Getting a Morphological Tree of Shapes for Multivariate Images: Paths, Traps, and Pitfalls

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At a Glance

- **Motivation.** The Tree of Shapes (ToS) provides a high-level representation of the image structure and has many applications (here, grain filter = denoising).
- **Objective.** Extend the ToS computation on color images.
- **Problem.** A natural tree does not exist for color images (it requires a total order).
- **Contribution.** Review of standard approaches and new leads to extend the ToS on colors and get a single structure representing the image.

Approach 1. Total (pre)order based Tree of Shapes (Standard)

Idea. Define a new total (pre)-order on colors.

<table>
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<tr>
<th>Rank transformation</th>
<th>Total order: lexicographical ordering</th>
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<tbody>
<tr>
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<td>Total pre-orders: luminance / chrominance in La<em>b</em>, RGB, HSL</td>
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Total (pre)order considered:
- Total order: lexicographical ordering
- Total pre-orders: luminance / chrominance in La*b*, RGB, HSL

Problems.
- Use an arbitrary choice of total ordering
- Many reconstruction policies with pre-orders that yield very different results (visible color artefacts)

Approach 2. Distance based Tree of Shapes (Standard)

Idea. Extend the ToS algorithm with a propagation to the closest level in the front.

In gray level. $\lambda_{\text{next}} = \lambda_{\text{current}} \pm 1$

Distance-based approach.

$\lambda_{\text{next}} = \arg \min_{\lambda} \|\lambda_{\text{current}} - \lambda\|_2$

Pros.
- Very natural extension of the gray-level algorithm
- Yield the same ToS in the gray-level case
- “Look” morphological and few color artefacts

Approach 3. The Graph of Shapes

Idea. Merge the individual ToS into a single structure based on the inclusion that yields a graph.

Pros.
- It is a graph (not a tree) \rightarrow filtering and reconstruction are more challenging
- Yield the same ToS in the gray-level case
- Single but rich structure (best denoising score)

Qualitative evaluation through denoising

- Approach 1. Pre-order based ToS. PSNR=38.23
- Approach 2. Distance-based ToS. PSNR=37.88
- Approach 3. Graph of Shapes. PSNR=39.98
