Salient Level Lines Selection Using the Mumford-Shah Functional

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Main Contributions

An efficient image simplification method:

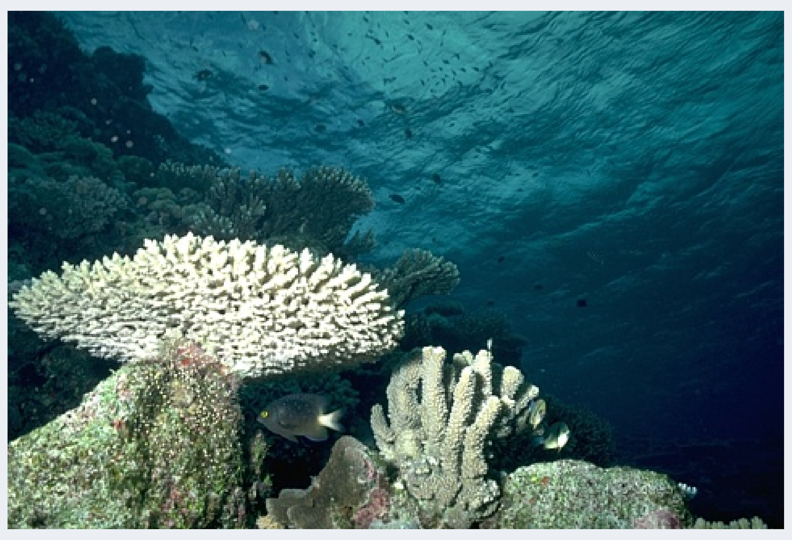
- **minimizing the Mumford-Shah functional on the tree of shapes [1];**
- **a fast greedy algorithm providing a local optimal solution.**

A hierarchical image simplification:

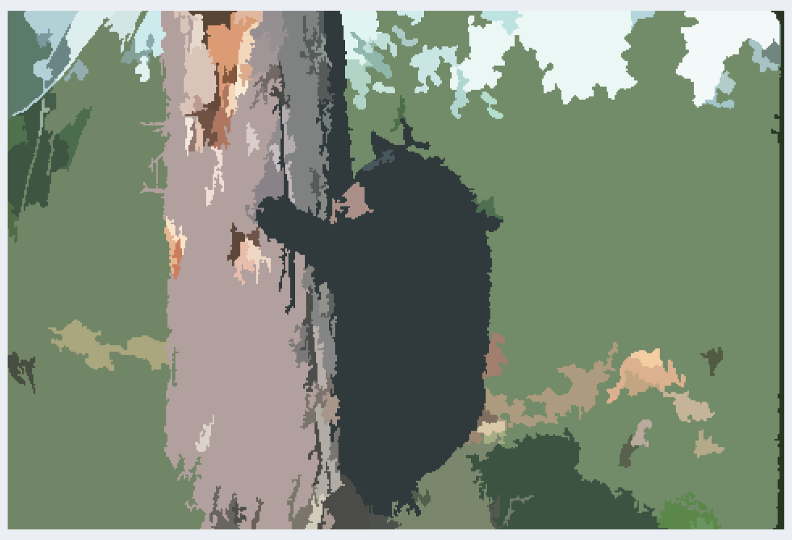
- **a variant of morphological shaping [2];**
- **a saliency map representing a hierarchical image simplification.**

Effective results: color images pre-segmentation

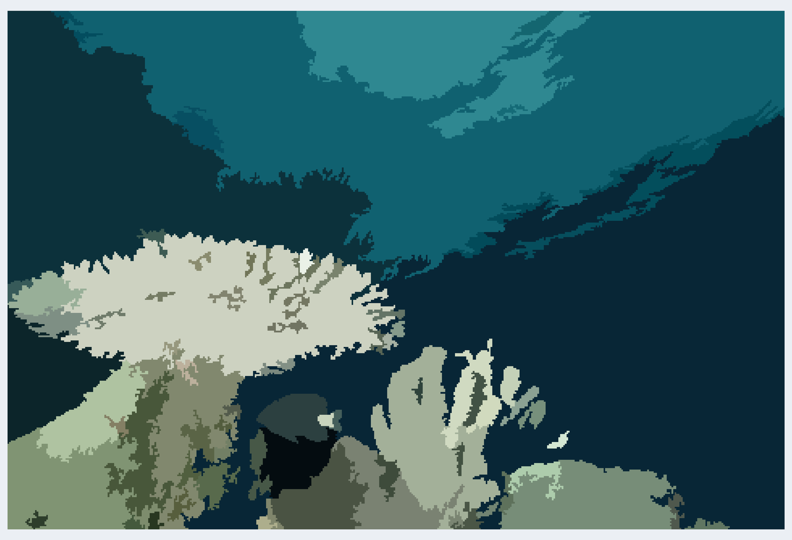
Input color image.



Input color image.



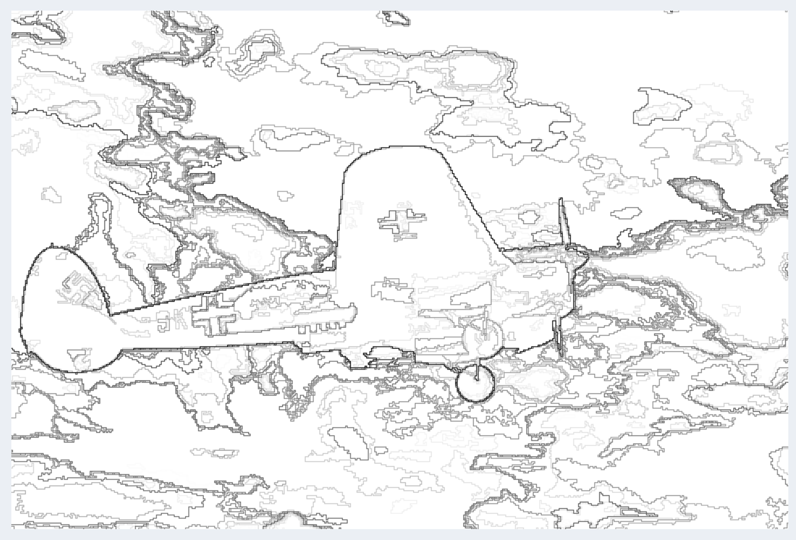
133 regions.



84 regions.

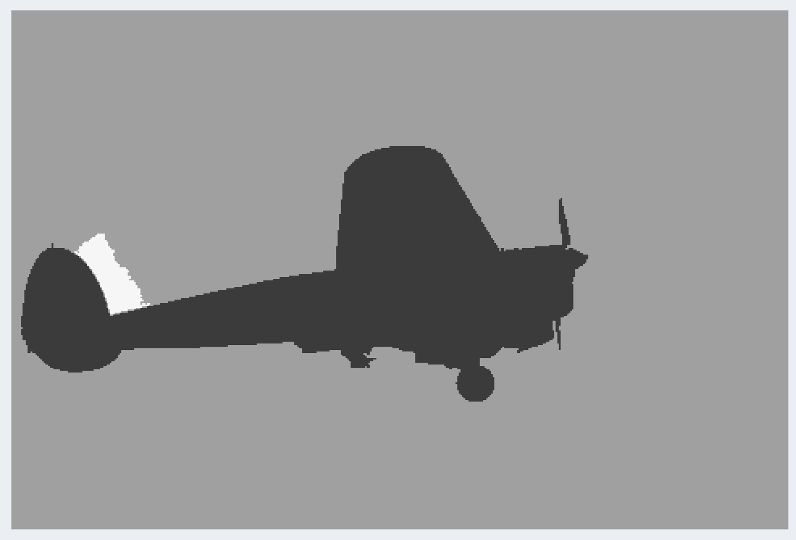
Effective results: hierarchical image simplification

Original image.



Saliency map.

Less simplified.



More simplified.

Basic idea

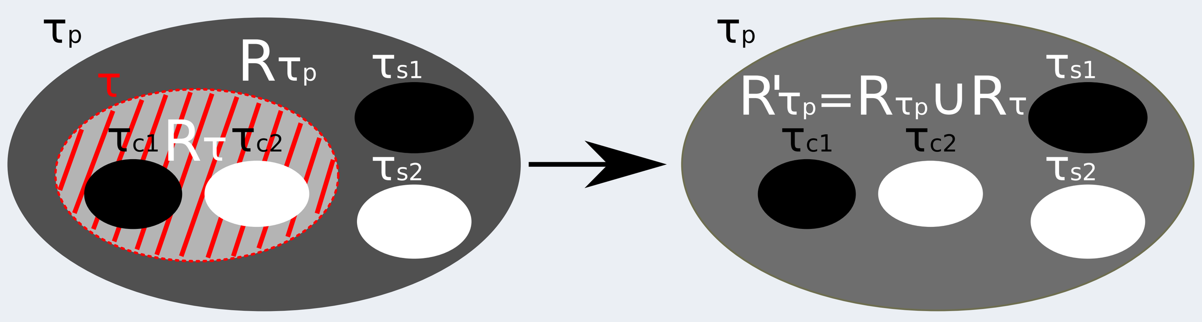
Minimize the piecewise-constant Mumford-Shah functional [3]...
...subordinated to the tree of shapes [1, 4] of an image f.

Piecewise-constant Mumford-Shah functional E:

$$E(f, \partial R) = \iint_R (\tilde{f}|_R - f)^2 \, dxdy + \nu \, |\partial R|$$

Tree of shapes $\mathcal{T}$: inclusion tree of level lines

Basic operation: level lines removal $\Rightarrow$ region merging



Removing $\tau \Rightarrow R'_{\tau_p} = R_{\tau_p} \cup R_\tau$, iff

$$\Delta E_\tau = \frac{S^2(f, R_\tau)}{|R_\tau|} + \frac{S^2(f, R_{\tau_p})}{|R_{\tau_p}|} - \frac{S^2(f, R'_{\tau_p})}{|R'_{\tau_p}|} - \nu |\partial \tau| \; < \; 0,$$

where $S(f, R) = \int_R f \, dxdy$.

Fast algorithm

Removing a node τ impacts its relatives $\Rightarrow$ the removal order is critical.

1) Algorithm of Ballester et al. [5] :

remove the node τ that decreases the most E

$\rightarrow$ prohibitive $O(N^2)$ time complexity.

2) Our proposed algorithm :

the removal order is based on a meaningfulness criterion...

... used meaningfulness: average of gradient magnitude along $\partial\tau$.

Input: tree $\mathcal{T}$ of image f; Output: simplified tree $\mathcal{T}'$

- *Initialization*: set $\mathcal{T}' = \mathcal{T}$.
- *Step 1*: sort $\{\tau | \tau \in \mathcal{T}\}$ in increasing order $\mathcal{O}$ of shape meaningfulness.
- *Step 2*: propagate $\{\tau\}$ following the order $\mathcal{O}$, remove τ, if $\Delta E_\tau < 0$.

$\mathcal{T}'$: a locally optimal solution of $\min_{\mathcal{T}'} E(f, \partial\mathcal{T}')$.

2 or 3 iterations of step 2 might be desired $\Rightarrow$ quasi-linear complexity

3) A variant of the proposed algorithm : compute $\nu_{\min}(\tau)$ instead of a fixed ν

- $\nu_{\min}$: a non-increasing attribute,
- Attribute filtering $\Rightarrow$ morphological shaping [2],
- Saliency map [2].

Comparison with the approach of Ballester et al. [5]

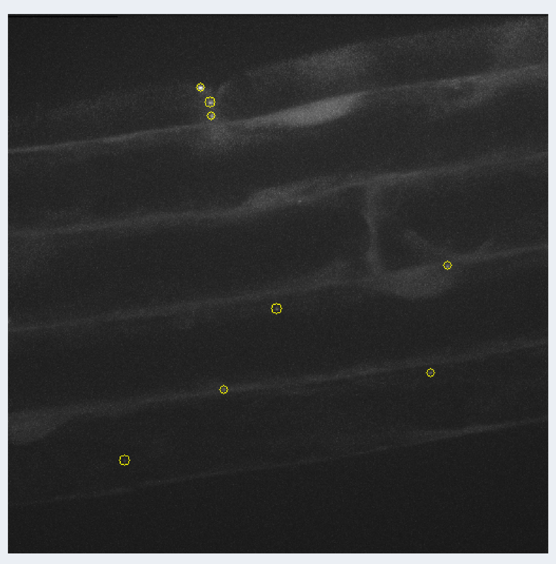
(a) Input image.

(b) Ballester, $\nu = 1\text{k}$, (t = 4s).

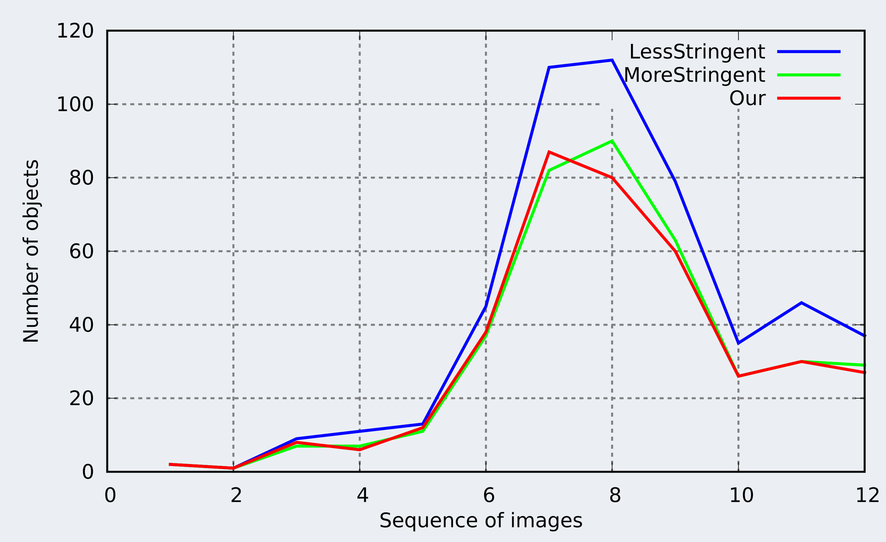
(c) Our, $\nu = 1\text{k}$, (t = 0.2s).

(a): $E = 1.24 \times 10^8$; (b): $E = 1.28 \times 10^7$, $\text{PSNR} = 27.3$; (c): $E = 1.15 \times 10^7$, $\text{PSNR} = 28.7$.

One application to autophagosome counting



Autophagosome counting.



Benchmark on a sequence of images.

Autophagosome in cellular images: small, round, bright objects.

References

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[4] T. Géraud, E. Carlinet, S. Crozet, and L. Najman, "A quasi-linear algorithm to compute the tree of shapes of nd images," in *ISMM*, pp. 98–110, 2013.

[5] C. Ballester, V. Caselles, L. Igual, and L. Garrido, "Level lines selection with variational models for segmentation and encoding," *JMIV*, vol. 27, pp. 5–27, 2007.