Gluing-Parallel-Symetric Poset and the Five Forbidden Structures

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Concurential theory

- Base structure: the **Posets**

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- Search to understand structures bigger and more complex

Concurential theory

- Base structure: the **Posets**
- Search to understand structures bigger and more complex
- Enumerate them

Poset ? Kesako ?

Definition

A **Poset** is a partiel order set.

- Reflexivity : $\forall x \in E, xRx$
- Transitivity : $x, y, z \in E, xRy \wedge yRz \implies xRz$
- Antisymmetric : $x, y \in E, xRy \wedge yRx \implies x = y$
- $\circ$: event
- $\longrightarrow$: precedence order

IPoset

Definition

A **poset with interfaces (IPoset)** is a poset together with two injected sets:

$$[n] \xrightarrow{s} P \xleftarrow{t} [m], \quad n, m \geq 0$$

such as the image of $s[n]$ is minimal and the image of $t[m]$ is maximal

- s is a starting interface and t is a terminating interface
- n is "unstarted" and m is "unfinished"

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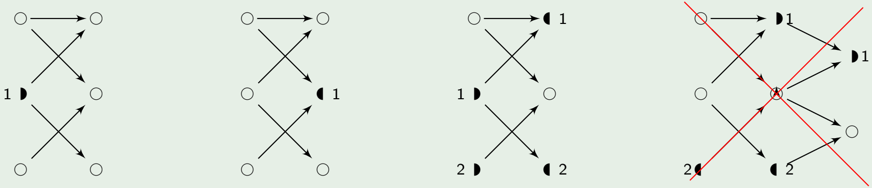
Examples



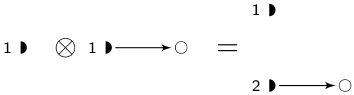
Parallel composition

Definition

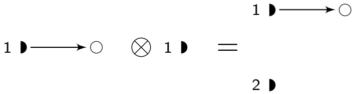
The **Parallel composition** of two GP-iposets $[n_1] \to (P_1) \leftarrow [m_1]$ and $[n_2] \to (P_2) \leftarrow [m_2]$ is defined by $[n_1 + n_2] \to [P_1 \otimes P_2] \leftarrow [m_1 + m_2]$

- Composition not commutative

Case 1:



Case 2:



Gluing composition

Definition

Only if $m_1 = n_2$, the **Gluing composition** of two iposets $[n_1] \xrightarrow{s_1} (P_1, <_1) \xleftarrow{t_1} [m_1]$ and $[n_2] \xrightarrow{s_2} (P_2, <_2) \xleftarrow{t_2} [m_2]$ is:

$$P_1 \triangleright P_2 = \begin{cases} (P_1 \sqcup P_2)/t_1(i) = s_2(i) \\ (<_1 \cup <_2 \cup (P_1 \ / \ t_1[m]) \times (P_2 \ / \ s_2[m]))^+ \end{cases}$$

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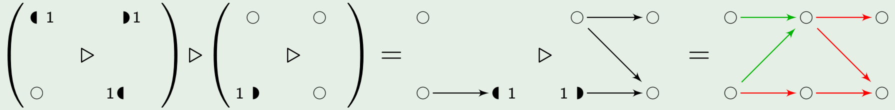
$$P_1 \triangleright P_2 = \begin{cases} (P_1 \sqcup P_2) / t_1(i) = s_2(i) \\ ({\color{red}<_1} \cup {\color{red}<_2} \cup {\color{green}(P_1 \ / \ t_1[m])} \times {\color{green}(P_2 \ / \ s_2[m])})^+ \end{cases}$$

Examples



GP-IPoset

Definition

An (i)poset is **Gluing-Parallel** (GP-(i)poset) if it is empty or if it is a finite combination of the following elements thanks to the *Parallel composition* and the *Gluing composition*.

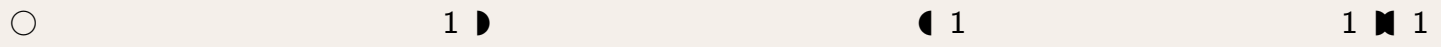
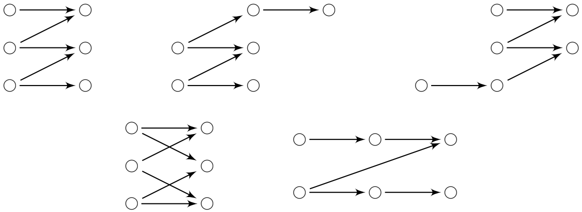


Figure: The four singletons

The following five posets are forbidden substructures for gp-posets:



Forbidden structures

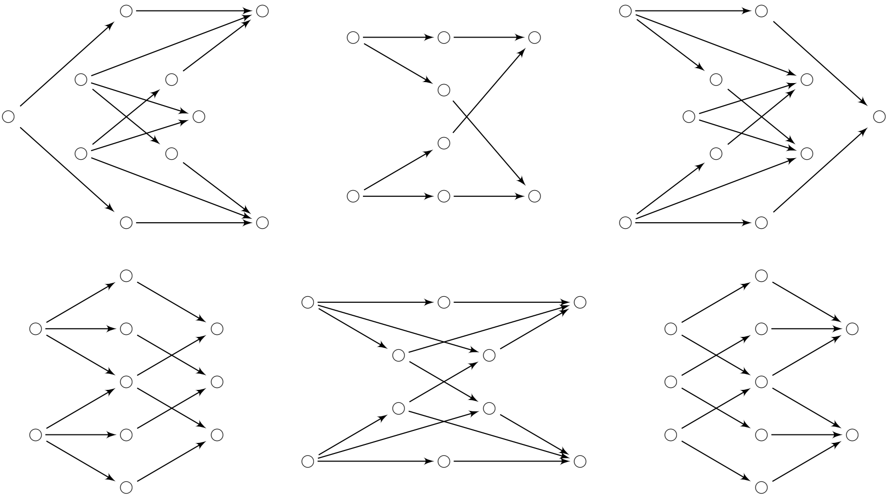
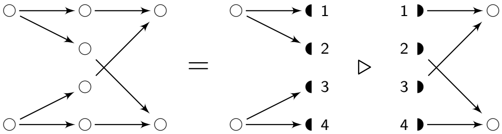


Figure: Additional forbidden substructures for gp-posets.

8-points forbidden structure



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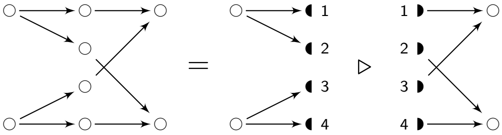

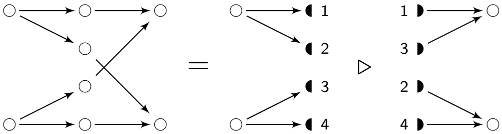

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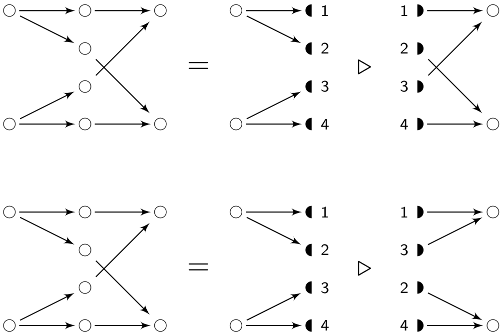

It is the same for the bigger forbidden structures: they are decomposable but interfaces are permuted in a wrong way.

GPS-Posets

Definition

An iposet is **gluing-parallel-symmetric** (**gps**) if it is empty or can be obtained from the elements:

- $\circ$
- $\blacktriangleleft\, 1$
- $1 \,\blacktriangleright$
- $1 \bowtie 1$
- $\begin{smallmatrix} 1 \bowtie 2 \\ 2 \bowtie 1 \end{smallmatrix}$ where $\begin{smallmatrix} 1 \bowtie 2 \\ 2 \bowtie 1 \end{smallmatrix} = (s, [2], t) : 2 \to 2$ is the non-trivial symmetry on 2.

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- We can define the gps-posets class in a similar way we defined the gps-iposets.
- gps-iposets contain all the gp-posets and also all the symmetries $n \to n$ for any n.

Forbidden Five

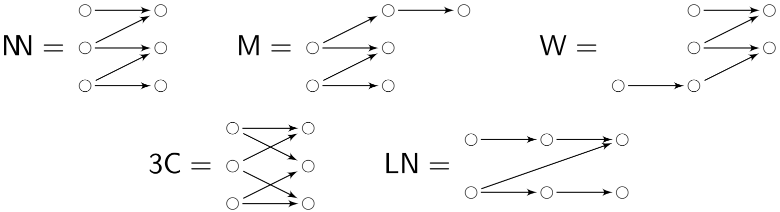

Figure: The five posets that are forbidden substructures

Conjecture

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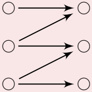
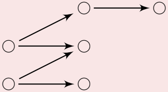
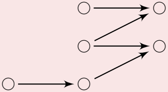
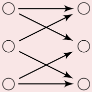
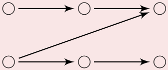
A poset is GPS if and only if it does not contain any of the five induced sub-structures:

NN = 

M = 

W = 

3C = 

LN = 

Work of this semester

Ziemianski theorem

Theorem

An poset P is GSP if and only if it admits an interval representation in a SP-poset V

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A **Serial-Parallel Poset** (**SP Poset**) if it is empty or can be obtained from the singleton $\{ \circ \}$ by finitely many applications of $*$ and $\otimes$.

Ziemianski theorem

Theorem

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Definition

A **Serial-Parallel Poset** (**SP Poset**) if it is empty or can be obtained from the singleton $\{\ \circ\ \}$ by finitely many applications of $*$ and $\otimes$.

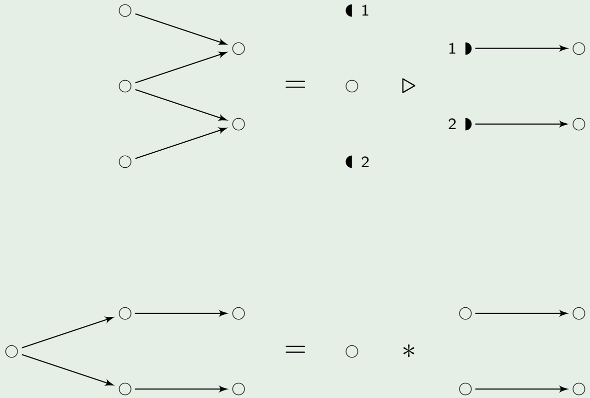
Definition

An **Interval representation** of a P poset in a V poset is a pair of functions $f, g : P \rightarrow V$ such that:

- $f(p) \leq g(p), \forall p \in P$
- $p <_P q \leftrightarrow g(p) <_V f(q), \forall p, q \in P$

Ziemianski theorem

Examples



- GPS $\Longleftrightarrow$ IO(SP)
- SP-Posets have good algebraic properties

Ziemianski theorem

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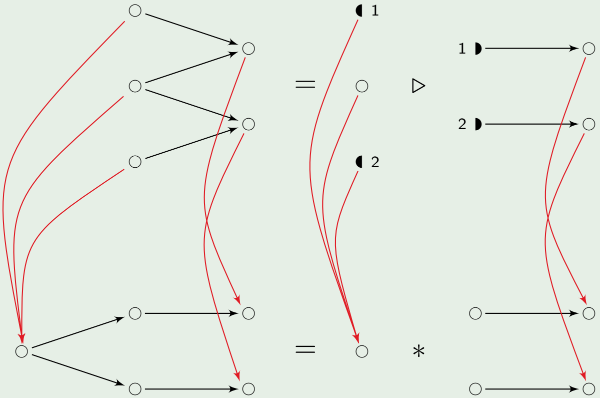

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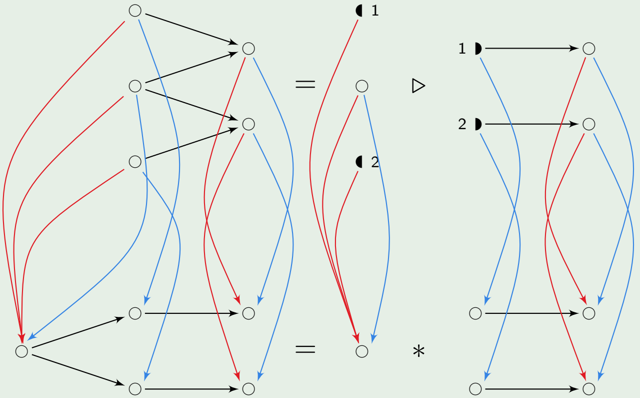

Ziemianski theorem

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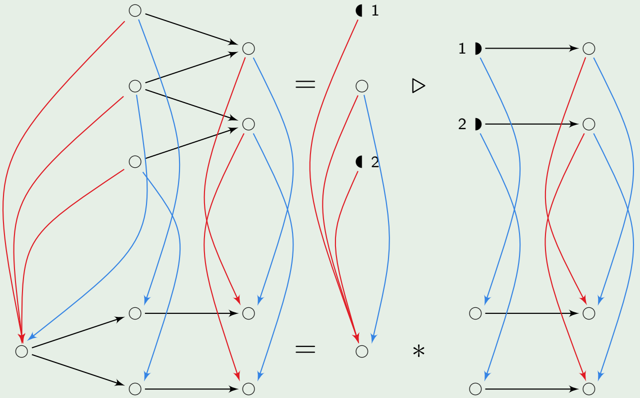

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IO(SP) of Forbidden Five structrures

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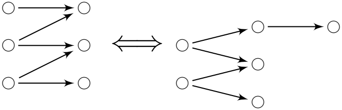

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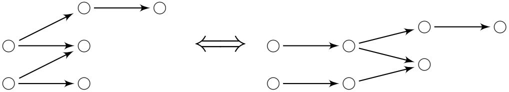

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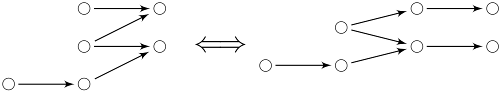

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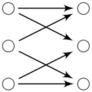
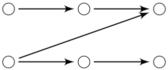
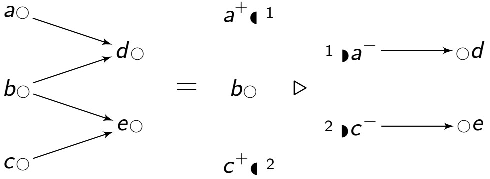
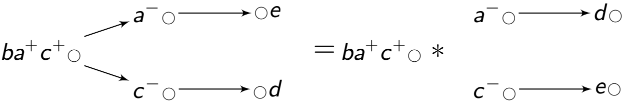


LN :



Issue





IMPOSSIBLE !! $c < f$ and $\forall x \in P/\{c\}|\ x \not> c$ and $x \not\leq c$

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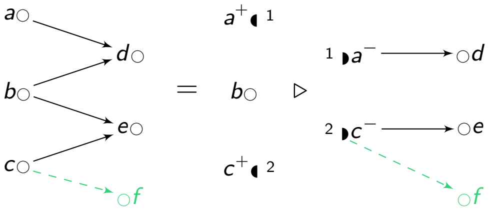
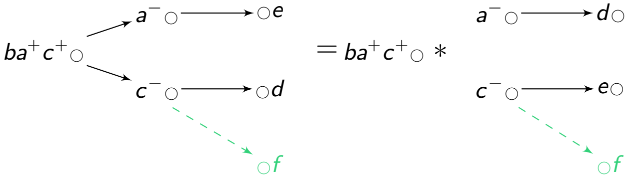
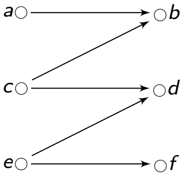
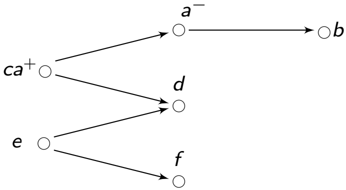




IMPOSSIBLE !! $c < f$ and $\forall x \in P/\{c\}|\ x \not> c$ and $x \not\leq c$

Solve





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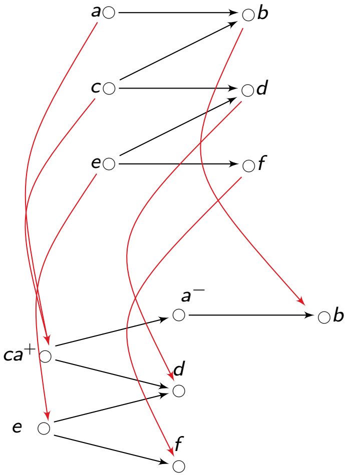

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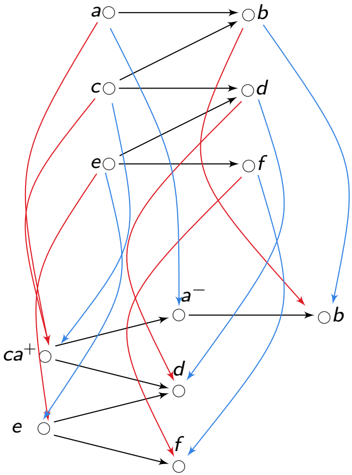

Conclusion

- Lack of "consistency" when adding a point
- Imprevisibility of a poset P in sp-interval
- Wrong way ?
- More hope in the track presented by Paul

References

Uli Fahrenberg, Christian Johansen, Georg Struth, and Krzysztof Ziemiański.
Posets with interfaces as a model for concurrency, 2022.

Olavi Äikäs, Uli Fahrenberg, Christian Johansen, and Krzysztof Ziemiański.
Generating posets with interfaces, 2022.