LRE
LABORATOIRE DE RECHERCHE DE L'EPITA

Solving 2-Player Games

Specialized data structure for Büchi game solving

Quentin Rataud, under the supervision of Philipp Schlehuber-Caissier

Seminar — July 2024

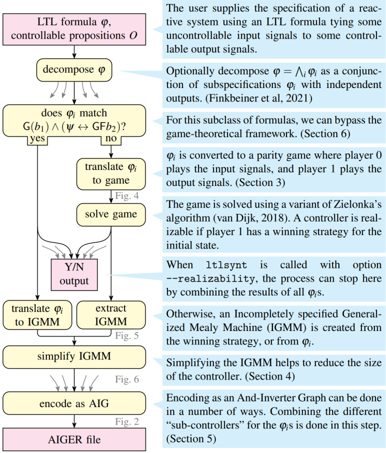

Figure: General outline of the process used by `ltlsynt` to solve the reactive synthesis problem. [2]

Input A sensor detects if a car is waiting right next to the traffic light (C) or not (!C)
Output The traffic light can either turn green (G) or turn red (R)

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- If no car is detected, the traffic light must be red
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- If a car is detected, the traffic light must eventually turn green

A smart traffic light

`ltlsynt` outline

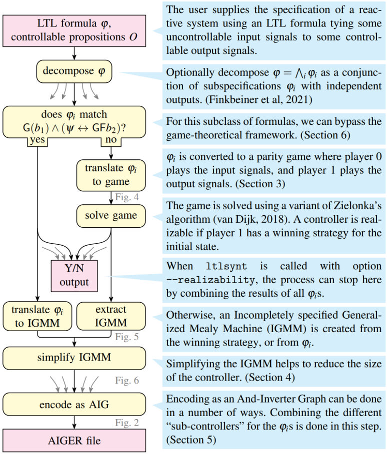

Figure: General outline of the process used by `ltlsynt` to solve the reactive synthesis problem. [2]

Input A sensor detects if a car is waiting right next to the traffic light (C) or not (!C)

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Input A sensor detects if a car is waiting right next to the traffic light (C) or not (!C)
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- $G(!C \implies R)$
- $G(G \implies X(R))$
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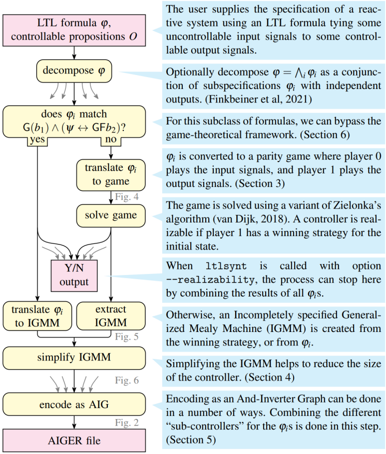

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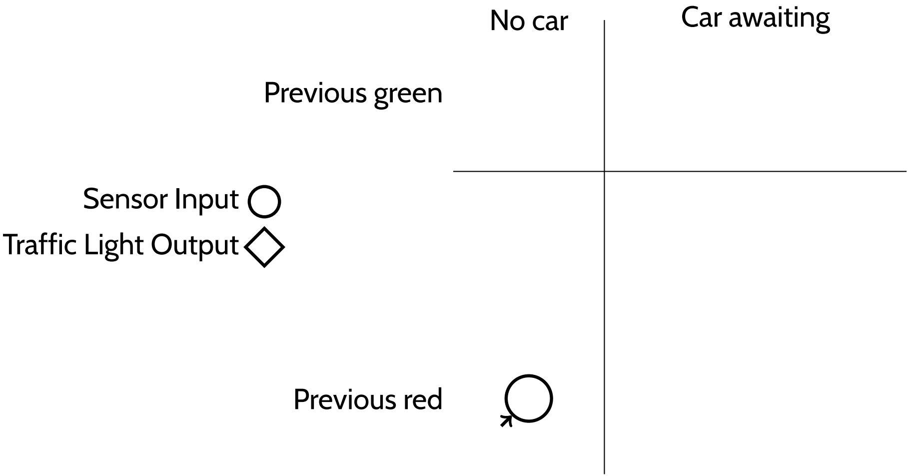
Turning LTL into a 2-Player Game



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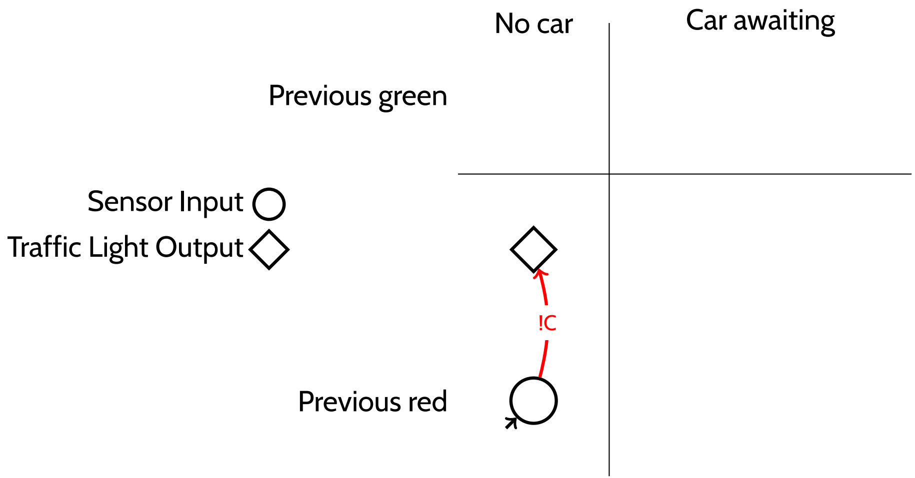

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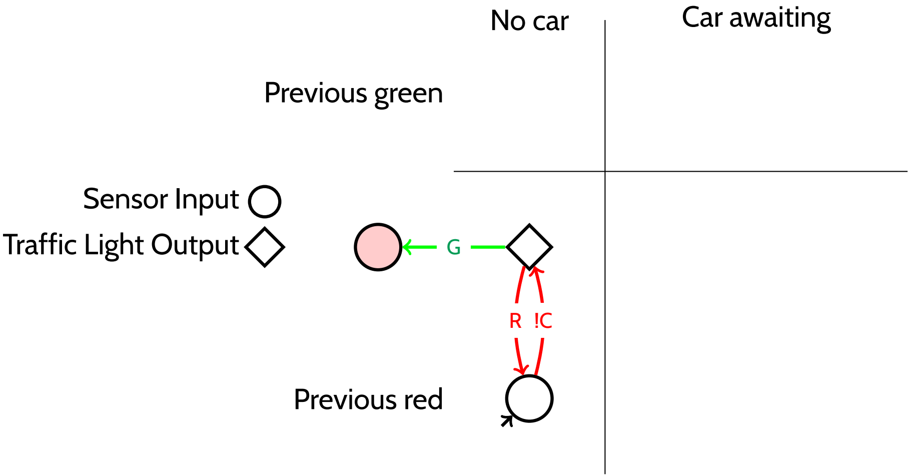

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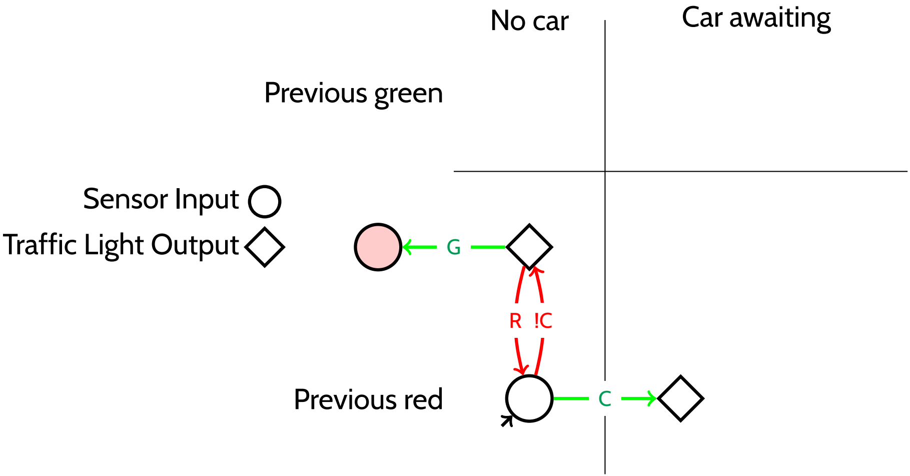
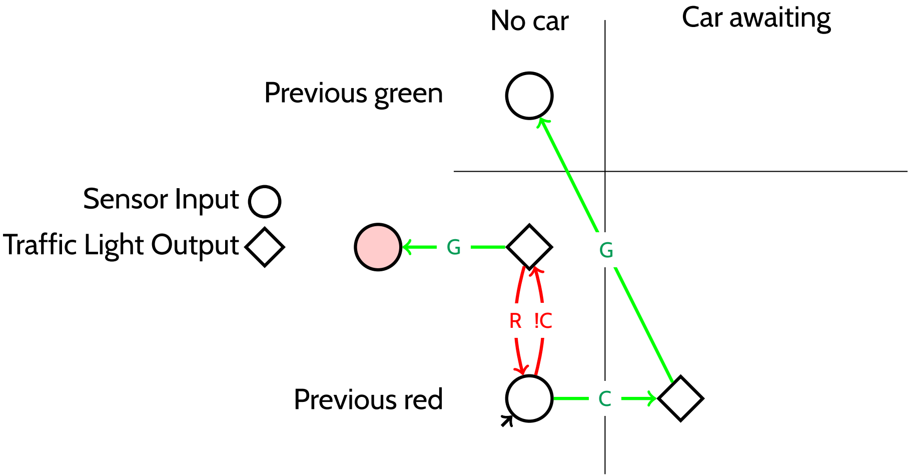


Figure: $G(!C \implies R)$ & $G(G \implies X(R))$ & $G(C \implies F(G))$



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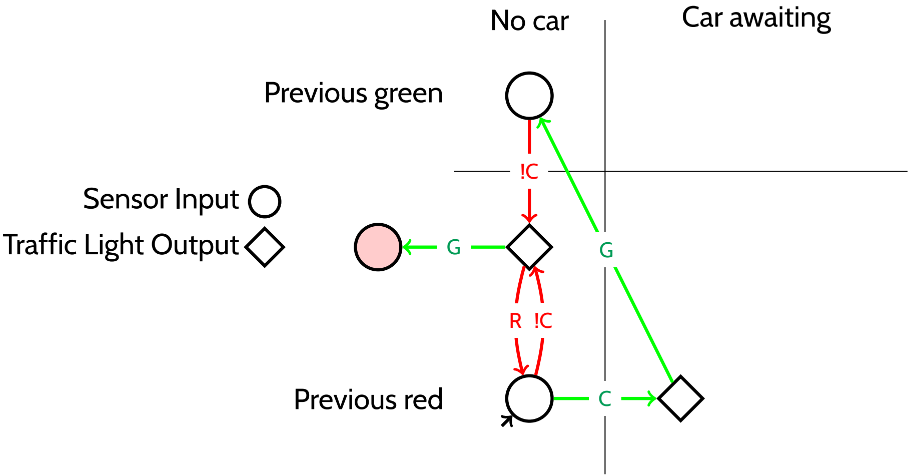

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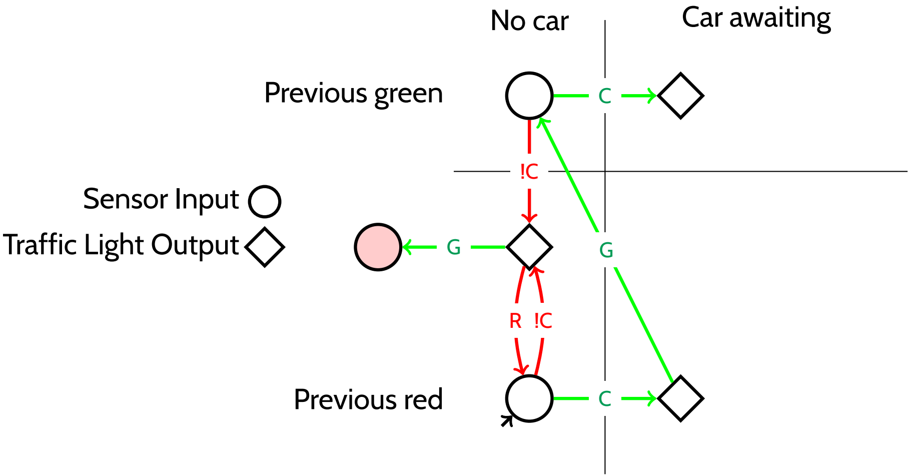

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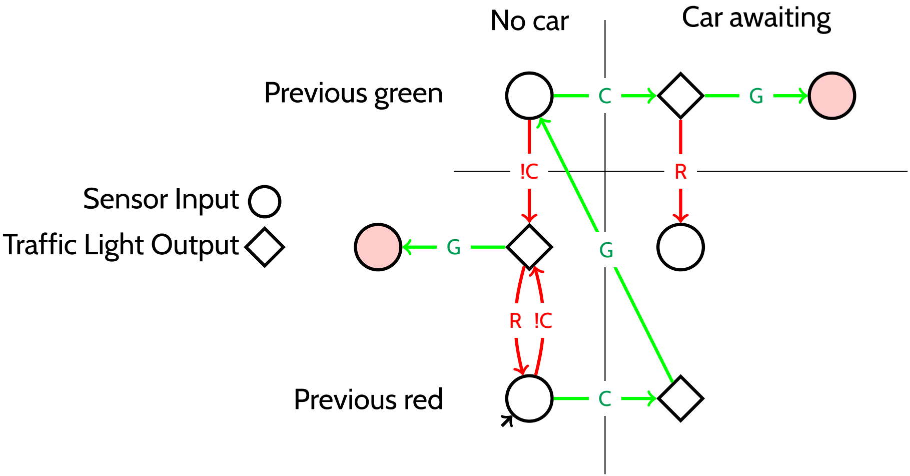

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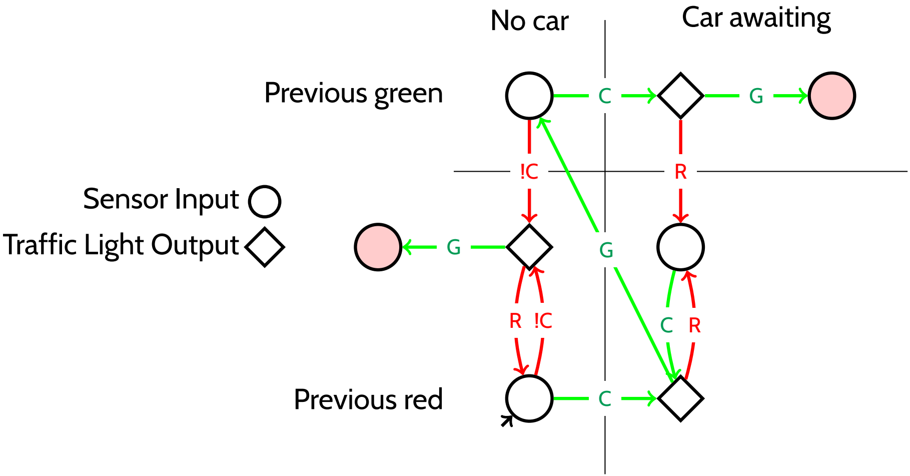

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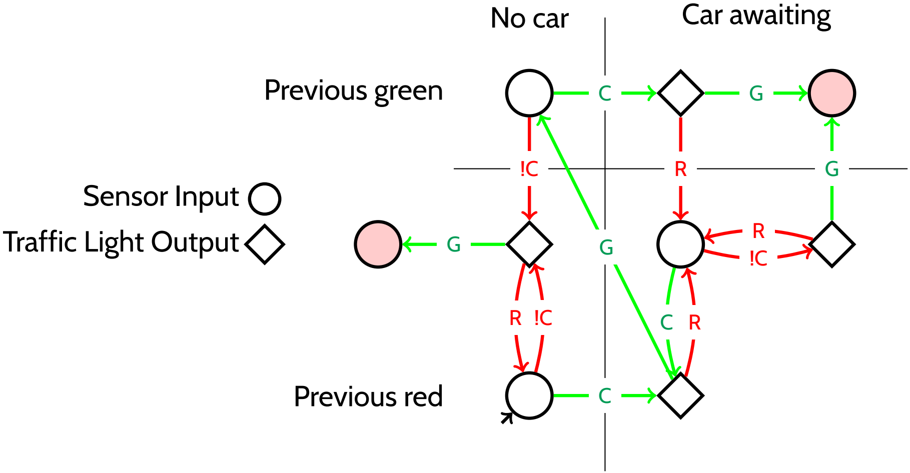

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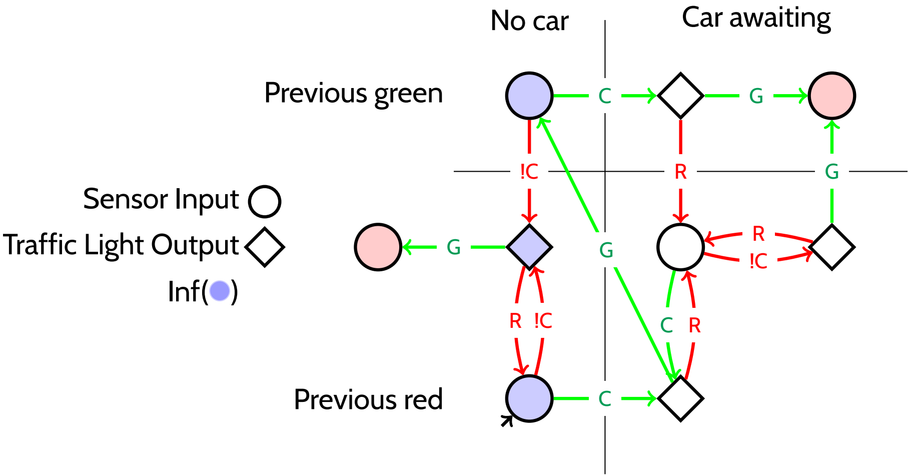

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A smart traffic light

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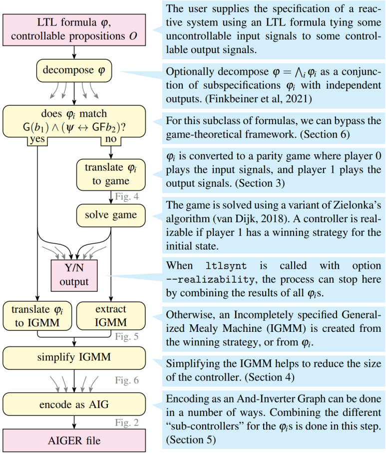

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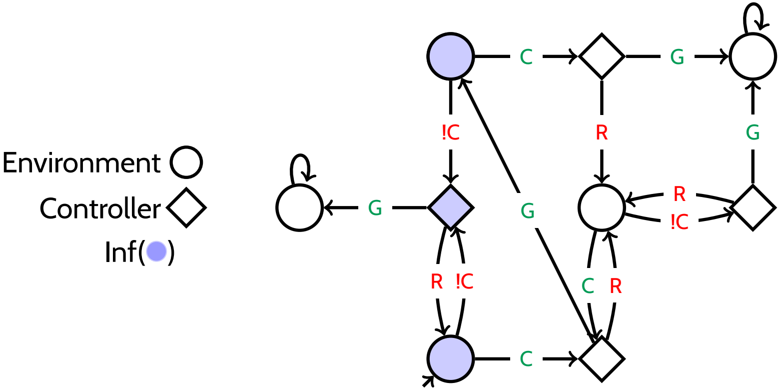
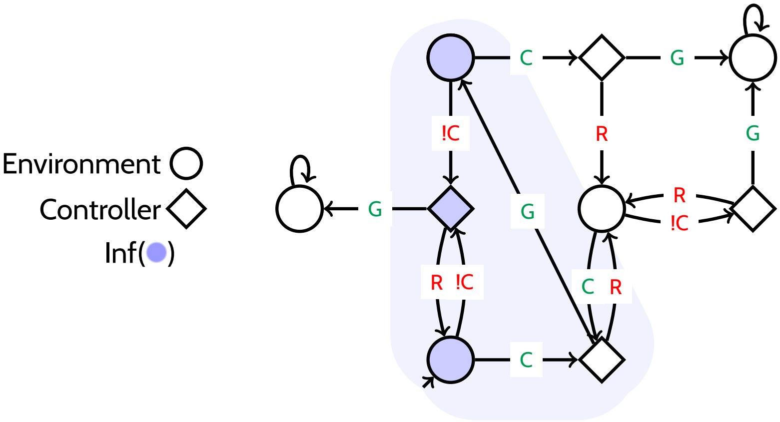


Figure: Solving a Büchi game



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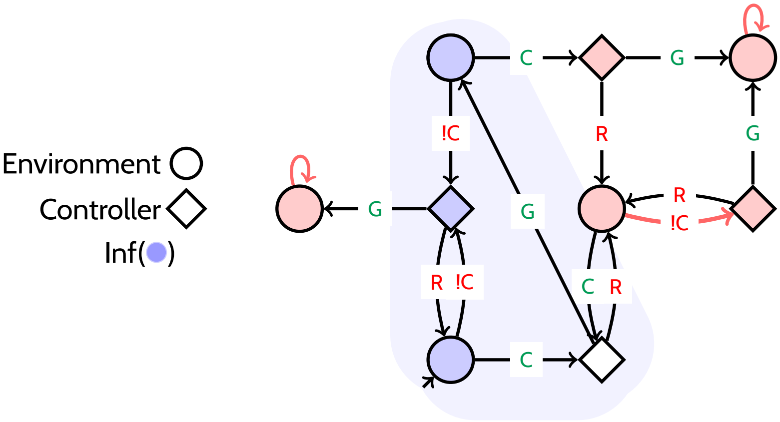

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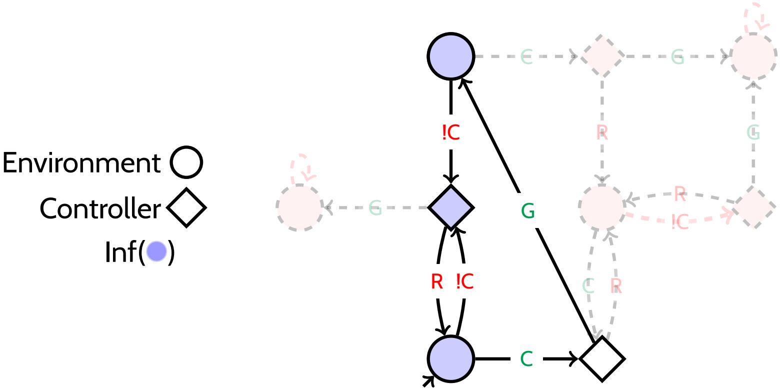

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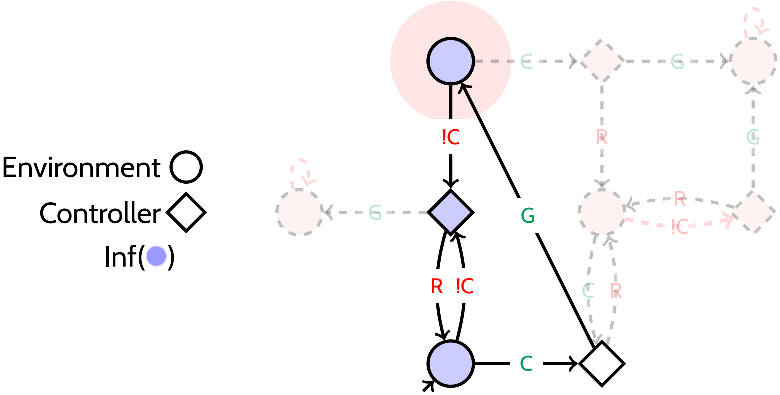

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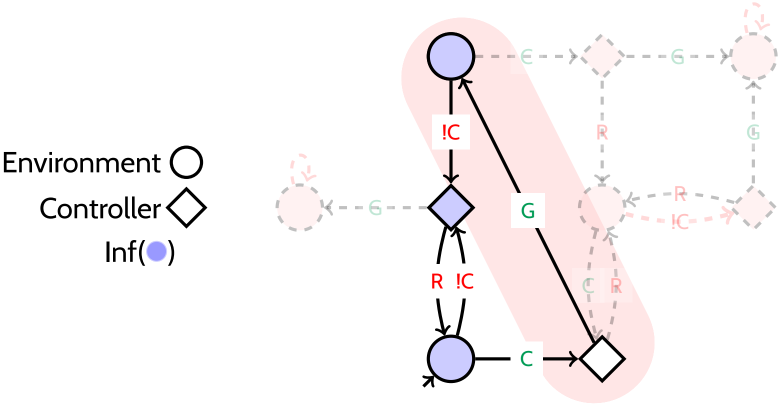

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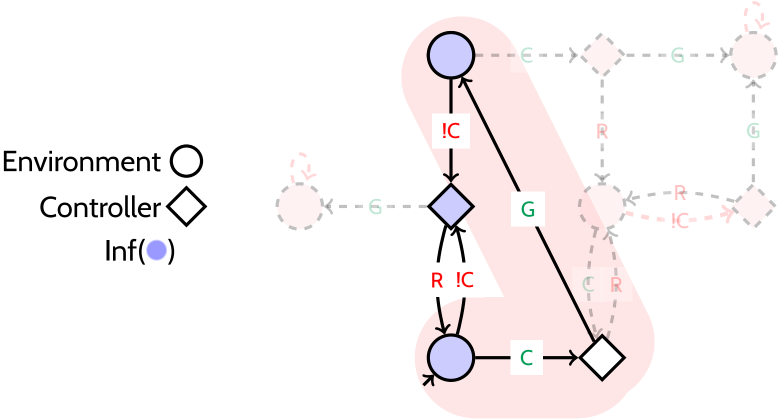

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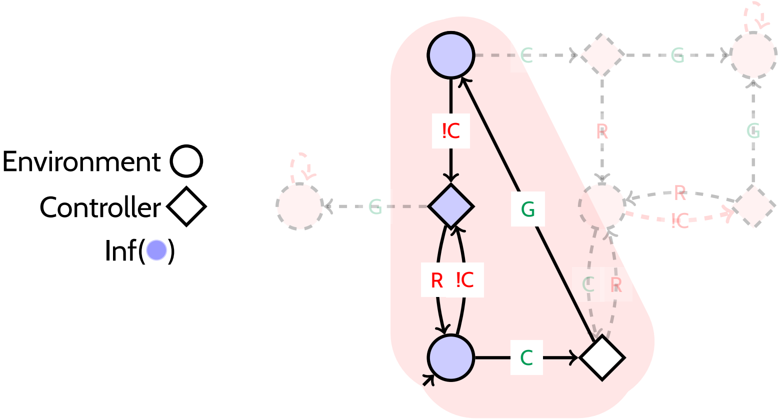

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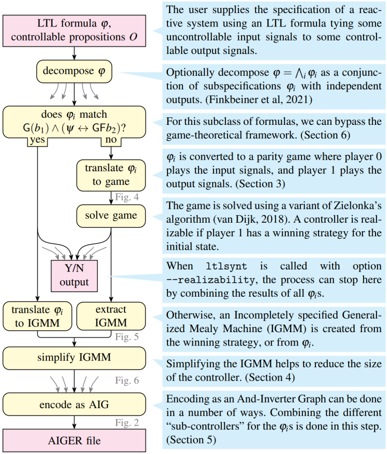

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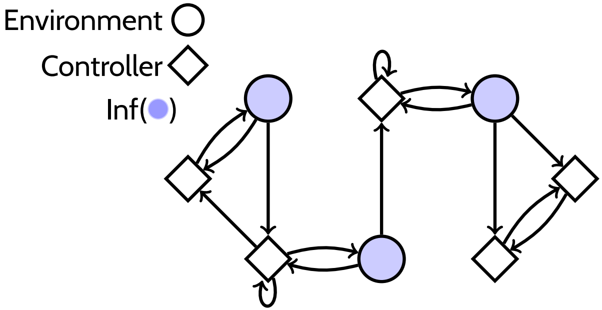


Figure: Solving a Büchi game

1 $R = \texttt{Attr}_1(B)$

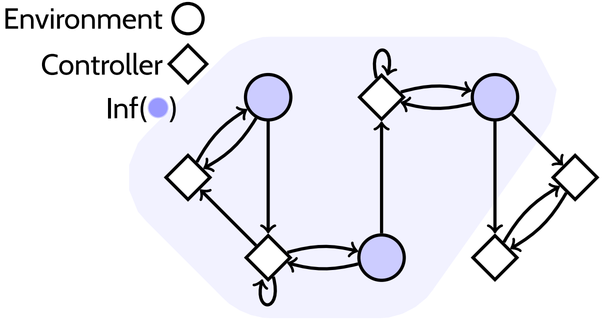
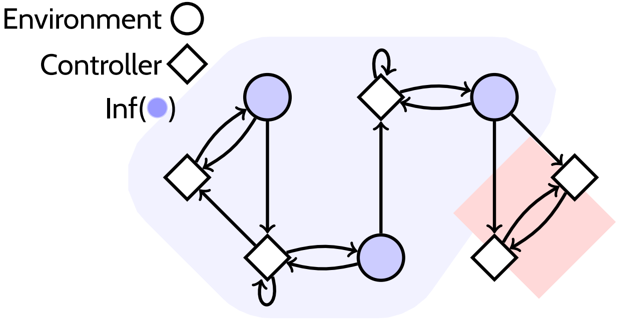


Figure: Solving a Büchi game

1. $R = \texttt{Attr}_1(B)$
2. $L = \texttt{Attr}_O(\bar{R})$



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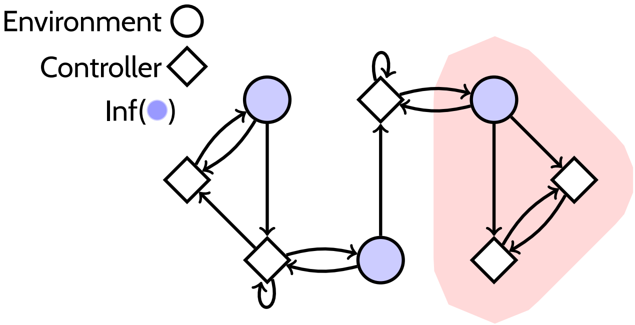

Figure: Solving a Büchi game

1. $R = \mathtt{Attr}_1(B)$
2. $L = \mathtt{Attr}_0(\bar{R})$
3. $G = G / L$

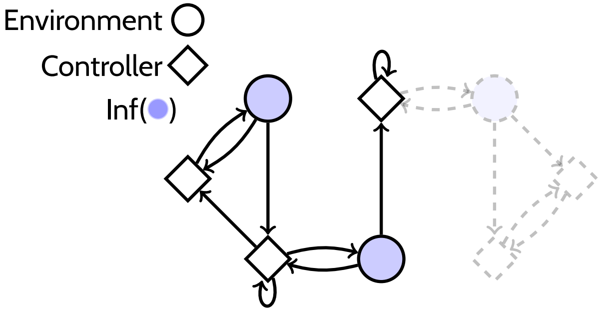

Figure: Solving a Büchi game

1. $R = \mathtt{Attr}_1(B)$
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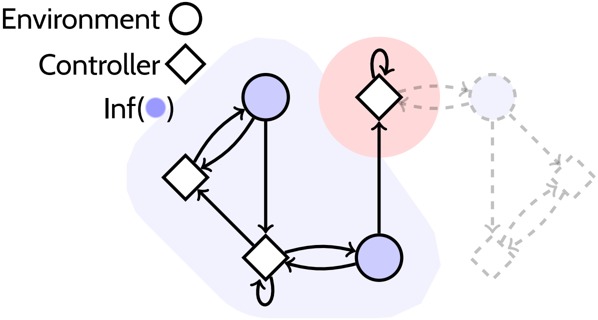
3. $G = G/L$
4. Repeat until fixed point is reached



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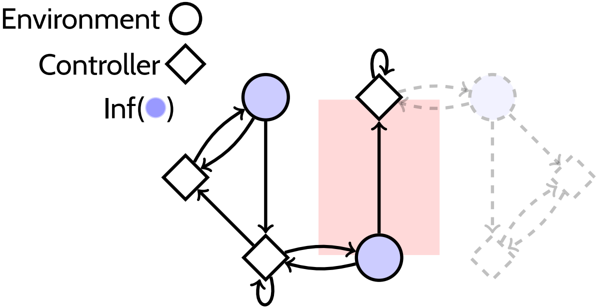

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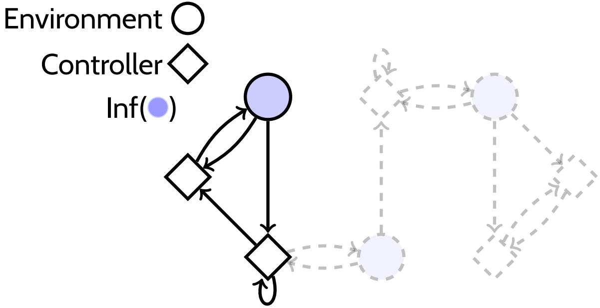

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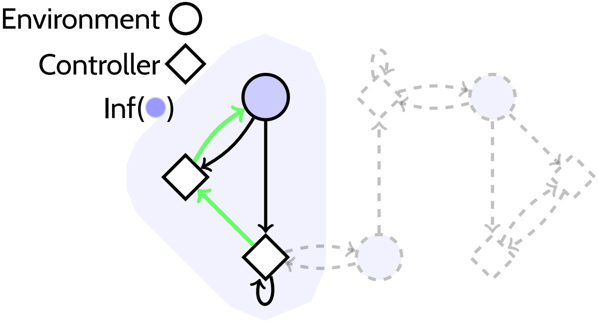

Figure: Solving a Büchi game

The data structure used to represent an arena needs to support:

- Deletion of arbitrary states and edges;
- Access to the data associated with any arbitrary state/edge;
- Traversal of all states/edges that are not yet removed.

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	`set`	`unordered_set`
Deletion	$O(\log(\|A\|))$	$O(1)$
Access	$O(\log(\|A\|))$	$O(1)$
Traversal	$O(\|A\|)$	$O(\|A\|)$

Where $|A|$ is the size of the *active* arena, and n is the total number of states.

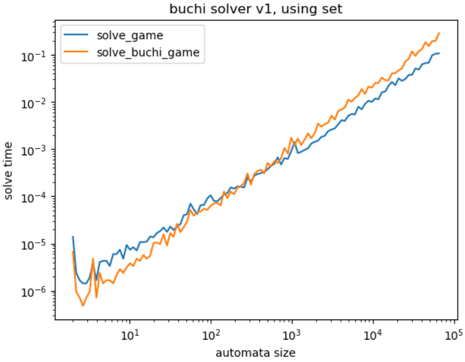

Figure: Using a `set` to represent an arena

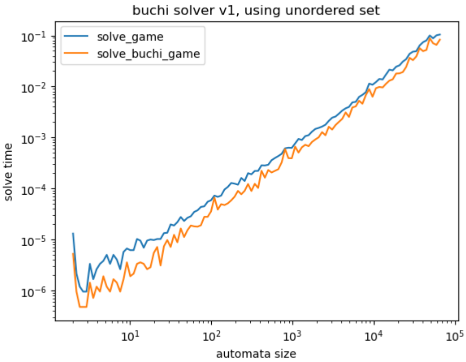

Figure: Using an `unordered_set` to represent an arena `unordered_set` have an ideal time complexity. However, they hide a strong hidden constant.

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	`set`	`unordered_set`	`vector<bool>`
Deletion	$O(\log(\|A\|))$	$O(1)$	$O(1)$
Access	$O(\log(\|A\|))$	$O(1)$	$O(1)$
Traversal	$O(\|A\|)$	$O(\|A\|)$	$O(n)$

Where $|A|$ is the size of the *active* arena, and n is the total number of states.

Customized Data Structure for Büchi Solving

`unordered_set` vs. `vector<bool>`

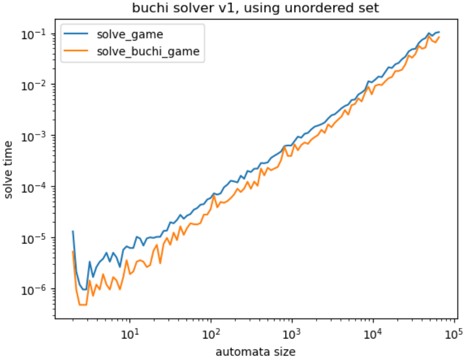


Figure: Using an `unordered_set` to represent an arena

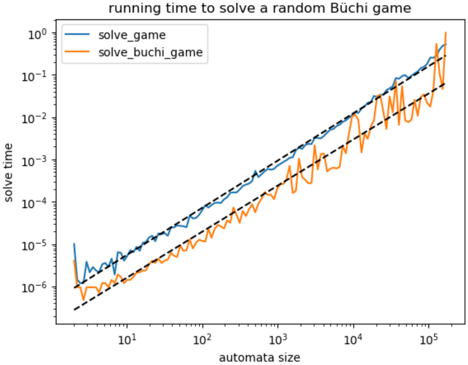
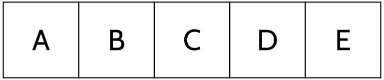


Figure: Using a `vector<bool>` to represent an arena

Customized Data Structure for Büchi Solving

Introducing `partitioned_dlist`

- We know in advance all the states we need to store, so we can store them statically.



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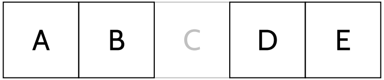
- We can mark a state if it is deleted using a single `boolean`



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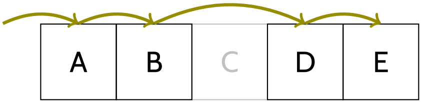
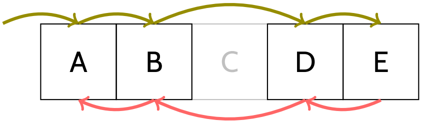
- We can mark a state if it is deleted using a single `boolean`
- To allow fast iteration over active elements, we use a pointer `next` to the next active element.



- We know in advance all the states we need to store, so we can store them statically.
- We can mark a state if it is deleted using a single `boolean`
- To allow fast iteration over active elements, we use a pointer `next` to the next active element.
- Deleting an element means the next of our previous becomes our next (we skip the element), so we also need a pointer `prev`.



Customized Data Structure for Büchi Solving

Complexity of `partitioned_dlist`

	`set`	`unordered_set`	`vector<bool>`	`partitioned_dlist`
Deletion	$O(\log(\|A\|))$	$O(1)$	$O(1)$	$O(1)$
Access	$O(\log(\|A\|))$	$O(1)$	$O(1)$	$O(1)$
Traversal	$O(\|A\|)$	$O(\|A\|)$	$O(n)$	$O(\|A\|)$

Where $|A|$ is the size of the *active* arena, and n is the total number of states.

Customized Data Structure for Büchi Solving

`vector<bool>` vs. `partitioned_dlist`

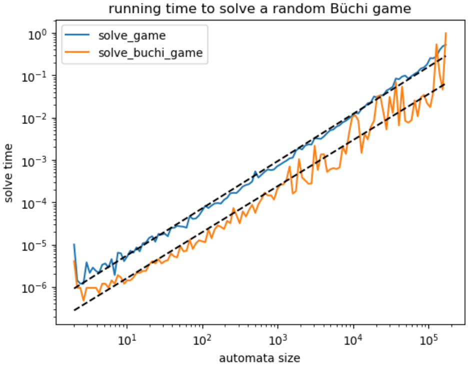
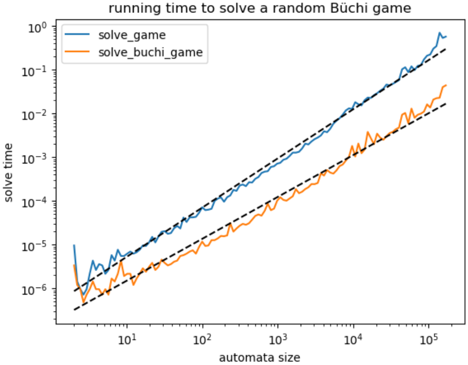


Figure: Using a `vector<bool>` to represent an arena



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Actual state of the Büchi Solver

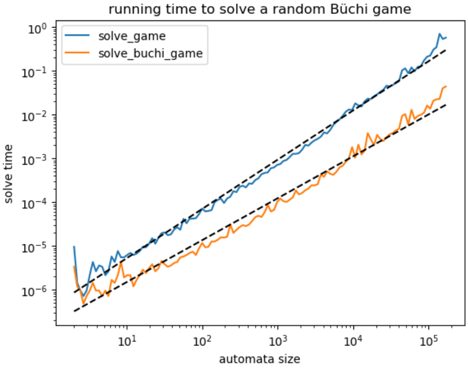


Figure: In double-log scale

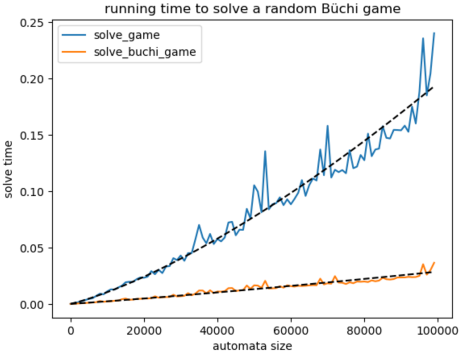


Figure: In linear scale

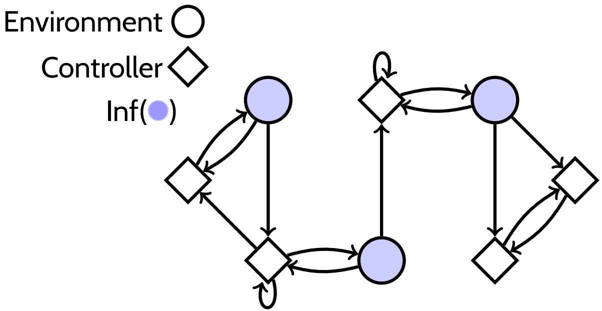


Figure: Decomposing an arena into strongly connected components

1. The solving time complexity may degenerate to $O(n^2)$

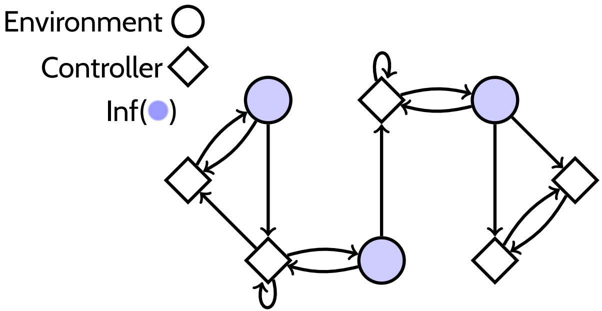


Figure: Decomposing an arena into strongly connected components

1. The solving time complexity may degenerate to $O(n^2)$
2. Decomposing into strongly connected components may help

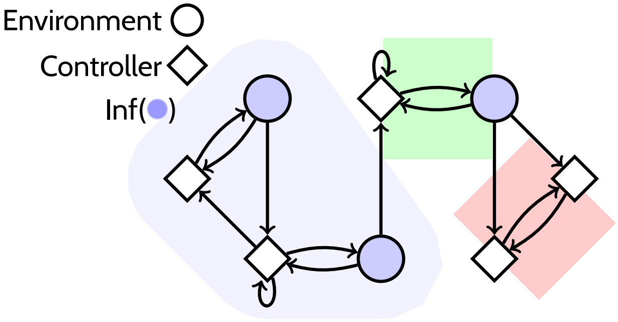
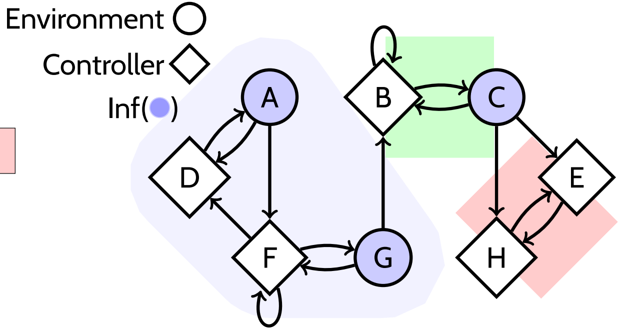


Figure: Decomposing an arena into strongly connected components

How to adapt our data structure to efficiently work only on some components?

A	B	C	D	E	F	G	H



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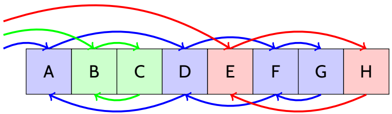

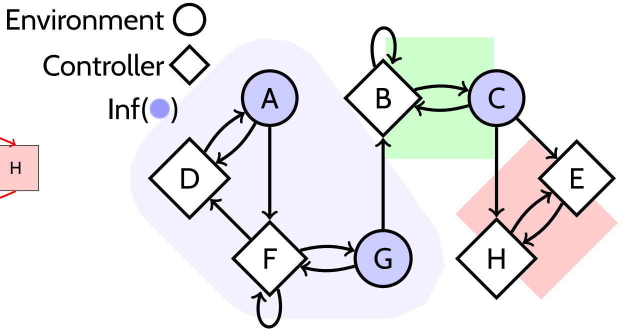

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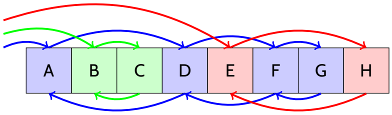
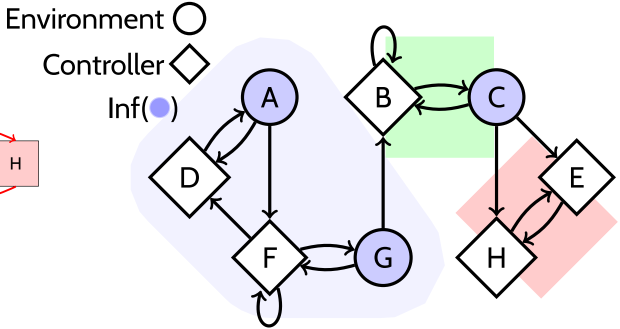


Multiple heads!



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