A Tree of Shapes for Multivariate Images

Edwin Carlinet

Under the direction of
Thierry Géraud and Jean Serra

EPITA Research and Development Laboratory (LRDE)
Laboratoire d'Informatique Gaspard-Monge (LIGM)
École doctorale MSTIC

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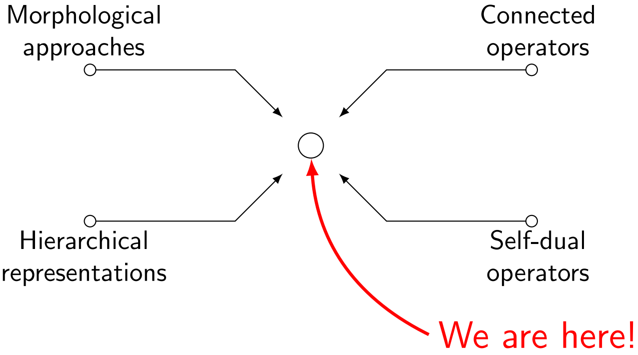
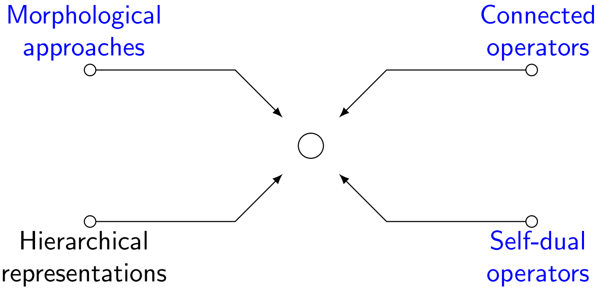
Context


Morphological approaches
Connected operators
Hierarchical representations
Self-dual operators
We are here!

Context


Morphological approaches
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Why is Mathematical Morphology (MM) interesting?

At the beginning.

Defined on binary images with *set theory* (dilation, erosion...)

→ Extended to grayscale images **through level set decomposition**.

Key property of morphological filters.

→ Contrast change invariance.

$$\varphi(G(u)) = G(\varphi(u)) \text{ for any increasing function } G$$

Useful property because morphological filters

→ enable to process low contrasted objects,

→ are quite robust to local changes of contrast.

On the need for contrast-invariance

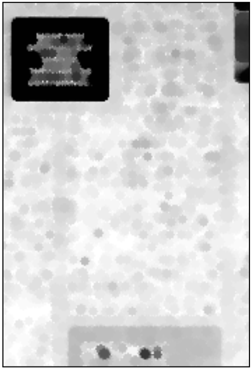

FIRST YOU GET PUBLICLY TRADED, THEN YOU DO COMPLIANCE.

ment suit outstanding against him by a former employee; two others have been dismissed—might scare off many investors. "He is very talented," says former J.C. Penney CEO Allen Questrom, a partner in private-equity firm Lee Equity Partners, "but he is a *meshugana* [read: nutty] type guy. It's very difficult to see him in a public sphere." (Charney, for his part, dismisses the talk about his reputation as "all tabloid.")

Charney's new bedfellows don't appear concerned, even if they're more likely to wear dark suits than American Apparel's colorful leggings. Endeavor's president, Jonathan Ledecky, is familiar with unusual financing vehicles; he founded U.S. Office Products, the once-hot roll-up that went bankrupt in 2001

SPAC Attack!

(a)



(b) Closing with SE

On the need for connected morphology.

Before 90's.

Morphology based on **Structuring Element**.
→ apriori about shapes we look for,
→ moves object boundaries.

From 90's.

Connected filtering (removing/preserving flat zones of the image).
→ does not move object boundaries.

On the need for connected morphology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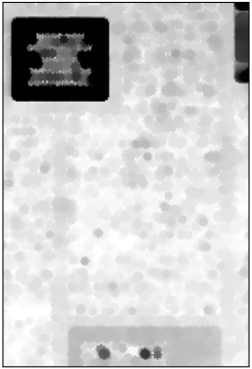
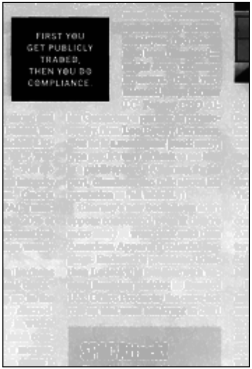


(b) Closing with a SE



(c) Area connected closing

On the need for self-duality.

Usage of MM filters depends on **prior knowledge**.

- *Pair of dual filters* (e.g., erosion/dilation, opening/closing. . .)
  $\rightarrow$ for light objects over dark background **xor** dark objects over light background.

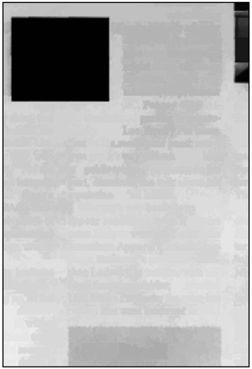
$$\varphi(\complement u) = \complement \psi(u)$$

- *Self-dual filters* (e.g. levelings, grain filters, . . .)
  $\rightarrow$ for light objects over dark background **and** dark objects over light background in a *symmetric way*.

$$\varphi(\complement u) = \complement \varphi(u)$$

On the need for constrast/inversion change invariance

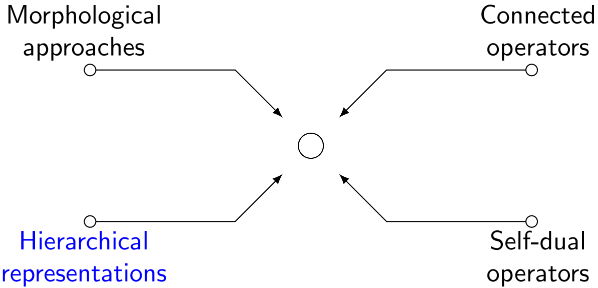
(a)



(b) Grain filter (self-dual)

(c) Residue

Context


Morphological approaches
Connected operators
Hierarchical representations
Self-dual operators

Why are hierarchical representations interesting?

Hierarchies are great:

- implicit multiscale analysis,
- (very) easy to compute/manipulate [GCCN13, CG14a, CG14b],
- versatile and efficient → many apps (see later).

Two kinds of hierarchies:

- Hierarchies of partitions

  VS

- Morphological trees

Hierarchies of partitions

Hierarchies of partitions → Hierarchical clustering, e.g.,

- hierarchy of quasi-flat zones,
- binary partition tree (BPT),
- watersheds,
- braids of partitions

Underlying requirement:

- **dissimilarity measure** between values,
- **region homogeneity**,
- functional. . .

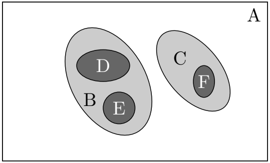
→ **extrinsic** parameter.

Morphological trees

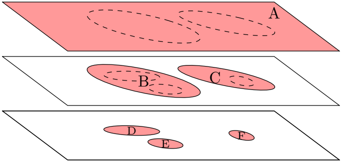
Morphological trees → Inclusion of the level sets.
e.g., min-tree, max-tree, Tree of Shapes (ToS)

Underlying requirement: **ordering** of values.

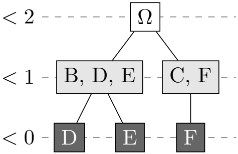
→ **intrinsic** parameter (the level).



(a) Original



(b) Lower level sets

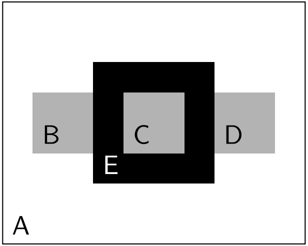


(c) Ex: Min-tree

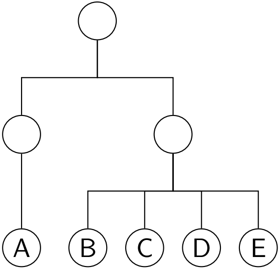
Comparison of both hierarchies (1/2)

Semantic differences illustrated

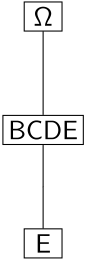
"A gray belt with a black belt buckle over a white shirt"



(a) Original



(b) A hierarchy of partition



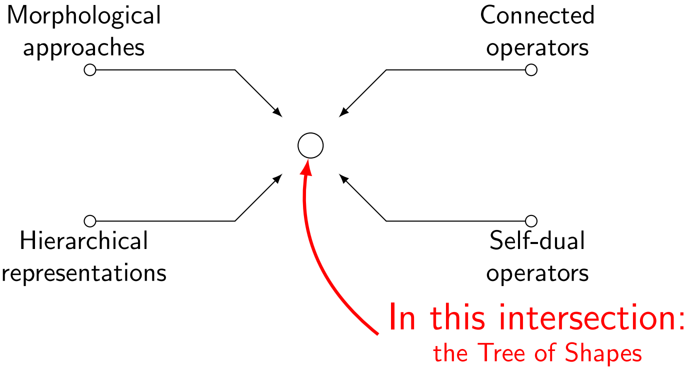
(c) Min-tree

The two hierarchies do not understand the scene in the same way.

Comparison of both hierarchies (2/2)

	Hier. of partitions	**Morpho. Trees**
Principle	Hierarchical clustering	Threshold decomposition
Construction	Merge **adjacent** regions	**Inclusion** of level sets
Grouping criterion	**Dissimilarity** measure	**Level** of the set
Cut in the hierarchy	Partition	Partial partition
Contrast change inv.	**No**	**Yes**
Contrast inversion inv.	**Yes**	**Yes for the ToS**

Context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In this intersection:
the Tree of Shapes

What is the ToS? (1/2)

Seen as a merge of min- and max- trees

Min-tree: connected components of lower level sets.

$$\Gamma^- = \{\, X \mid X \in \mathcal{CC}([u \leq \lambda]\,\}_\lambda$$

Max-tree: connected components of upper level sets.

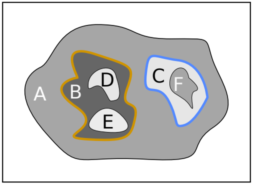
$$\Gamma^+ = \{\, X \mid X \in \mathcal{CC}([u \geq \lambda]\,\}_\lambda$$

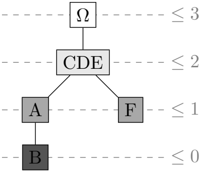
ToS: hole-filled ($\mathcal{H}$) connected components of lower/upper level sets.

$$\Gamma = \{\, \mathcal{H}(X) \mid X \in \Gamma^+ \cup \Gamma^- \,\}$$

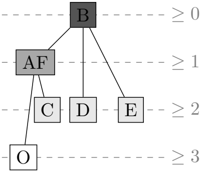
What is the ToS? (1/2)

Seen as a merge of min- and max- trees

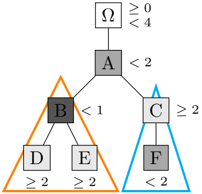




(a) Min-tree



(b) Max-tree

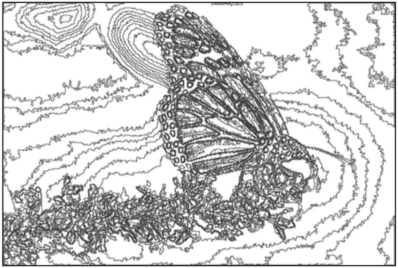


(c) ToS

The Tree of shapes (ToS) of u, formed by cavity-filled connected components of the level sets (self-dual representation)

What is the ToS? (2/2)

As the inclusion tree of level lines

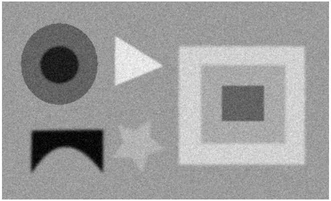
u and its level lines (every 5 levels)

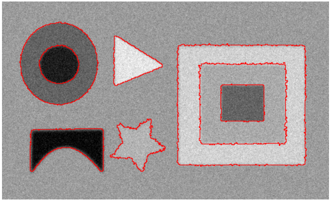
- The ToS encodes the inclusion of the image level lines,
- They are the contours of shapes.

Important properties for many apps → let's see !

Some applications with the ToS (1/2)

Object detection [XGN12].



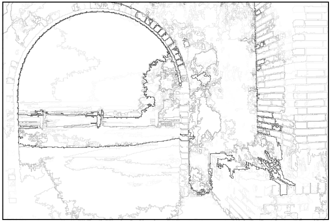


Segmentation & Simplification [XGN13].

Some applications with the ToS (2/2)

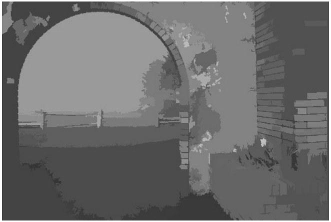
Hierarchical segmentation [XGN15].

(a) Original



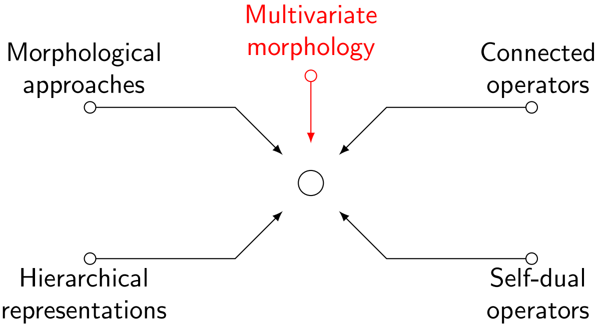
(b) Saliency map

(c) Fine segmentation



(d) Coarse segmentation

Context

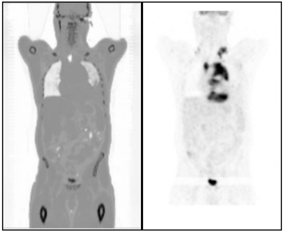

Multivariate morphology
Morphological approaches
Connected operators
Hierarchical representations
Self-dual operators

Motivations

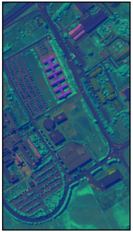
Why do we need multivariate morphology?

- Photography → natural images (colors),
- Medical imaging → multimodal images,
- Satellite imaging → multi/hyper-spectral images,
- RGB-D images, . . .

(a) Natural image



(b) PET + CT



(c) Pavia (3 bands as RGB)

Morphological trees & multivariate images

There is no "natural" ordering of vectors (multivariate data).

$\rightarrow$ The ToS is **not defined** for multivariate images :(

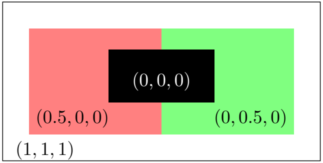
Problem: We *still* want a ToS for multivariate images!

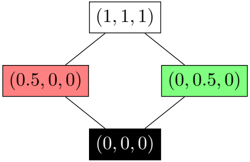
Outline

What about colors (1/2) ?

From a level set point of view...

Problem. Level sets from partial order overlap, $\Rightarrow$ a DAG.





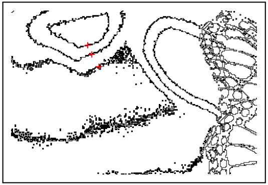
$[u \leq (0.5, 0, 0)]$ and $[u \leq (0, 0.5, 0)]$ are overlapping (via $[u \leq (0, 0, 0)]$)

$\rightarrow$ Not a tree

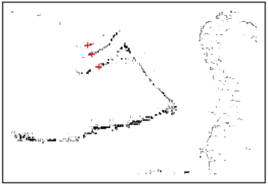
What about colors (2/2) ?

From a level line point of view...

(a) Original image



(b) Iso-levels at three points on the gray level image.



(c) Iso-colors at the same three points on the color image.

In gray levels. iso-levels form closed curves from which an inclusion tree makes sense.

In colors. iso-colors are not closed. → No inclusion tree

What people may do? (1/2)

1. Pretend it is a gray level image.
$\rightarrow$ And just loose important geometric information

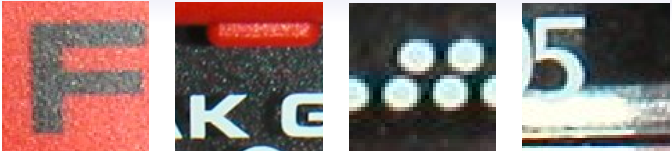
2. Do a marginal processing.
$\rightarrow$ And just hope everything is going to be alright on boundaries

3. Impose a total ordering on values.
$\rightarrow$ Might be OK but many total orderings and reconstruction strategies exist (more or less sensible).

4. Compute the Component-Graph [PN09] extended to Shapes
$\rightarrow$ And just take a week off if you want it computed on the whole image

Comparing on image simplification with classical approaches

Original Crops



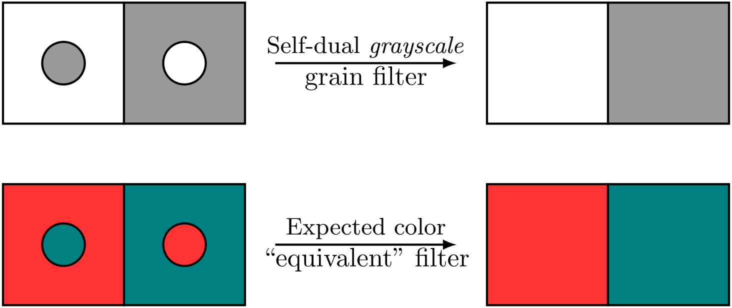
Standard Approaches

Please wait.
Processing. . .

Gray-level | Total-preorder [LE12] | Marginal (3 trees) | Component-graph

Ours

Do we really need to care about **ordering** values?
And, do we really need to care about **values themselves**?



→ Only the inclusion matters.

Outline

General Overview

Btw. do we really need to care about **ordering** values?
and do we really need to care about **values themselves**?

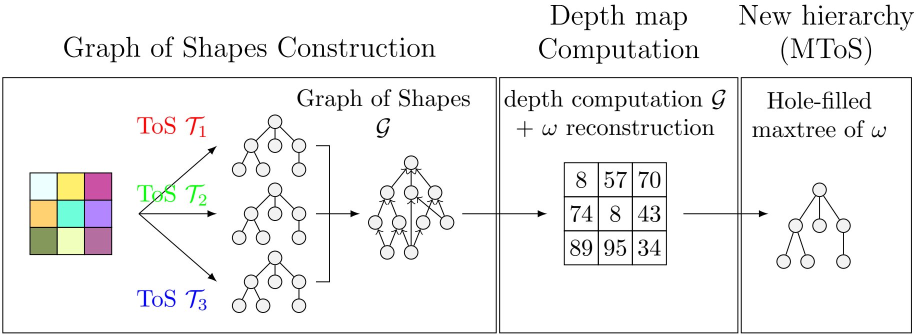
$\rightarrow$ Only the inclusion matters.

Key Idea.
Ordering the pixels w.r.t. to their **level of inclusion** instead of their values.

Level of inclusion = **depth** in the "hierarchy"

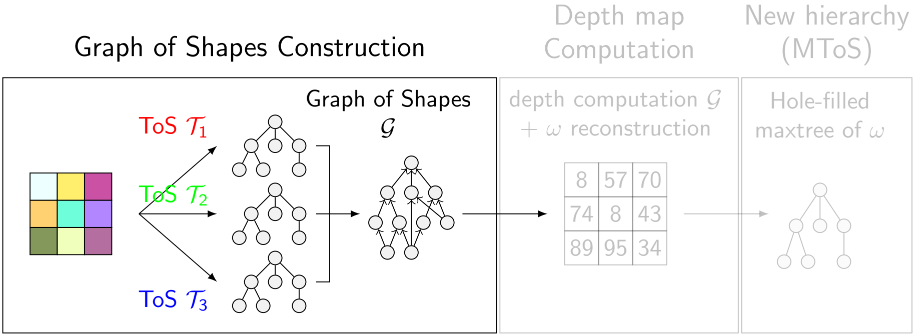
General overview

Scheme of the method

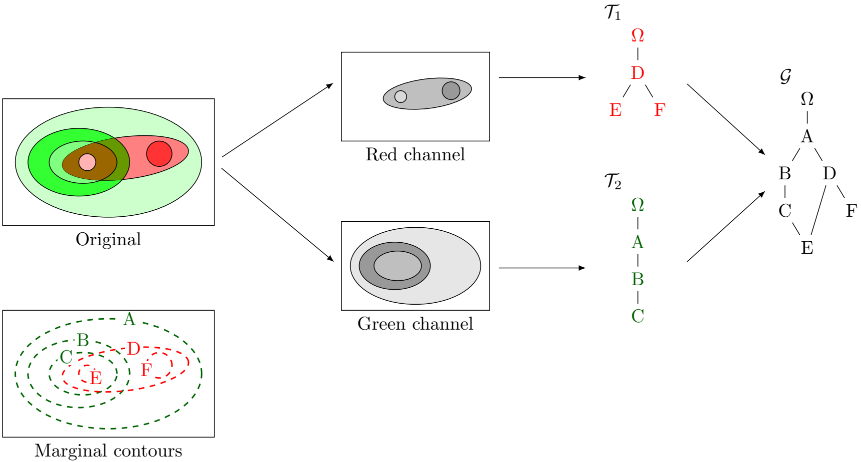


General Overview

Scheme of the method

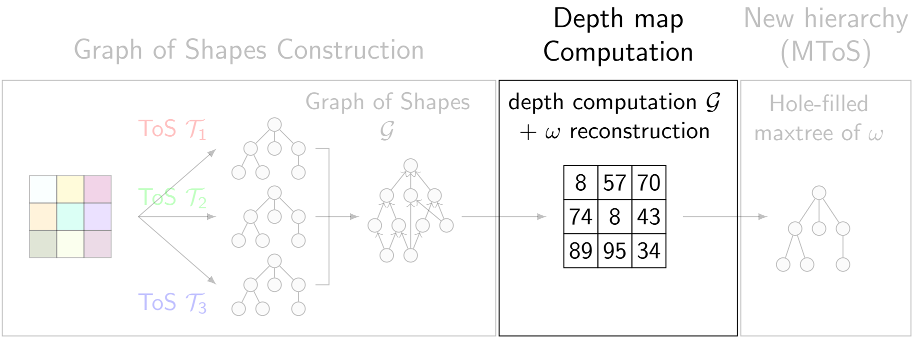


1. Get the primary shape set $\mathcal{S}$ from the marginal ToS.
2. Compute the Graph of Shapes $\mathcal{G} = (\mathcal{S}, \subseteq)$


Original
Marginal contours
A
B
C
D
E
F
Red channel
Green channel
$\mathcal{T}_1$
Ω
D
E
F
$\mathcal{T}_2$
Ω
A
B
C
$\mathcal{G}$
Ω
A
B
D
C
F
E

General Overview

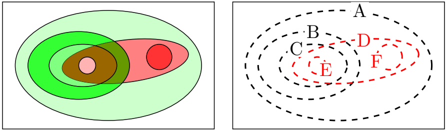
Scheme of the method

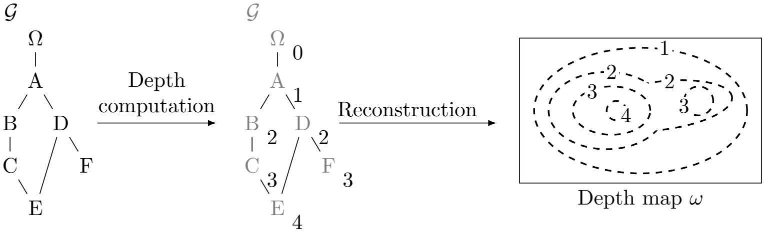


1. Compute the depth attribute over $\mathcal{G}$,
2. Reconstruct the depth map ω (back in the image space *again*).

$\omega(x)$ stands for:

> *The number of marginal level lines (that are nested) along the path from the border to the deepest shape that contains x.*

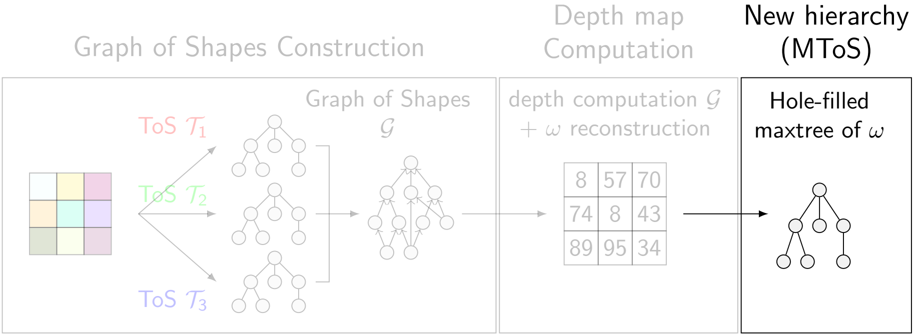

A
B
C
D
E
F


$\mathcal{G}$
Ω
A
B
D
C
F
E
Depth computation
$\mathcal{G}$
Ω 0
A 1
B 2
D 2
C 3
F 3
E 4
Reconstruction
1
2
2
3
3
4
Depth map ω

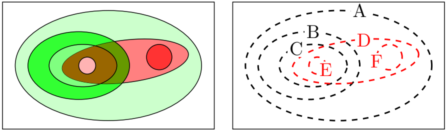
Some examples of depth map

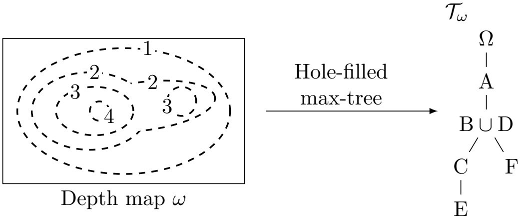
General Overview

Scheme of the method



1. Compute the cavity-filled maxtree of ω (new hierarchy $\rightarrow$ MToS).


A
B
C
D
E
F


1
2
2
3
3
4
Depth map ω
Hole-filled
max-tree
$\mathcal{T}_\omega$
Ω
A
$B \cup D$
C
F
E

Some properties

- The MToS defines a set of shapes **S** that satisfies:

(P1) Tree structure.

(P2) Maximal shape preservation.

It implies the Scalar ToS equivalence if u is scalar.

(P3) Marginal contrast change invariance.

Invariant to any increasing functions applied independently to u's channels.

(P4) Marginal contrast inversion invariance.

Invariant to any inversion applied independently to u's channels.

(Q) A "well-formed" tree. Not degenerated and $\#nodes \simeq \#pixels$.

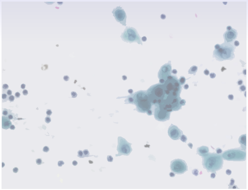
The ToS is not defined for multivariate images!

$\rightarrow$ The MToS is definable by some expected properties.
(marginal contrast change/inversion invariance)

$\rightarrow$ The MToS is definitely a morphological tree.

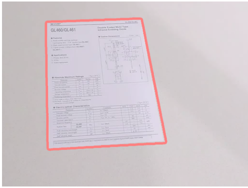
Outline

1. Grain Filters

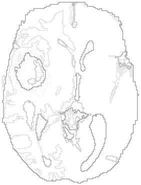


Shapings

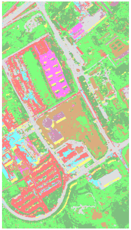
2. Interactive segmentation



3. Smartphone document detection



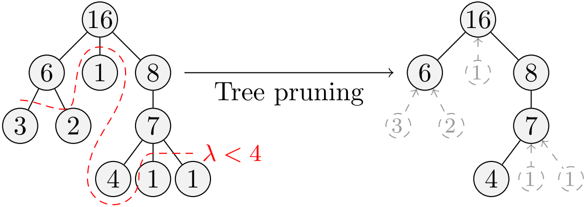
Hierarchical segmentation



Hyperspectral classification

"Color" Grain filters [CM02](1/2)

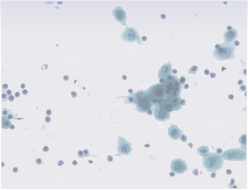
Method overview



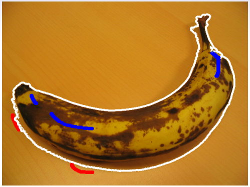
1. Compute the size attribute over the tree.
2. Threshold and collapse.

"Color" Grain filters (2/2): Document layout extraction

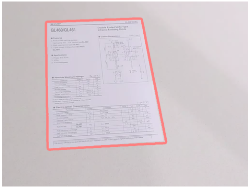
1. Grain Filters



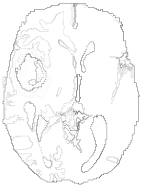
Shapings



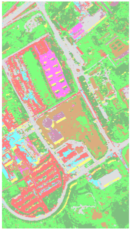
2. Interactive segmentation



3. Smartphone document detection



Hierarchical segmentation



Hyperspectral classification

MToS-based interactive segmentation

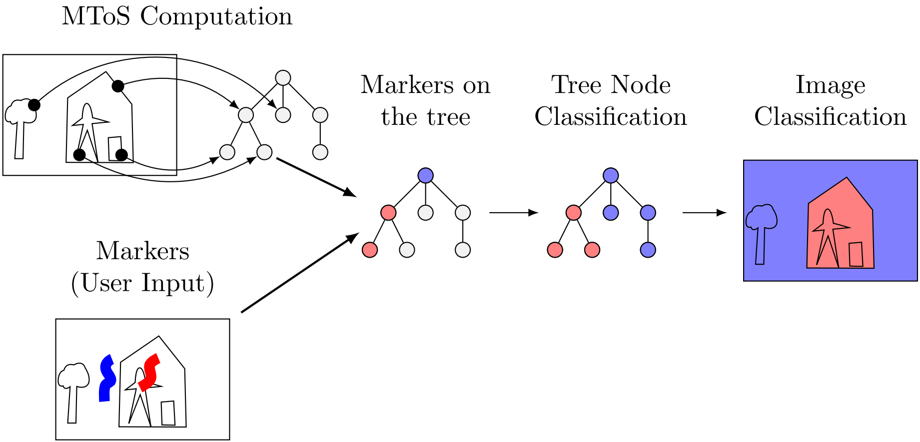
Base statement.

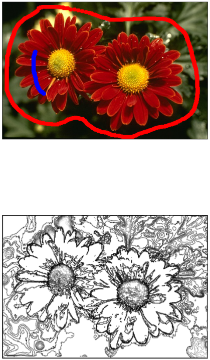
"Significant contours of objects in images coincide with segments of the image level lines"

Key Idea.

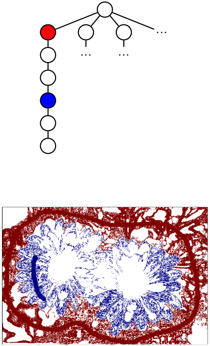
Classification of the level sets (shapes) to get a segmentation of the image.

Scheme of the method

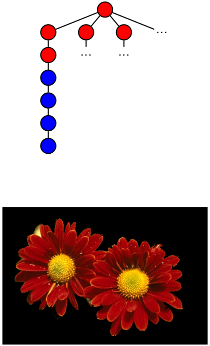




Input and
user scribbles

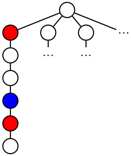


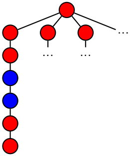
Node labeling
from scribbles

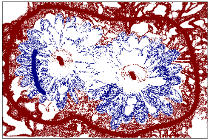


Tag propagation
and segmentation

Note: *Even if shapes are hole-free components, user can gets objects with holes if he wants to.*





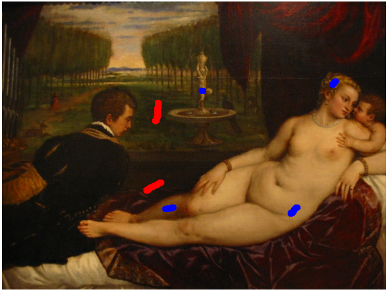


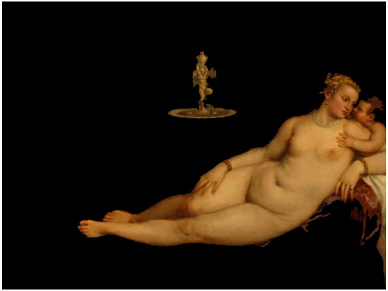
Input and user scribbles

Node labeling from scribbles

Tag propagation and segmentation

Some results





MToS-based interactive segmentation

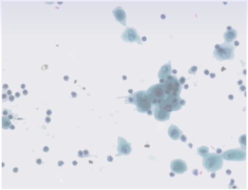
Our proposal.

Morphological method based on the classification of level sets through the MToS.

Pros.

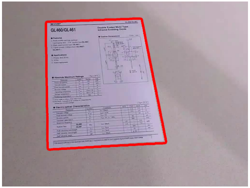
- No statistical learning and modeling of the background/foreground color distribution.
- $\rightarrow$ Few input scribbles required (compared with other state-of-the-art methods)

1. Grain Filters

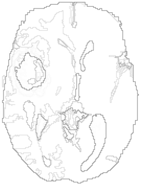


Shapings

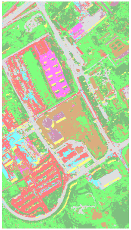
2. Interactive segmentation



3. Smartphone document detection

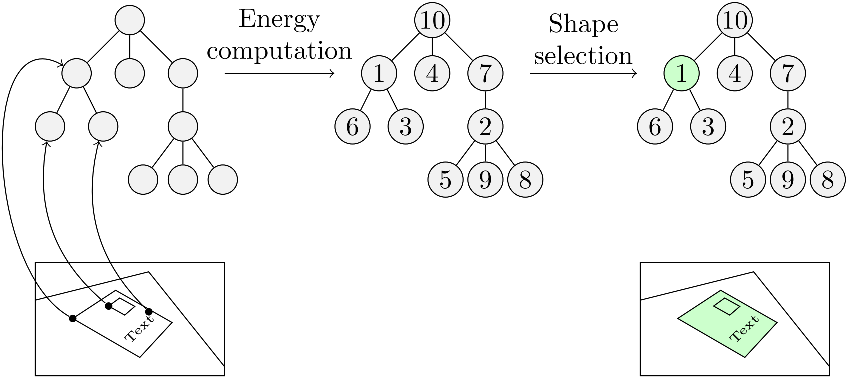


Hierarchical segmentation



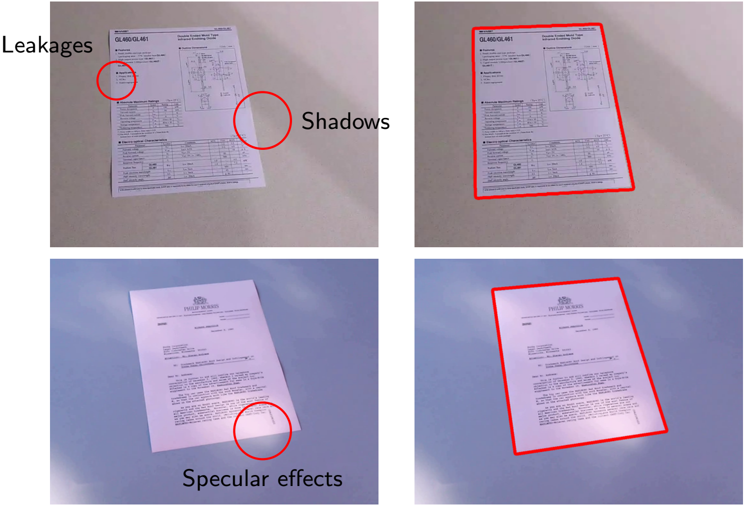
Hyperspectral classification

Document detection in videos (ICDAR SmartDoc'15)

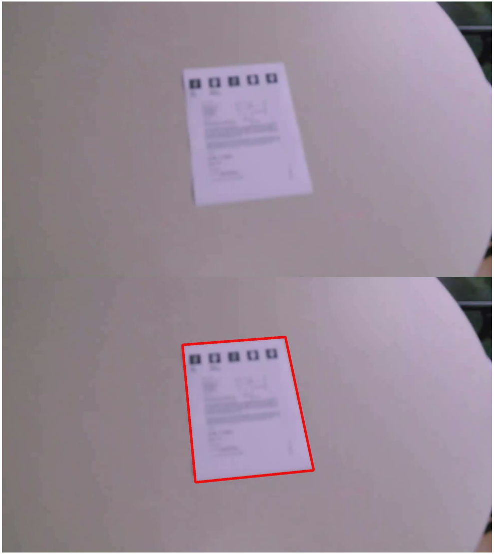


1. Valuate a two-term energy adpated to the object to detect. Here:
    - How much the shape boundary fits a quadrilateral
    - How "textured" the object is (texts & graphics)
2. Retrieve the shape with the lowest energy.

Document detection in videos (ICDAR SmartDoc'15)



Document detection in videos (ICDAR SmartDoc'15)



Document detection in videos (ICDAR SmartDoc'15)

Ranking	Method	Jaccard Index	Confidence Interval
1	LRDE (ours)	0.9716	[0.9710, 0.9721]
2	ISPL-CVML	0.9658	[0.9649, 0.9667]
3	SmartEngines	0.9548	[0.9533, 0.9562]
4	NetEase	0.8820	[0.8790, 0.8850]
5	A2iA run 2	0.8090	[0.8049, 0.8132]
6	A2iA run 1	0.7788	[0.7745, 0.7831]
7	RPPDI-UPE	0.7408	[0.7359, 0.7456]
7	SEECS-NUST	0.7393	[0.7353, 0.7432]

$\rightarrow$ Winner of the competition

Outline

Conclusion (1/4)

One-line problem.
Definitions of the ToS are not reusable
(neither as a merge of min-/max-trees, nor as the inclusion of level lines).

Key Idea.
Ordering pixels w.r.t. their level of **inclusion**
(no more on values).

Key result.
The definition of a MToS based on morphological properties
(the ones of the ToS once generalized).

Conclusion (2/4)

Extra results (in the PhD report):

- proofs relying on cubical complexes,
- quasi-linear algorithms for the ToS and MToS,
- algorithms for context-based attribute computation,
- ...

Conclusion (3/4)

Validation. Both qualitative and quantitative validation on many apps.

Reproductible research:

`http://publications.lrde.epita.fr/carlinet.15.itip`
→ Source code and binaries for the MToS.

Perspectives.

- Robustness (noise, “dynamics”, . . .)
- Study the properties of MToS-based filters (idempotence. . .).
- Idea extendable to “Multivariate” min- and max- trees.
- Some other applications. . .

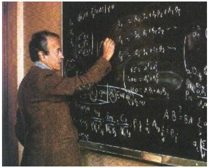
Conclusion (4/4)

Properties important for Computer Vision because the MToS:

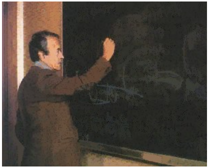
- enables multiscale analysis of the image,
- is a non-local representation of the image (level lines may spread over the entire image),
- is robust to **local** changes of contrast (*i.e.* changes of illumination),
- has no *prior knowledge* of the contrast “direction”.

First three images share the same MToS (level lines on the right image)

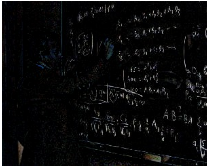
Questions?



(a) Original



(b) Multivariate Opening



(c) Residue

An extra app.: blackboard cleaning

List of publications I

Journal papers

- Edwin Carlinet and Thierry Géraud. "A Comparative Review of Component Tree Computation Algorithms". In: ITIP. Vol 23. (2014), pp. 3885–3895.
- Edwin Carlinet and Thierry Géraud. "MToS: A Tree of Shapes for Multivariate Images". In: ITIP. Vol 24. (2015), pp. 5330–5342.

Conference papers

- Edwin Carlinet and Thierry Géraud. "A comparison of many max-tree computation algorithms". In: ISMM. 2013, pp. 73–85.
- Edwin Carlinet and Thierry Géraud. "A Morphological Tree of Shapes for Color Images". In: ICPR. 2014, pp. 1133–1137.
- Edwin Carlinet and Thierry Géraud. "Getting a morphological Tree of Shapes for Multivariate Images: Paths, Traps and Pitfalls". In: ICIP. 2014, pp. 615–619.
- Edwin Carlinet and Thierry Géraud. "Traitement dimages multivariées avec larbre des formes." GT GeoDis (Reims Image 2014). In French. Nov. 2014.
- Edwin Carlinet and Thierry Géraud. "A Color Tree of Shapes with Illustrations on Filtering, Simplification, and Segmentation". In: ISMM. 2015, pp. 363–374.
- Edwin Carlinet and Thierry Géraud. "Morphological Object Picking Based on the Color Tree of Shapes". In: IPTA. 2015.
- Edwin Carlinet and Thierry Géraud. "Une approche morphologique de segmentation interactive avec l'arbre des formes couleur". GRETSI. 2015.

List of publications II

Co-authored papers

- T. Géraud, E. Carlinet, S. Crozet, and L. Najman. "A quasi-linear algorithm to compute the tree of shapes of n-D images." In: ISMM. 2013, pp. 98–110.
- R. Levillain, T. Géraud, L. Najman, and E. Carlinet. "Practical Genericity: Writing Image Processing Algorithms Both Reusable and Efficient". In: CIARP. 2014, pp. 70–79.
- Y. Xu, E. Carlinet, T. Géraud, and L. Najman. "Meaningful disjoint level lines selection". In: ICIP. 2014, pp. 2938–2942.
- T. Géraud, E. Carlinet, and S. Crozet. "Self-Duality and Digital Topology: Links Between the Morphological Tree of Shapes and Well-Composed Gray-Level Images". In: ISMM. 2015, pp. 573–584.
- Y. Xu, E. Carlinet, T. Géraud, and L. Najman. "Efficient Computation of Attributes and Saliency Maps on Tree-Based Image Representations". In: ISMM. 2015, pp. 693–704.

[AL07] E. Aptoula and S. Lefèvre.
A comparative study on multivariate mathematical morphology.
Pattern Recognition, 40(11):2914–2929, 2007.

[CG14a] E. Carlinet and T. Géraud.
A comparative review of component tree computation algorithms.
IEEE Transactions on Image Processing, 23(9):3885–3895, September 2014.

[CG14b] S. Crozet and T. Géraud.
A first parallel algorithm to compute the morphological tree of shapes of *n*D images.
In *Proc. of IEEE Intl. Conf. on Image Processing (ICIP)*, pages 2933–2937, Paris, France, 2014.

[CM02] V. Caselles and P. Monasse.
Grain filters.
Journal of Mathematic Imaging and Vision, 17(3):249–270, November 2002.

[GCCN13] T. Géraud, E. Carlinet, S/ Crozet, and L. Najman.
A quasi-linear algorithm to compute the tree of shapes of *n*D images.
In *Proc. of Intl. Symp. on Mathematical Morphology (ISMM)*, volume 7883 of *LNCS*, pages 98–110, Heidelberg, 2013. Springer.

[LE12] O. Lézoray and A. Elmoataz.
Nonlocal and multivariate mathematical morphology.
In *Proc. of IEEE Intl. Conf. on Image Processing (ICIP)*, pages 129–132, Orlando, USA, 2012.

[PN09] Nicolas Passat and Benoît Naegel.
An extension of component-trees to partial orders.
In *Proc. of IEEE Intl. Conf. on Image Processing (ICIP)*, pages 3933–3936. IEEE Press, 2009.

[XGN12] Yongchao Xu, Thierry Géraud, and Laurent Najman.
Context-based energy estimator: Application to object segmentation on the tree of shapes.
In *Proc. of IEEE Intl. Conf. on Image Processing (ICIP)*, pages 1577–1580, 2012.

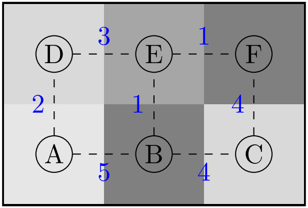
[XGN13] Y. Xu, T. Géraud, and L. Najman.
Salient level lines selection using the Mumford-Shah functional.
In *Proc. of IEEE Intl. Conf. on Image Processing (ICIP)*, pages 1227–1231, Merlbourne, Australia, 2013.

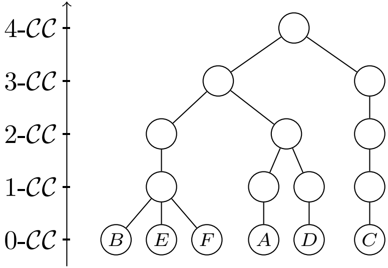
[XGN15] Yongchao Xu, Thierry Géraud, and Laurent Najman.
Hierarchical image simplification and segmentation based on mumford-shah-salient level line selection.
Pattern Recognition Letters, 2015.
submitted.

Hierarchies of partitions (1/2)

Hierarchies of partitions → Hierarchical clustering
e.g., hierarchy of quasi-flat zones, binary partition tree (BPT)...

Underlying requirement: **dissimilarity measure** between values.

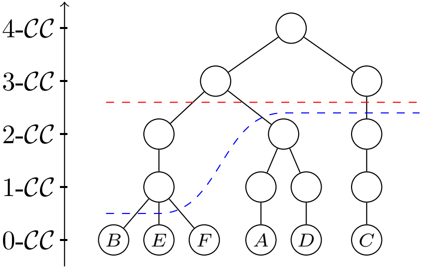


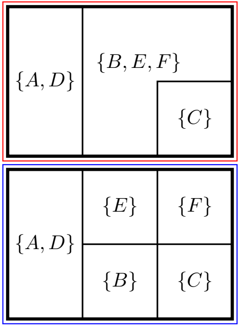


The hierarchy of quasi-flat zones

Hierarchies of partitions (2/2)

- Any cut in the hierarchy is a partition (segmentation) of the image.
  $\rightarrow$ everything is object (no distinction object/background)



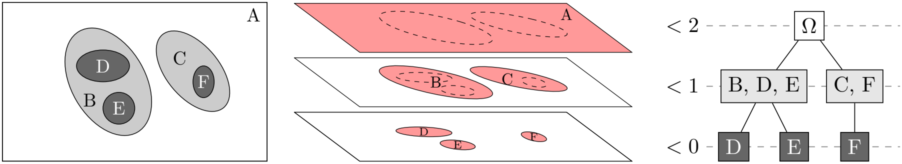


Horizontal and non-horizontal cuts in the hierarchy.

Morphological trees (1/2)

Morphological trees → Inclusion of the level sets.
e.g., min-tree, max-tree, Tree of Shapes (ToS)

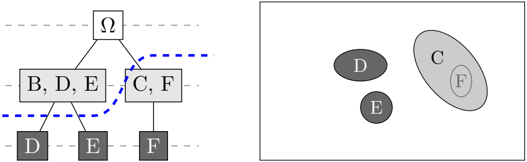
Underlying requirement: **ordering** of values.



(a) Original (b) Lower level sets (c) Min-tree

Morphological trees (2/2)

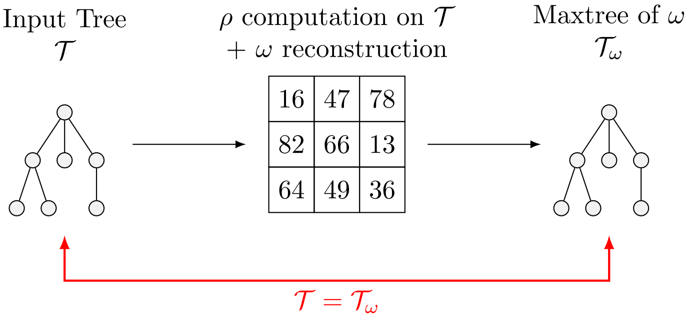
- Any cut in the tree is a partial partition of the image.
  $\rightarrow$ selection of objects (distinction object/background)



A cut in the tree and its corresponding objects.

Rationale (1/2)

Idea 1. $\mathcal{T}+$ dec. attribute ρ + restitution ω_ρ + Maxtree $\mathcal{T}_{\omega_\rho} = \mathcal{T}$



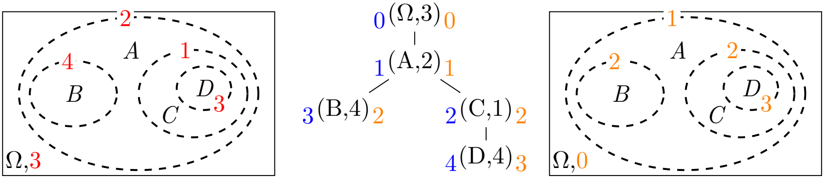
Rationale (2/2)

Idea 1. $\mathcal{T}$+ dec. attribute ρ + restitution ω_ρ + Maxtree $\mathcal{T}_{\omega_\rho} = \mathcal{T}$

Idea 2. u level lines $= \omega_{TV}$ level lines (TV from the border).

$= \omega_{CV}$ level lines (Counted variations).

$\rightarrow$ ToS of u = Maxtree of ω_{CV}



(a) u and its level lines. (b) The ToS of u and ρ_{CV} (orange). (c) The level lines of ω_{CV}.

Conclusion. Use the depth attribute on $\mathcal{G}$ and reconstruct.

$\omega_{CV}(x)$ stands for:

> *The number of marginal level lines (that are nested) along the path from the border to the deepest shape that contains x.*

Differences with "Shape" component-graphs

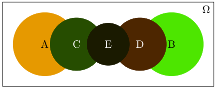
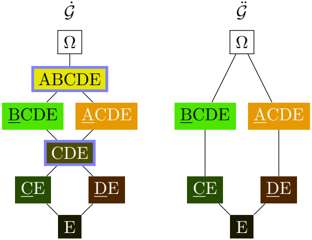
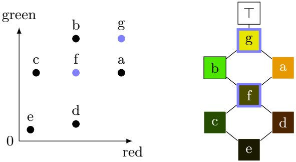


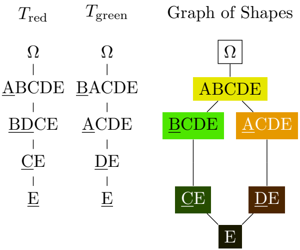
Image u



"Shape" Component graphs

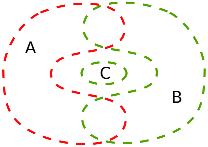


Lattice of the values

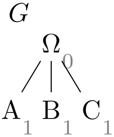


The graph of shapes

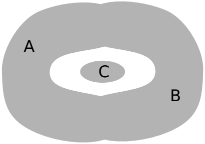
On the need of the saturation



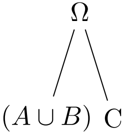
(a) Original.



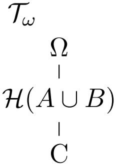
(b) Marginal ToS and GoS.



(c) ω map.



(d) Maxtree of ω (w/o cavity filling).

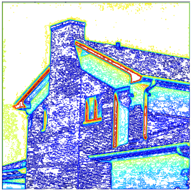


(e) Final MToS (with cavity filling).

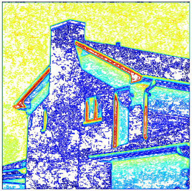
Effect of noise

(a) House

(b) House (red channel) + Gaussian Noise ($\sigma = 20$, green channel)



(c) Level lines of the tos of (a). Level lines: 24k, avg. depth: 37, max. depth: 124.

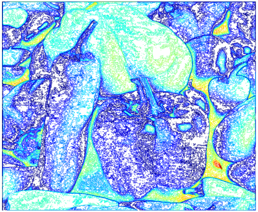


(d) Level lines of the ctos of (b). Level lines: 48k, avg. depth: 48, max. depth: 127.

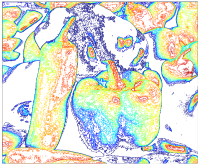
Effect of the dynamic

(a) Peppers (only red/green)

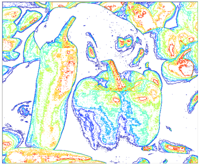
(b) Peppers (only red/green) with green sub-quant. to 10 levels



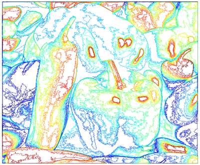
(c) Level lines of the red channel of (a) and (b)



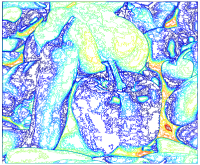
(d) Level lines of the green channel (a)



(e) Level lines of the green channel (b)



(f) Level lines of the ctos of (a)



(g) Level lines of the ctos of (b)