Integrating Mathematical Morphology within Deep Convolutional Neural Networks

Alexandre Kirszenberg

September 30th, 2019

Given an image, detect the outline of objects automatically.

Chen et al., “Rethinking Atrous Convolution for Semantic Image Segmentation”

An introduction to Image Segmentation

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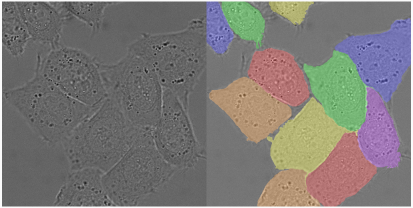
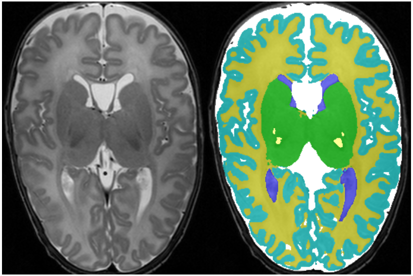
Image Segmentation: Application to Medical Imaging



HeLa cells



Axial adult brain

Ronneberger, Fischer, and Brox, “U-Net: Convolutional Networks for Biomedical Image Segmentation”

Xu, Géraud, and Bloch, “From neonatal to adult brain MR image segmentation in a few seconds using 3D-like fully convolutional network and transfer learning”

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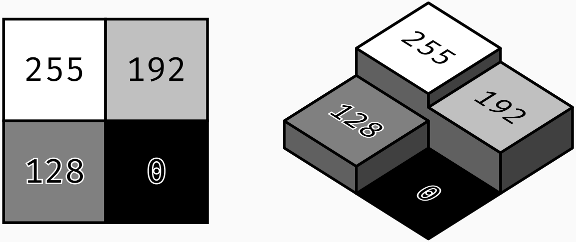

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Morphological Filters

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- These filters are the composition of two operations:
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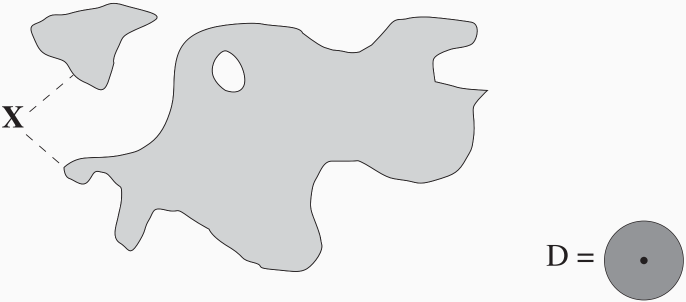
- They take a structuring element as parameter.

Morphological Filters: an example



A set X and a structuring element D

Serra and Vincent, "An overview of morphological filtering"

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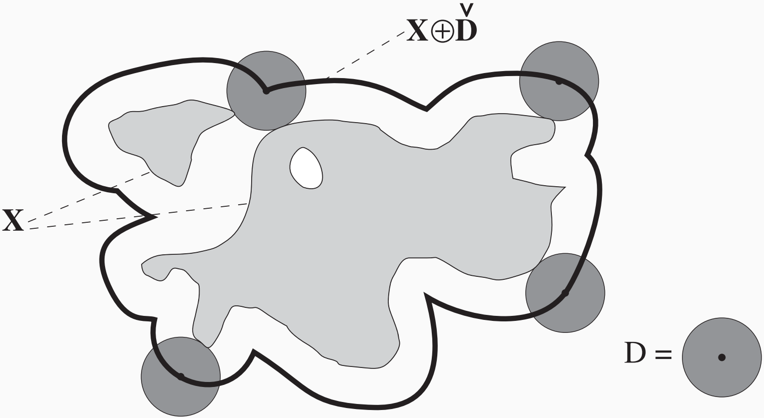

The dilation of *X* by *D*

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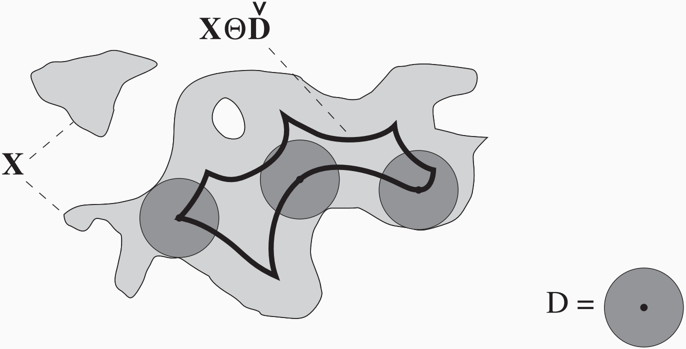
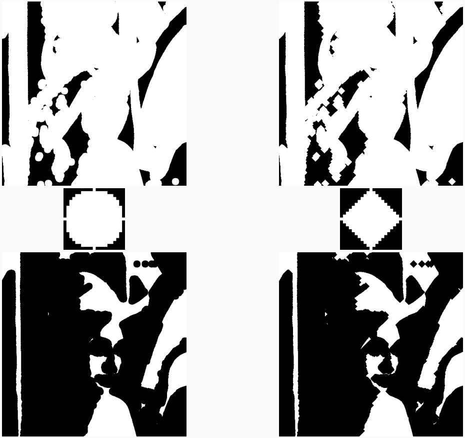


The erosion of *X* by *D*

Serra and Vincent, "An overview of morphological filtering"

Morphological Filters: with an actual image

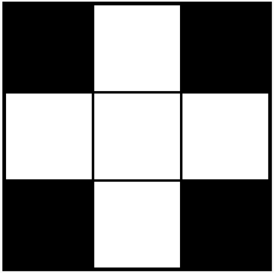
Dilation δ_C Dilation δ_D



Erosion ϵ_C Erosion ϵ_D

From Binary Dilation to Grayscale Dilation

$$(f \oplus b)(x) = \sup_{y \in E} [f(y) + b(x - y)]$$

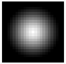


0	1	0
1	1	1
0	1	0

-∞	0	-∞
0	0	0
-∞	0	-∞

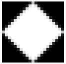
Grayscale Filters

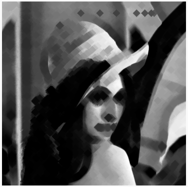
Dilation δ_C



Erosion ϵ_C

Dilation δ_D





Erosion ϵ_D

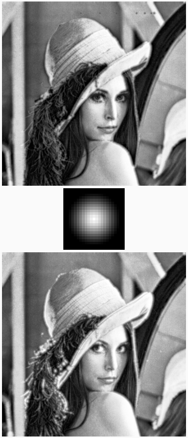
Higher Order Filters

We can combine these filters to create higher order operators:

- The opening $\gamma = \delta \circ \epsilon$.
- The closing $\phi = \epsilon \circ \delta$.

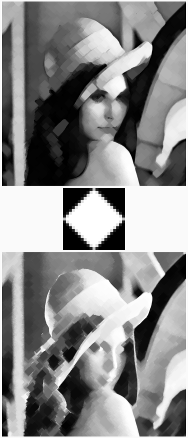
Higher Order Filters: an example

Opening γ_C



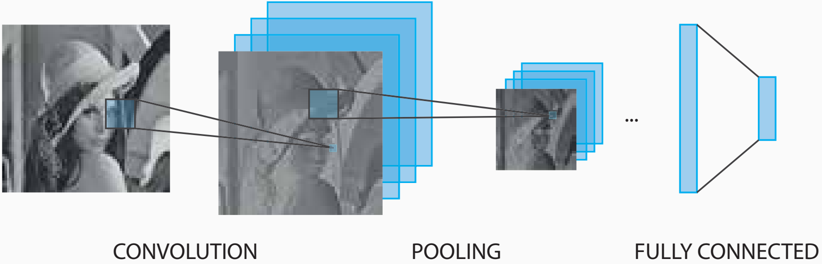
Closing ϕ_C

Opening γ_D



Closing ϕ_D

Convolutional Neural Networks



$$(f * g)(t) \triangleq \int_{-\infty}^{\infty} f(\tau) g(t - \tau) \, d\tau$$

Lecun et al., "Gradient-based learning applied to document recognition"

Masci, Angulo, and Schmidhuber, "A Learning Framework for Morphological Operators using Counter-Harmonic Mean"

A Fixed Dilation Layer

- Rather straightforward to implement with Tensorflow.[2]

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- We first experimented with fixed binary structuring elements (e.g. a vertical cross, an X).
- As such, we had to apply the previously mentioned transformation to our structuring elements.

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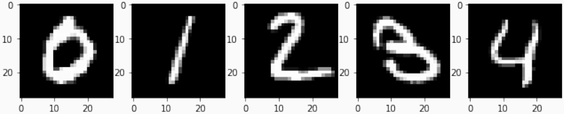
The Data

- For the purpose of these experiments, we used the MNIST Handwritten Digits[4] dataset.



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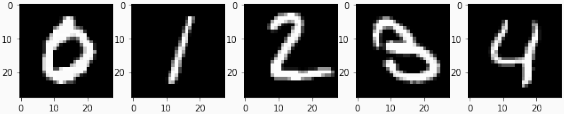

- 28×28 grayscale, $60\,000$ training images, $10\,000$ test images.

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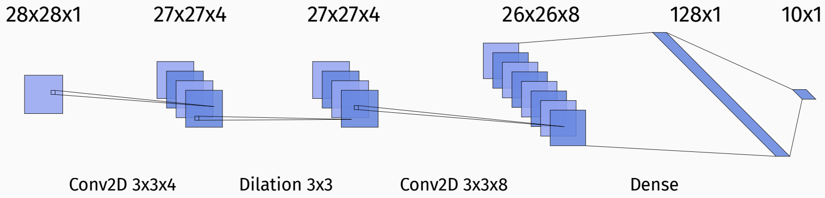
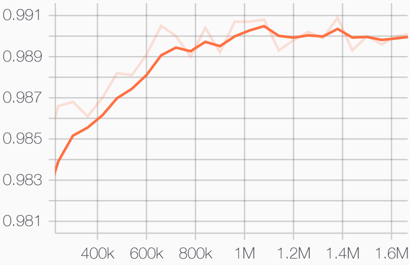
Testing a Dilation Layer on MNIST: Architecture



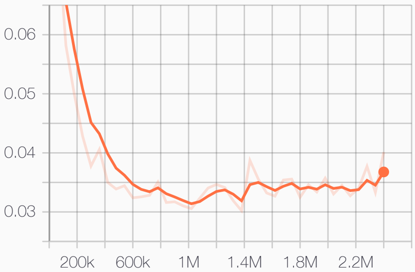
Also tested with a single level of convolution + dilation.

Testing a Dilation Layer on MNIST: Training

Trained over 40 epochs, with a batch size of 128.



Accuracy on the test set $a_{max} = 0.9914$



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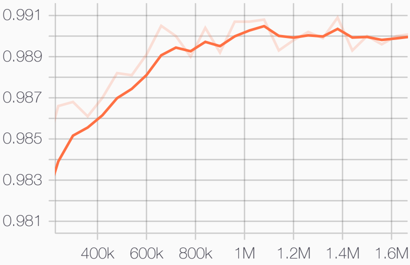

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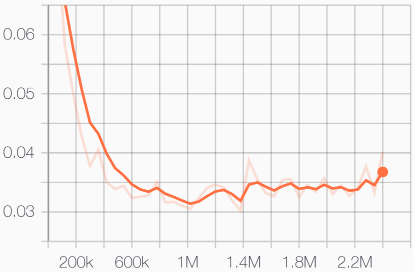

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A more classical architecture with a MaxPooling layer reaches similar scores: $a_{max} = 0.9923$, $l_{min} = 0.0252$ (~150 epochs).

Learning a Structuring Element

- Tensorflow[5] itself already implements the operations of Grayscale Dilation and Erosion[6]:

$$(f \oplus b)(x) = \sup_{y \in E}[f(y) + b(x - y)]$$

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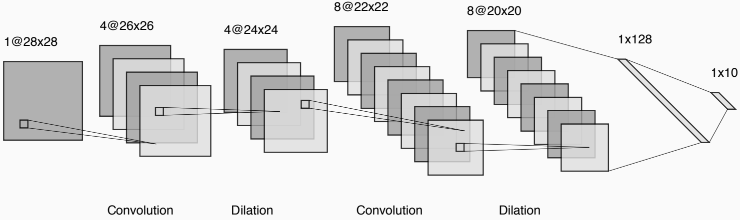

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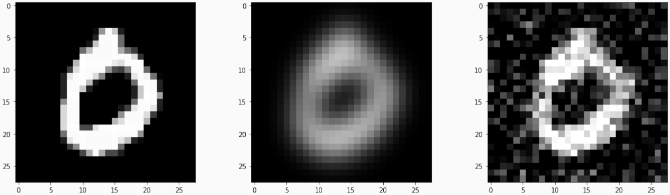

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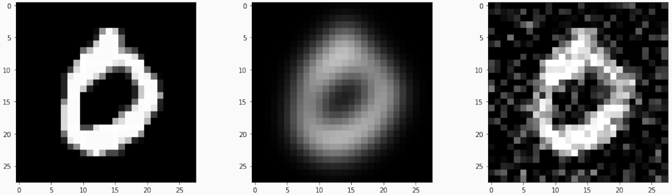
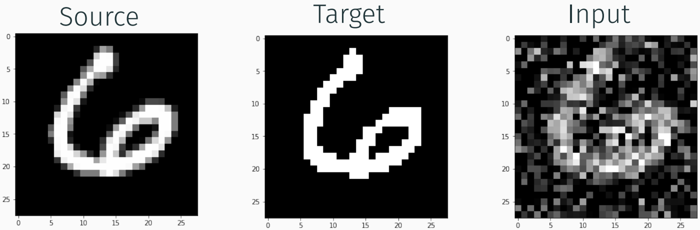


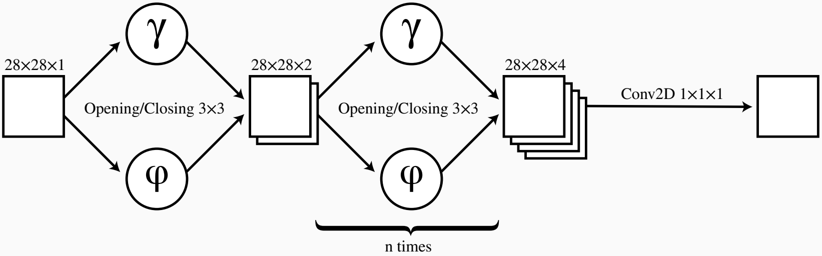
- Dilation layers do not appear to outperform similarly-shaped convolution layers.

  Val. acc. of 0.9792 vs 0.9819 after ~20 epochs, resp.

We keep our MNIST dataset, but this time we want to classify each input pixel into binary classes.



We can already get some results with very few parameters.



Total number of parameters: 135

Comparing Results

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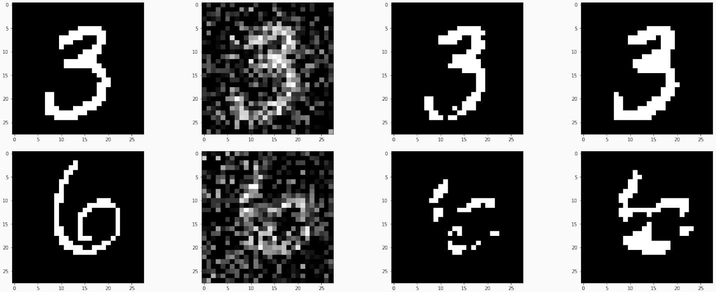

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- These experiments showed no significant benefit in leveraging morphological filters within the structure of standard convolutional neural networks.
- However, the "toy" challenge we set ourselves does not map to a real world problem.
- There is still much experimentation to be done in the field!

PConv

- We started experimenting with PConv layers[8]:

$$PConv(f; w, P)(x) = \frac{(f^{P+1} * w)(x)}{(f^P * w)(x)}$$

[8] Masci, Angulo, and Schmidhuber, "A Learning Framework for Morphological Operators using Counter-Harmonic Mean".

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  - $P < 0$ is a pseudo-erosion.
  - $P > 0$ is a pseudo-dilation.

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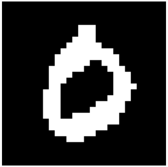
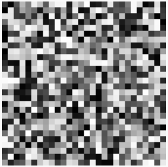
A first PConv result

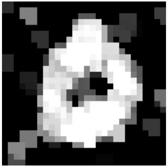


Target



Input

$P = -0.6663$



$P = 4.2216$

$P = -1.2166$

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- …with a bunch of edge cases involving `NaN`.
- The original paper proposes alternating between learning P and the weights w.

What's next?

- Move to a real problem, e.g. dHCP.[9]

[9] http://www.developingconnectome.org/project/

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- Move to a real problem, e.g. dHCP.[9]
- This will allow us to experiment with integrating morphological filters within much more complex architectures.

[9] http://www.developingconnectome.org/project/

References

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– ."A Learning Framework for Morphological Operators using Counter-Harmonic Mean". In: *CoRR* abs/1212.2546 (2012). arXiv: 1212.2546. URL: http://arxiv.org/abs/1212.2546.

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Thanks!

Any questions?

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Annex A: Building a Visualization Framework

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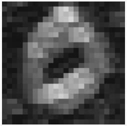
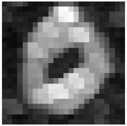
- We needed a way to inspect the output of each layer as well.
- We built a Keras Callback that saves weights and outputs after every batch.
- After training, these weights and outputs are transformed into image sequences:


Init


Batch 64


Batch 96


Batch 160


Batch 512