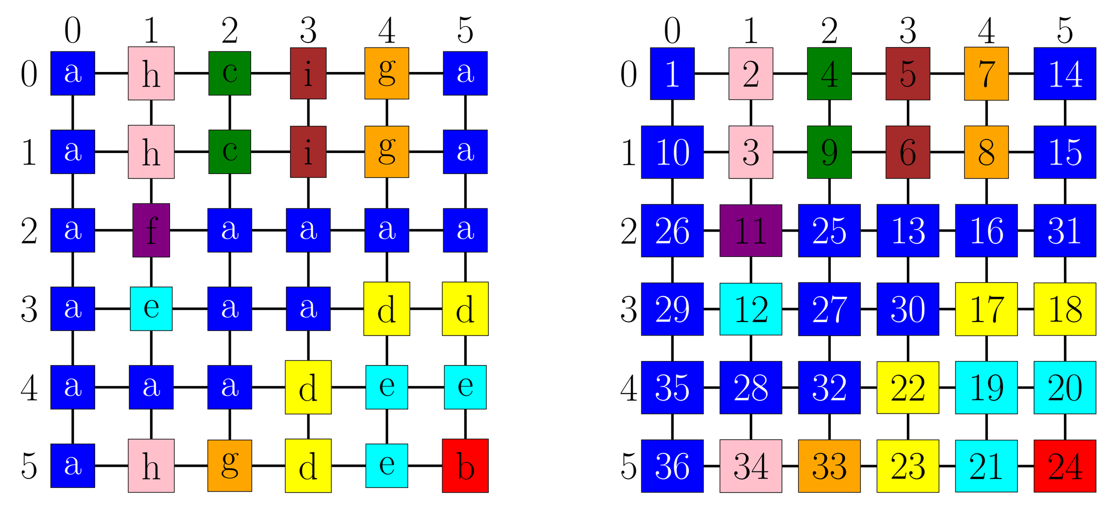
```
Input: (Ω, ⌢), (V, ⩽), F : Ω → V
Output: T = (Θ, ⊲)
Build nodes, points, status, nb_nodes, index, progress
v_min := ⋀^⩽ V
Choose x_min ∈ Ω such that F(x_min) = v_min
points[v_min].add(x_min)
progress[v_min] := true
Flood(v_min)
```

Function Flood

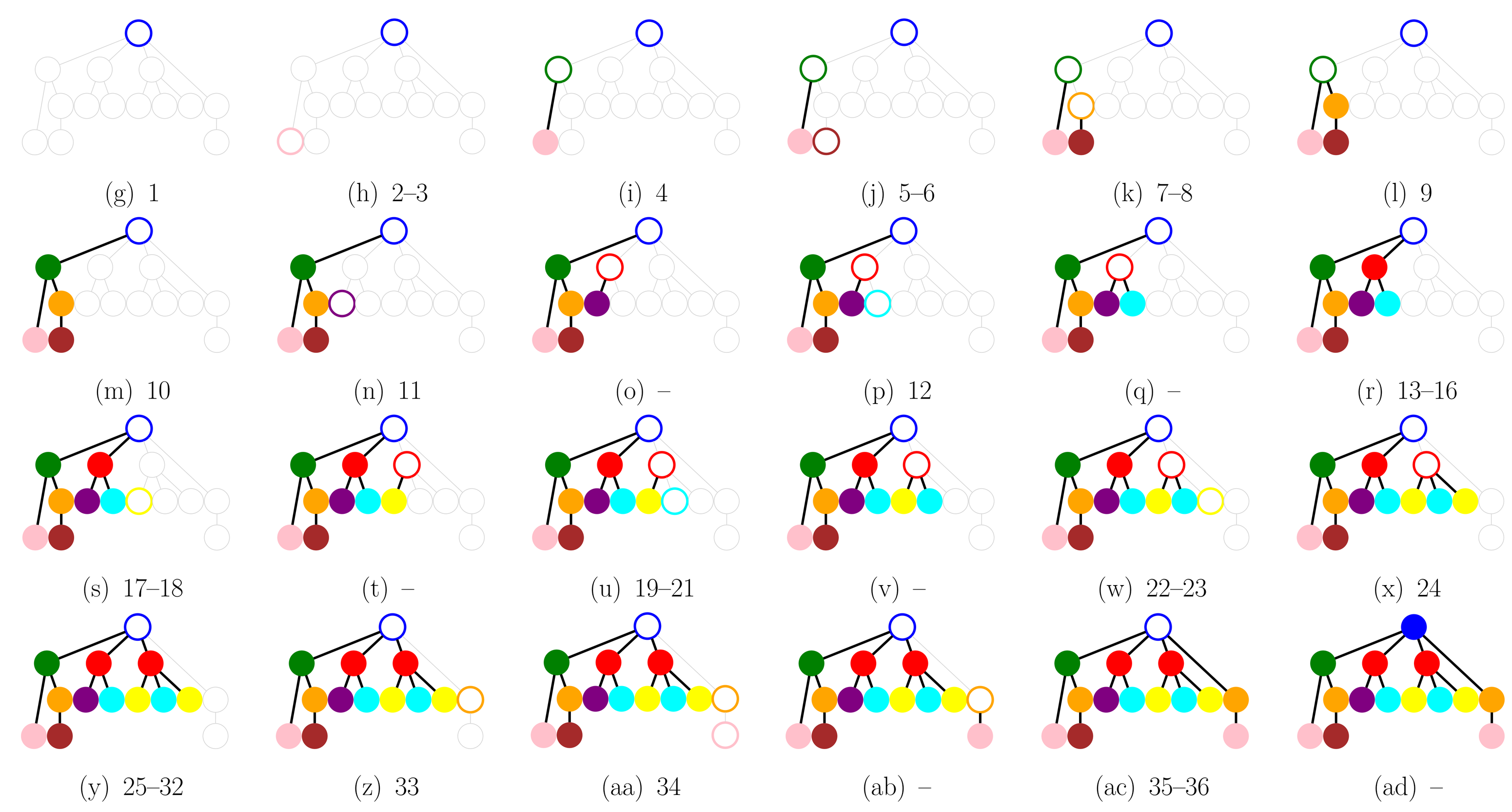
```
Input: u ∈ V: current level
Output: w ∈ V: value of the parent node of the root of the built (partial) MCT at value u
while !(points[u].empty()) do
    x := points[u].remove()
    if index[u] > nb_nodes[u] then
        nb_nodes[u] := index[u]
        X := create_node()                                  // new node in Θ
        nodes[u].insert(X)
    if F(x) ≠ u then
        w := F(x)
        points[w].add(x)
        progress[w] := true
        while u < w do  w := Flood(w)
    else
        status[x] := index[u]
        nodes[u][index[u]].add_to_proper_part(x)
        foreach y ⌢ x do
            w := F(y)
            if status[y] = −1 then
                if u ⩽ w then  ŵ := w
                else  ŵ := ⋀^⩽{u, w}
                points[ŵ].add(y)
                status[y] := 0
                progress[ŵ] := true
                while u < ŵ do  ŵ := Flood(ŵ)
if u = v_min then
    w := ε
else
    w := ⋁^⩽{w′ ∈ V | w′ < u}
    while progress[w] = false do  w := ⋁^⩽{w′ ∈ V | w′ < w}
    create_edge(nodes[u][index[u]],nodes[w][index[w]])          // new edge in ⊲
progress[u] = false
index[u]++
```

Running the Construction Algorithm

Construction of the MCT of an image $\mathcal{F} : \Omega \to \mathbb{V}$. At a current stage: a plain coloured node is fully built; a contour-colored node is under construction; a non-colored node has not been considered yet; a black edge is built; a light gray edge is not built.



(e) $\mathcal{F} : \Omega \to \mathbb{V}$ (f) Order of processing of Ω

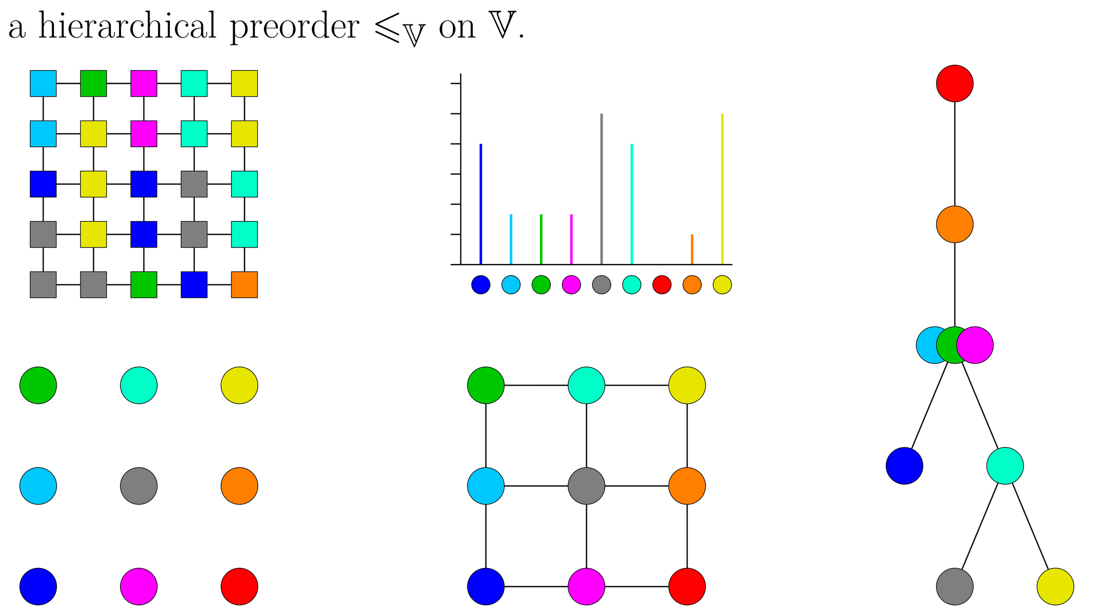

(g) 1 (h) 2–3 (i) 4 (j) 5–6 (k) 7–8 (l) 9
(m) 10 (n) 11 (o) – (p) 12 (q) – (r) 13–16
(s) 17–18 (t) – (u) 19–21 (v) – (w) 22–23 (x) 24
(y) 25–32 (z) 33 (aa) 34 (ab) – (ac) 35–36 (ad) –

Hierarchical order construction (1/2): (Pre)ordering the value set

Building the MCT requires a hierarchical order on $\mathbb{V}$.
First, we can build a hierarchical preorder $\leqslant_{\mathbb{V}}$ on $\mathbb{V}$. This can be done by building the CT of an "image" composed by the value set. It is only required that $\mathbb{V}$ be endowed with:

- an adjacency $\frown_{\mathbb{V}}$, allowing to map a graph structure on $\mathbb{V}$;
- a function $\delta_{\mathbb{V}} : \mathbb{V} \to \mathbb{N}$, allowing to associate to each element of $\mathbb{V}$ a value within the totally ordered set $(\mathbb{N}, \leq)$.

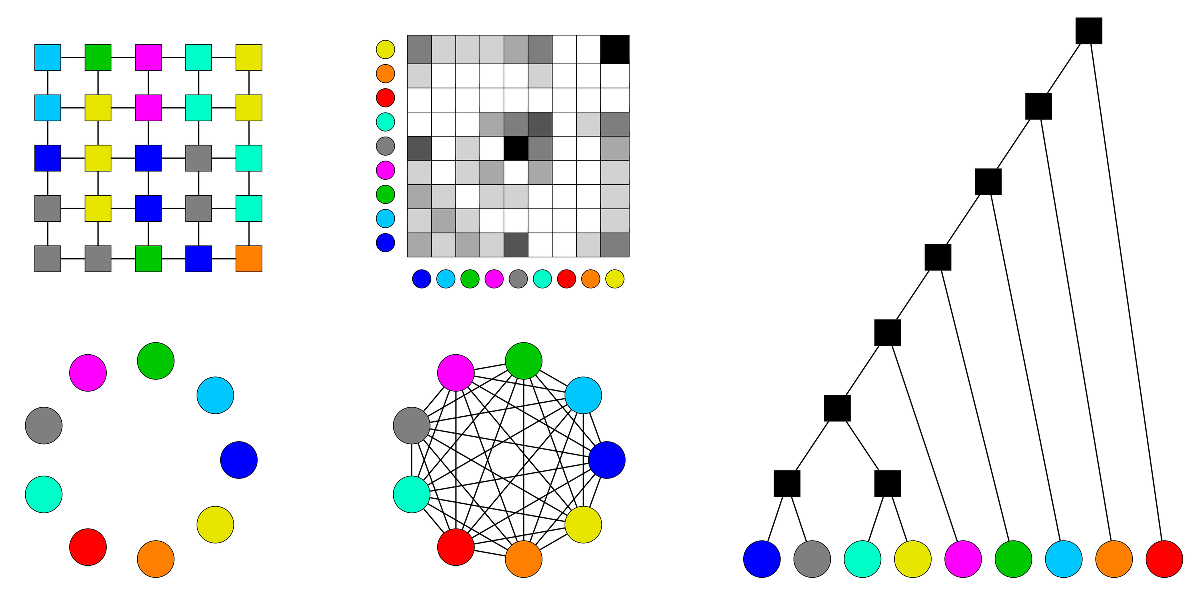
The CT of $\delta_{\mathbb{V}}$ is a hierarchical preorder $\leqslant_{\mathbb{V}}$ on $\mathbb{V}$.


(ae) (af) (ag)

Hierarchical order construction (2/2): Ordering the enriched value set

Second, we can build hierarchical order $\leqslant_{\mathbb{W}}$ on $\mathbb{W} \supset \mathbb{V}$ so that the elements of $\mathbb{V}$ are the maximal elements with respect to $\leqslant_{\mathbb{W}}$.
This can be done by building the binary partition tree (BPT) [2] of an "image" composed by the value set $\leqslant_{\mathbb{V}}$. It is only required that $\mathbb{V}$ be endowed with:

- an adjacency $\frown_{\mathbb{V}}$, allowing to map a graph structure on $\mathbb{V}$;
- a priority function $\delta_{\frown_{\mathbb{V}}} : \frown_{\mathbb{V}} \to \mathbb{N}$, allowing to determine the couples of nodes to be merged in priority.


(ah) (ai) (aj)

References

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