

# FAST DISJOINT LEVEL LINES SELECTION

Yongchao Xu<sup>1,2</sup>, Thierry Géraud<sup>1,2</sup>

Laurent Najman<sup>2</sup>

<sup>1</sup> EPITA Research and Development Laboratory  
(LRDE)  
14-16, rue Voltaire  
FR-94276 Le Kremlin-Bicêtre, France

<sup>2</sup> Université Paris-Est,  
Laboratoire d'Informatique Gaspard-Monge (LIGM),  
A3SI, ESIEE Paris, Cité Descartes, BP 99  
FR-93162 Noisy-le-Grand, France

## ABSTRACT

Many methods relying on the morphological notion of *shapes*, (i.e., connected components of level sets) have been proved to be very useful for pattern analysis and recognition. Selecting meaningful level lines (boundaries of level sets) yields to simplify images while preserving salient structures. Many image simplification and/or segmentation methods are driven by the optimization of an energy functional, for instance the Mumford-Shah functional. In this article, we propose an efficient shape-based morphological filtering that very quickly compute to a locally (subordinated to the tree of shapes) optimal solution of the piecewise-constant Mumford-Shah functional. Experimental results demonstrate the efficiency, usefulness, and robustness of our method, when applied to image simplification, pre-segmentation, and detection of affine regions with viewpoint changes.

**Index Terms**— Level lines, Tree of shapes, Energy minimization, Pre-segmentation, Morphological shaping.

## 1. INTRODUCTION

In natural images, meaningful contours are usually smooth and well-contrasted. Recently, many authors claim that significant contours of objects in images coincide with segments of the image level lines [1]. Each connected level line is the contour of a *level set*, or *shape*, a connected set of pixels without holes. The inclusion relationship of level sets allows for representing an image by a tree, called a *tree of shapes* [2], which is invariant to contrast changes. Image simplification or segmentation can then be defined by selecting meaningful level lines in that tree. That subject has been investigated in the past ten years by [3, 4, 5, 6]. In [7] Lu et al. propose also a tree simplification method for image simplification purpose using the binary partition tree [8] and a knee function.

Following the seminal work of Mumford and Shah [9], finding relevant contours is often tackled thanks to an energy-based approach, as a compromise between some image-driven force (image contrast along contours, data fidelity, etc.) and the regularity of contours. Minimizing the Mumford-Shah functional tends to find a simplified or segmented image

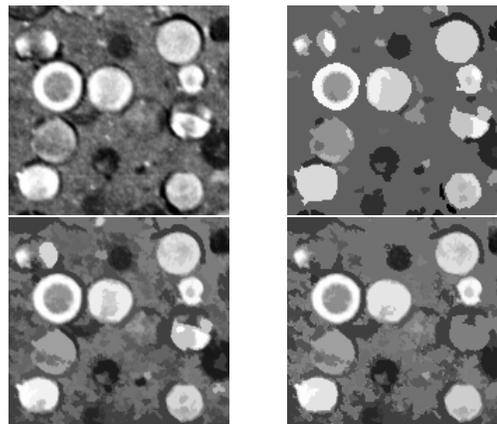


Fig. 1: Image in introduction.

into regions. Curve evolution methods [10, 11] are usually used to solve such an energy minimization problem. They have solid theoretical foundations, yet they are often computationally expensive.

In this paper we propose to formalize the piecewise-constant Mumford-shah functional on an image, subordinated to the tree of shapes of this image. The selection of the salient level lines corresponds to a meaningful locally optimal solution of the energy minimization problem. The main contribution is the proposition of an efficient greedy algorithm which takes into account the meaningfulness of the set of level lines. Simply put, a level line is easier to remove when it has a low degree of meaningfulness and when it favors a great decreasing of energy. Our algorithm drives very fast to a relevant local optimum in the sense that no more level lines can be removed while decreasing energy. The reason why we claim that we reach a relevant optimum is that meaningful level lines are hard to be removed during the proposed process. Note that our method actually belongs to the class of *morphological shapings* described in [12].

In [13], the authors proposed an efficient greedy algorithm to minimize the Mumford-Shah functional on a certain hierarchy, which leads to a global optimal segmentation on that

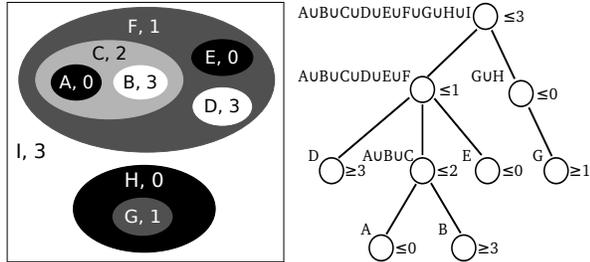


Fig. 2: An image (left) and its tree of shapes (right).

hierarchy. In [14], the authors gave a detailed review of the tree (including the tree of shapes) filtering strategies. The works in [15] and [16] are closest to what we propose here. They both select meaningful level lines for image simplification and segmentation purpose using the piecewise-constant Mumford-Shah functional. In [15] the whole image domain is initially considered as a single region; level lines of the tree of shapes are browsed from root to leaves and are successively removed until the functional cannot decrease anymore. However, this top-down decision is based upon a non-significant energy variation since it is computed from the very few pixels lying between a shape and their immediate sub-shapes. Actually, our work is related to the one described in [16], where at each removal step, the level line which decreases the most the functional is selected. As a consequence, the iterative process of [16] requires not only to compute a lot of information to be able to update the functional value after each level line suppression, but also to find at each step, among all remaining level lines, the one candidate to the next removal. Hence [16] is computationally expensive, while what we propose here is fast.

The rest of this paper is organized as follows. Some background information about the Mumford-Shah functional and the tree of shapes is provided in Section 2. Our proposed method is detailed in Section ???. In Section ???, we present some experimental results. We then conclude and give some perspectives in Section 4.

## 2. BACKGROUND

### 2.1. The Tree of Shapes

For any  $\lambda \in \mathbb{R}$  or  $\mathbb{Z}$ , the upper level sets  $\mathcal{X}_\lambda$  and lower level sets  $\mathcal{X}^\lambda$  of an image  $f$  are respectively defined by  $\mathcal{X}_\lambda(f) = \{p \in \Omega \mid f(p) \geq \lambda\}$  and  $\mathcal{X}^\lambda(f) = \{p \in \Omega \mid f(p) \leq \lambda\}$ . Both upper and lower level sets have a natural inclusion structure:  $\forall \lambda_1 \leq \lambda_2, \mathcal{X}_{\lambda_1} \supseteq \mathcal{X}_{\lambda_2}$  and  $\mathcal{X}^{\lambda_1} \subseteq \mathcal{X}^{\lambda_2}$ , which leads to two distinct and dual representations of an image, the max-tree and the min-tree [17].

Another tree has been introduced in [2]. A *shape* is defined as a connected component of an upper or lower level

set where its holes have been filled in. Thanks to the inclusion relationship of both kinds of level sets, the set of shapes gives a unique tree, called *tree of shapes*. This tree features an interesting property: it is invariant to contrast changes. To put it differently, it is a self-dual, non-redundant, and complete representation of an image. Furthermore, such a tree inherently embeds a morphological scale-space (the parent of a node/shape is a larger shape). An example on a simple image is depicted in Fig. 2.

### 2.2. Computation algorithm

## 3. DISJOINT LEVEL LINES SELECTION

Each node of the topographic map represents a connected component without holes, the boundaries of the connected components are the level lines. It has been shown that significant contours of objects in images coincide with segments of the level lines [1]. In natural images, the number of level lines is in the same order of number of pixels, which make the topographic map difficult to be visualized. In fact, many level lines share some parts in common, which is to say that two neighboring level lines in the topographic map may only differ in a few pixel edges (in the case such that the level lines are materialized into pixel edges, i.e., 1-faces). In this section, we provide an efficient algorithm to select a set of disjoint level lines from the topographic map, such that any two selected level lines do not intersect. This algorithm yields a simplified image  $f'$  reconstructed from those selected level lines. The main structure of the topographic map of the original image  $f$  can be easily visualized from this simplified image  $f'$ . In fact, this simplified image is a well-composed image which is usually obtained by doubling the image size. In our case, the generated well-composed image  $f'$  has the same size as the original image  $f$ . The core of this algorithm will be detailed in Section 3.1. Then in Section 3.2, we will depict the algorithm and illustrate several rules of disjoint level lines selection.

### 3.1. Incompatible nodes preparation

The core of the algorithm for selecting a set of disjoint level lines relies on an image defined on the nodes that we call *last\_not\_allowed*, that encodes for each node  $\mathcal{N}$  the highest ancestor node  $\mathcal{N}_a$  for which they still share some pixel edges. When this image is available, for each level line  $\partial\mathcal{N}$ , we are able to predict a set of incompatible nodes of this node  $\mathcal{N}$ . In fact, if a node  $\mathcal{N}$  is selected, we cannot select its ancestor nodes  $\mathcal{N}_a$  till the one encoded by the image *last\_not\_allowed*( $\mathcal{N}$ ), and the descendant nodes  $\mathcal{N}_d$  for which the selected node  $\mathcal{N}$  is in the subbranch starting from  $\mathcal{N}_d$  till the ancestor node given by *last\_not\_allowed*( $\mathcal{N}_d$ ) cannot be selected either. Based on this principle, we are able to select a set of nodes such that any pair of selected nodes are not incompatible, i.e., they are disjoint level lines.

Now let us show how to compute the image *last\_not\_allowed*. To compute it efficiently, we need another image *depth* storing the depth of each node on the tree (starting from 0 for the root node). For each node  $\mathcal{N}$  apart from the root node:

$$depth(\mathcal{N}) = depth(parent(\mathcal{N})) + 1 \quad (1)$$

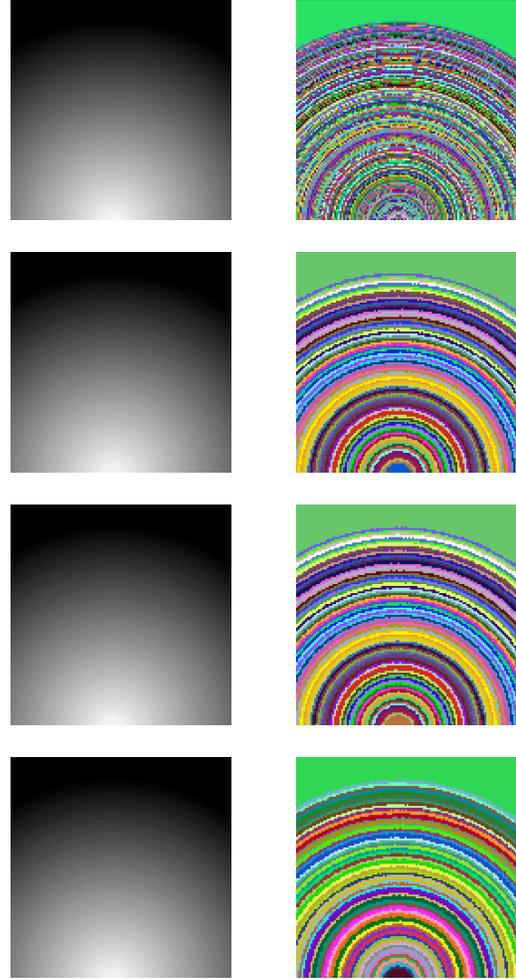
And thanks to the image *appear* and *vanish* used in Section ??, the computation of the image *last\_not\_allowed* can be achieved with the following process: For each pixel edge  $e$ , let the node  $a = appear(e)$  and  $v = vanish(e)$  be respectively the first node where  $e$  is on its boundary and the first node where  $e$  is no longer on its boundary. Let  $v_c$  be the child of  $v$  being also an ancestor node of  $a$ . We can update the image *last\_not\_allowed* for the nodes on the sub-branch starting from  $a$  to  $v'$ , which cannot be lower than  $v'$ . That is to say the *depth* of the nodes of the image *last\_not\_allowed* on  $a$  to  $v'$  cannot be smaller than  $depth(v')$ . The algorithm to compute this image *last\_not\_allowed* is depicted in Algorithm ??, where *vec\_nodes* stands for the sorted set of nodes of the topographic map in the tree descending order, and  $N_{\mathcal{N}}$  is the total number of nodes.

### 3.2. Final disjoint nodes selection

Once the image *last\_not\_allowed* is available, we are able to select a set of nodes such that any pair of nodes are compatible. Now let us detail how to process the choices according to a pre-computed order of nodes selection, such as top-down or bottom-up order. We propagate all the nodes in the pre-computed order, for each node  $\mathcal{N}$ , we perform two operations:

- 1) Check if there is an ancestor node being incompatible with  $\mathcal{N}$  has already been selected. This verification is based on the image *last\_not\_allowed*.
- 2) If none of the incompatible ancestor nodes given by the image *last\_not\_allowed*( $\mathcal{N}$ ) is selected, then select this node  $\mathcal{N}$ , and disable all its incompatible ancestor nodes. This step makes sure that we don't have to check the incompatibility with the descendants for a given node. Otherwise, do nothing.

Finally, the algorithm for the disjoint level lines selection is depicted in Algorithm ???. We have experienced three different orders for disjoint level lines selection: top-down selection order (from root node to the leaf nodes), bottom-up (from the leaf nodes to the root node), and meaningfulness decreasing order, e.g., average of gradient's magnitude along the level lines defined by Eq (??). Two examples of such disjoint level lines selection are illustrated in Figure 5 and Figure 4, where a grain filter [?] is also applied to not select level lines of regions being too small. The main structure of the topographic map can be easily visualized through the simplified image reconstructed from the set of disjoint level lines.



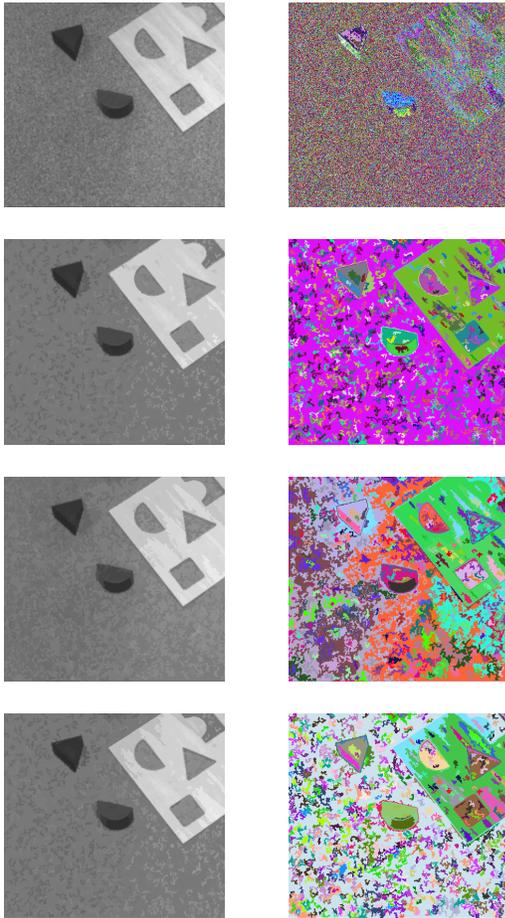
**Fig. 3:** Illustration of the disjoint level lines selection with different orders. From top to down: original image, bottom-up, top-down, average of gradient's magnitude decreasing. Left: grayscale image; Right: corresponding randomly colored image.

#### 4. CONCLUSION AND PERSPECTIVES

In this paper, we presented an efficient morphological shaping to salient level lines selection, based on the minimization of the piecewise-constant Mumford-Shah functional. Our major contribution is to rely on a *meaningful* ordering of level lines in order to minimize this energy functional on the tree of shapes. As a consequence, the proposed greedy algorithm converges to a relevant local optimum very quickly compared with the similar work of Ballester et al.. We have shown that the proposed method allows for strongly simplifying images while preserving their salient structures. We have seen that a strong property of our proposal is its robustness to noise and to viewpoint changes. Furthermore simplification results can be used as pre-segmentations that are suitable for object recognition, scene analysis, or practical shape matching [18]. The authors are currently investigating some applications of the proposed simplification method. In addition, a major perspective of this work is to rely on shape-based morphology [12] to make this method hierarchical.

#### 5. REFERENCES

- [1] V. Caselles, B. Coll, and J. Morel, "Topographic maps and local contrast changes in natural images," *Intl. Journal of Computer Vision*, vol. 33, no. 1, pp. 5–27, 1999.
- [2] P. Monasse and F. Guichard, "Fast computation of a contrast-invariant image representation," *IEEE Trans. on Image Processing*, vol. 9, no. 5, pp. 860–872, 2000.
- [3] A. Pardo, "Semantic image segmentation using morphological tools," in *Proc. of ICIP*, 2002, pp. 745–748.
- [4] F. Cao, P. Musé, and F. Sur, "Extracting meaningful curves from images," *JMIV*, vol. 22, pp. 159–181, 2005.
- [5] J. Cardelino, G. Randall, M. Bertalmio, and V. Caselles, "Region based segmentation using the tree of shapes," in *Proc. of ICIP*, 2006, pp. 2421–2424.
- [6] Y. Xu, T. Géraud, and L. Najman, "Context-based energy estimator: Application to object segmentation on the tree of shapes," in *ICIP*, 2012, pp. 1577–1580.
- [7] H. Lu, J.C. Woods, and M. Ghanbari, "Binary partition tree analysis based on region evolution and its application to tree simplification," *IEEE Transactions on Image Processing*, vol. 16, no. 4, pp. 1131–1138, 2007.
- [8] P. Salembier and L. Garrido, "Binary partition tree as an efficient representation for image processing, segmentation and information retrieval," *IEEE Trans. on Image Processing*, vol. 9, no. 4, pp. 561–576, 2000.
- [9] D. Mumford and J. Shah, "Optimal approximations by piecewise smooth functions and associated variational



**Fig. 4:** Another illustration of the disjoint level lines selection with different orders. From top to down: original image, bottom-up, top-down, average of gradient's magnitude decreasing. Left: grayscale image; Right: corresponding randomly colorized image.



**Fig. 5:** An image in result

problems,” *Communications on Pure and Applied Mathematics*, vol. 42, no. 5, pp. 577–685, 1989.

- [10] J.A. Sethian, *Level Set Methods and Fast Marching Methods*, Cambridge University Press, 1999.
- [11] T.F. Chan and L.A. Vese, “Active contours without edges,” *IEEE Transactions on Image Processing*, vol. 10, no. 2, pp. 266–277, 2001.
- [12] Y. Xu, T. Géraud, and L. Najman, “Morphological Filtering in Shape Spaces: Applications using Tree-Based Image Representations,” in *ICPR*, 2012, pp. 485–488.
- [13] L. Guigues, J. P. Cocquerez, and H. L. Men, “Scale-sets image analysis,” *International Journal of Computer Vision*, vol. 68, no. 3, pp. 289–317, 2006.
- [14] P. Salembier and M. H. F. Wilkinson, “Connected operators,” *IEEE Signal Processing Mag.*, vol. 26, no. 6, pp. 136–157, 2009.
- [15] Y. Pan, “Top-down image segmentation using the mumford-shah functional and level set image representation,” in *Proc. of ICASSP*, 2009, pp. 1241–1244.
- [16] C. Ballester, V. Caselles, L. Igual, and L. Garrido, “Level lines selection with variational models for segmentation and encoding,” *Journal of Mathematical Imaging and Vision*, vol. 27, pp. 5–27, 2007.
- [17] P. Salembier, A. Oliveras, and L. Garrido, “Antiextensive connected operators for image and sequence processing,” *IEEE Transactions on Image Processing*, vol. 7, no. 4, pp. 555–570, 1998.
- [18] J. L. Lisani, L. Moisan, P. Monasse, and J. M. Morel, “On the theory of planar shape,” *SIAM Multiscale Modeling and Simulation*, vol. 1, no. 1, pp. 1–24, 2003.