

A Color Tree of Shapes with Illustrations on Filtering, Simplification, and Segmentation

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Context

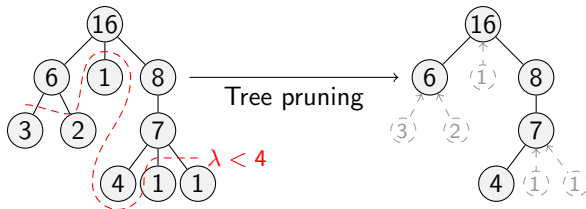
About morphological tree representations:

- versatile and efficient → many apps;
- (very) easy to compute/manipulate [5, 2, 4],
- implicit multiscale analysis,
- some of them feature (very) desirable properties:
 - contrast change invariance,
 - self-duality. . .

Not convinced? Let's see. . .

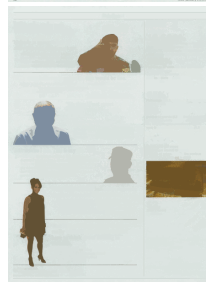
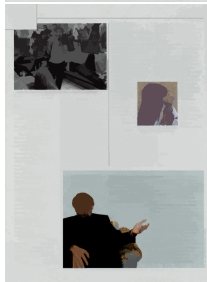
Grain filters [3](1/2)

Method overview



1. Compute the size attribute over the tree.
2. Threshold and collapse.

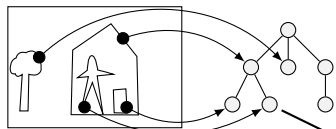
Grain filters (2/2): Document layout extraction



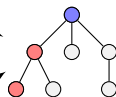
Interactive segmentation (1/2)

Method overview

Color ToS Computation



Markers on the tree



Tree Node Classification

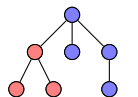
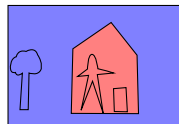
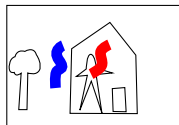


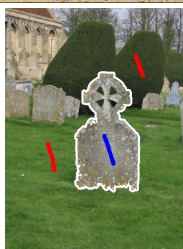
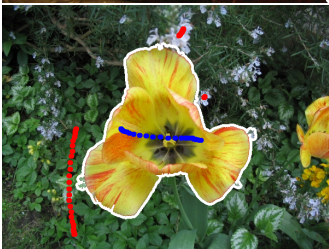
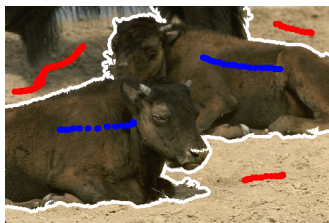
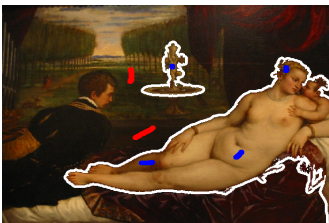
Image Classification



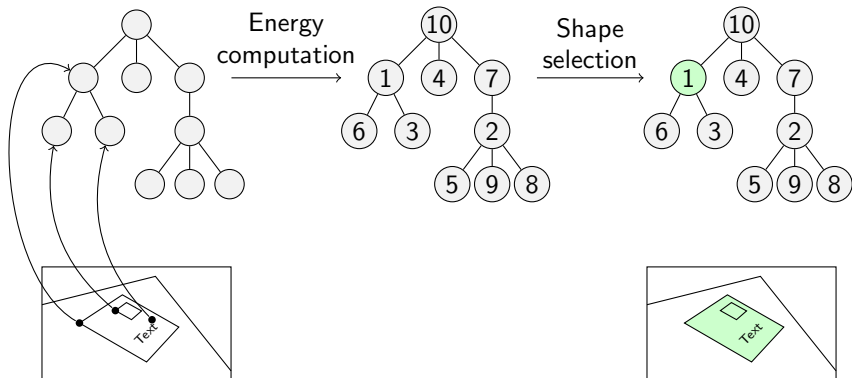
Markers (User Input)



Interactive segmentation (2/2)



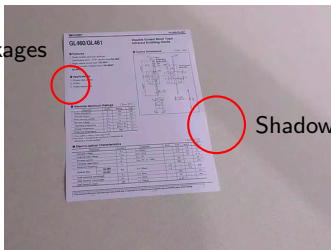
Document detection in videos (ICDAR SmartDoc'15)



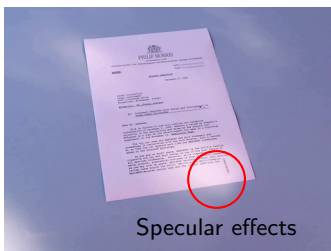
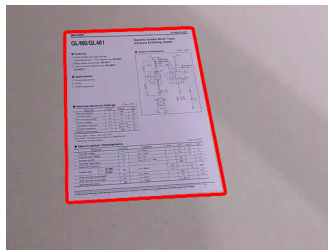
1. Evaluate an energy adapted to the object to detect.
2. Retrieve the shape with the lowest energy.

Document detection in videos (ICDAR SmartDoc'15)

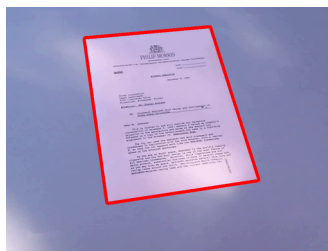
Leakages



Shadows



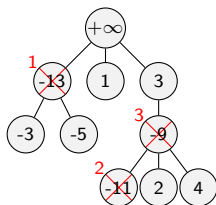
Specular effects



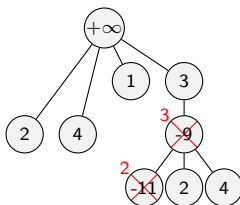
Natural Image Simplification[7]

Principle. Mumford-Shah energy optimization constrained to the tree.

Δ Energy

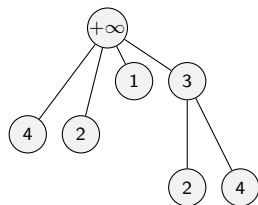


Iteration 1



...

Iteration n

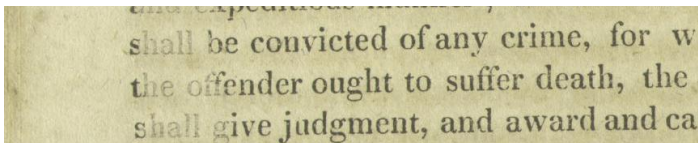


Natural Image Simplification[7]



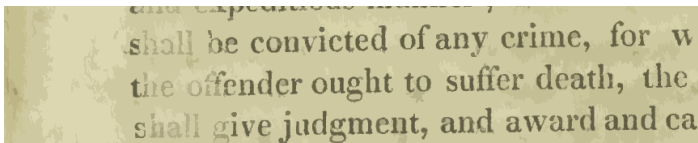
Image simplification: the simplified images have less than 100 nodes (original: $\sim 80k$ nodes)

Document Image Simplification[7]



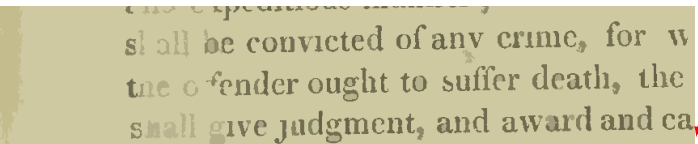
(a) Original (113k nodes).

nodes
 $\div 100$



(b) Strong simplification (1000 nodes).

nodes
 $\div 1000$



(c) Drastic simplification (285 nodes).

These applications use a single image
representation:

The Color Tree of Shapes

Outline

What for?

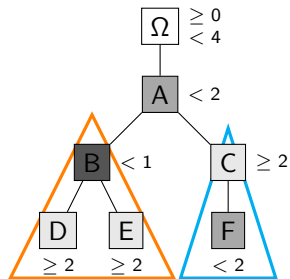
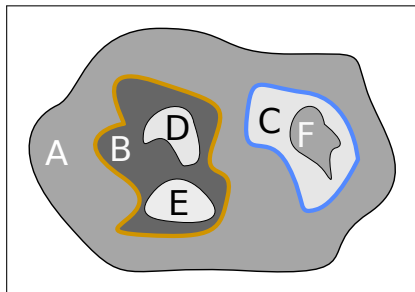
Why is a Color ToS challenging?

Proposal for a Color ToS

Comparison and Conclusion

What is the Tree of Shapes? (1/2)

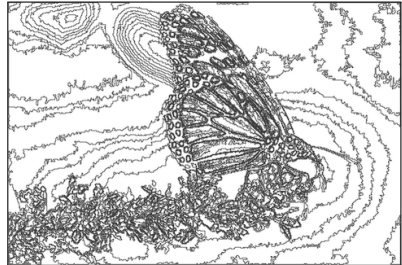
As the fusion of the min- and max- trees



The Tree of shapes (ToS) of u , formed by cavity-filled connected components of the min- and max- trees (self-dual representation)

What is the Tree of Shapes? (2/2)

As the inclusion tree of the level lines



u and its level lines (every 5 levels)

- The ToS also encodes the inclusion of the image level lines,
- They are the contours of shapes.

Properties of the ToS

We have:

- Invariance by **contrast change**:

$$T(g(u)) = T(u) \text{ for any increasing function } g$$

→ it handles low-contrasted objects

- Invariance by **contrast inversion**:

$$T(\mathbb{C}u) = T(u)$$

→ it represents light objects over dark background and the contrary, in a symmetric way

- A way to get self-dual connected operators:

→ they do not shift object boundaries

We would like the same “kind” of properties
for color images.

→ Yet, the ToS requires a **total** order on colors (does one make sense?)

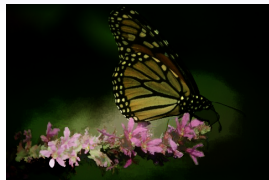
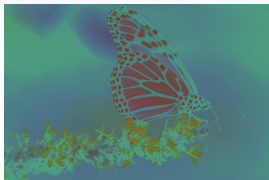
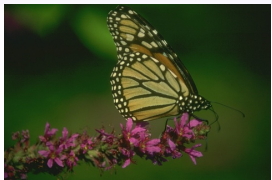
Outline

What for?

Why is a Color ToS challenging?

Proposal for a Color ToS

Comparison and Conclusion



Independent
Marginal contrast
change & inversion.

Local contrast change

What do these images have in common?

They share an exact same representation:
the [Color Tree of Shapes](#)

General Overview

What do we want?

- Given $\mathcal{M} = \{\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_n\}$, where $(\mathcal{S}_i, \subseteq)$ is a tree, we note $\mathcal{S} = \bigcup \mathcal{S}_i$ the primary shape set.
- We aim at defining a new set of shapes **S** such that:

(P1) **Tree structure**: every two shapes are either nested or disjoint.

(P2) **Maximal shape preservation**: any shape that does not overlap with any other shape should exist in the final shape set.

*It implies the **Scalar ToS equivalence** if u is scalar.*

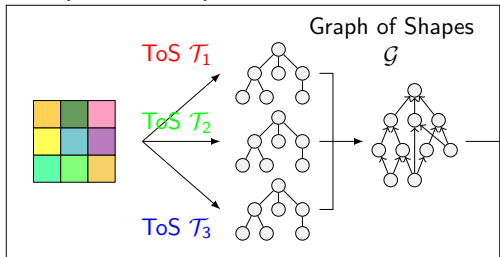
(P3) **Marginal contrast change/inversion invariance**: invariant to any strictly monotonic functions applied independently to u 's channels.

(Q) A “well-formed” tree: $\#nodes \simeq \#pixels$ and not degenerated.

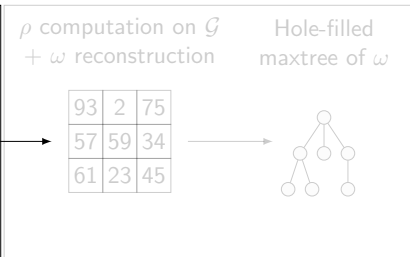
General Overview

Scheme of the method

Graph of Shapes Construction



Tree Extraction

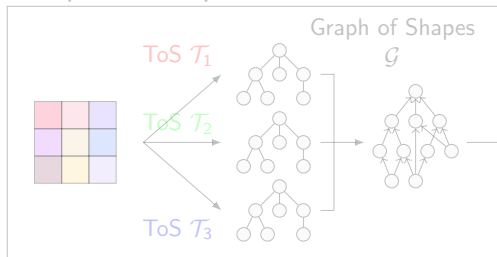


1. Get the primary shape set \mathcal{S} from the marginal ToS.
2. Compute the Graph of Shapes $\mathcal{G} = (\mathcal{S}, \subseteq)$

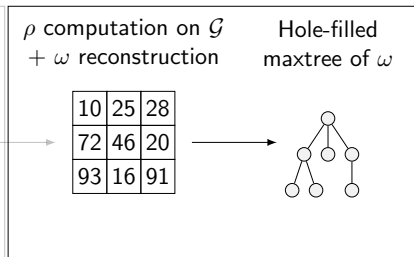
General Overview

Scheme of the method

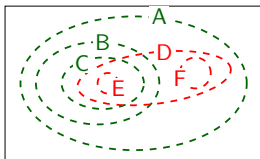
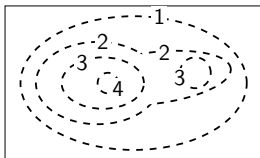
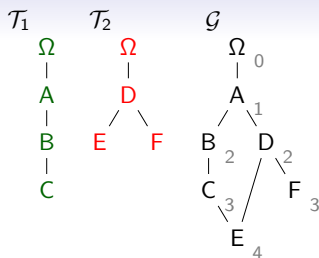
Graph of Shapes Construction



Tree Extraction



1. Compute the depth attribute ρ over \mathcal{G} ,
2. Reconstruct the attribute map ω (in the image space),
3. Compute the cavity-filled maxtree of ω

(a) Input u (c) ω map(b) Graph of Shapes + ρ (d) Cavity-filled Maxtree \mathcal{T}_ω

Justification

There is no magic!

In gray level:

The ToS of u is related to the maxtree of the depth map (cf. paper).

Furthermore. . .

It fulfills the properties. (Proofs in an upcoming paper)

You can get effective results. (you've already seen that!)

Outline

What for?

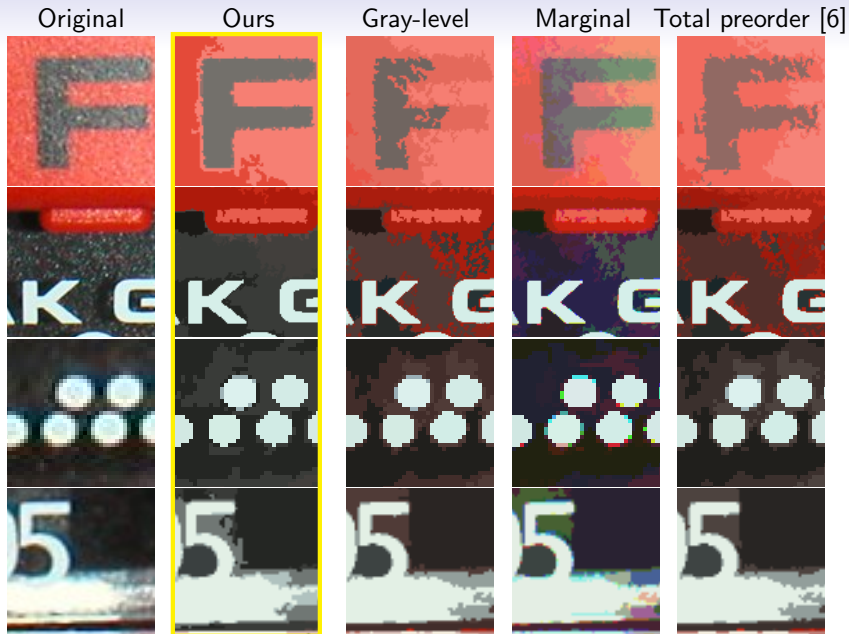
Why is a Color ToS challenging?

Proposal for a Color ToS

Comparison and Conclusion

Comparing on image simplification with classical approaches





Conclusion (1/2)

Key Idea. A method where the ordering is not based on colors (values), but on inclusion of shapes (components).

What has been done?

1. A proposal for a Color Tree of Shapes
2. An a-posteriori validation: get convincing results for simplification, segmentation. . .

Conclusion (2/2)

Perspectives: Use it!

Reproducible research:

`http://publications.lrde.epita.fr/carlinet.15.itip`
→ Source code, binaries, and extra results.

By the way... It's quite fast (2s on a 512×512 pixels image).



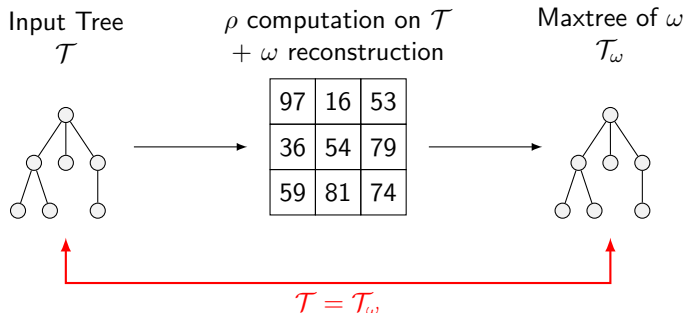
Plant a tree!

Questions?

- [1] E. Aptoula and S. Lefèvre.
A comparative study on multivariate mathematical morphology.
Pattern Recognition, 40(11):2914–2929, 2007.
- [2] E. Carlinet and T. Géraud.
A comparative review of component tree computation algorithms.
IEEE Transactions on Image Processing, 23(9):3885–3895, September 2014.
- [3] V. Caselles and P. Monasse.
Grain filters.
Journal of Mathematic Imaging and Vision, 17(3):249–270, November 2002.
- [4] S. Crozet and T. Géraud.
A first parallel algorithm to compute the morphological tree of shapes of nD images.
In *Proc. of IEEE Intl. Conf. on Image Processing (ICIP)*, pages 2933–2937, Paris, France, 2014.
- [5] T. Géraud, E. Carlinet, S/ Crozet, and L. Najman.
A quasi-linear algorithm to compute the tree of shapes of nD images.
In *Proc. of Intl. Symp. on Mathematical Morphology (ISMM)*, volume 7883 of LNCS, pages 98–110, Heidelberg, 2013. Springer.
- [6] O. Lézoray and A. Elmoataz.
Nonlocal and multivariate mathematical morphology.
In *Proc. of IEEE Intl. Conf. on Image Processing (ICIP)*, pages 129–132, Orlando, USA, 2012.
- [7] Y. Xu, T. Géraud, and L. Najman.
Salient level lines selection using the Mumford-Shah functional.
In *Proc. of IEEE Intl. Conf. on Image Processing (ICIP)*, pages 1227–1231, Merlbourne, Australia, 2013.

Rationale (1/2)

Idea 1. $\mathcal{T} + \text{dec. attribute } \rho + \text{restitution } \omega_\rho + \text{Maxtree } \mathcal{T}_{\omega_\rho} = \mathcal{T}$

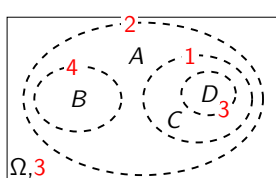


Rationale (2/2)

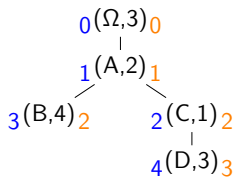
Idea 1. $\mathcal{T} + \text{dec. attribute } \rho + \text{restitution } \omega_\rho + \text{Maxtree } \mathcal{T}_{\omega_\rho} = \mathcal{T}$

Idea 2. u level lines = ω_{TV} level lines (TV from the border).
 = ω_{CV} level lines (Counted variations).

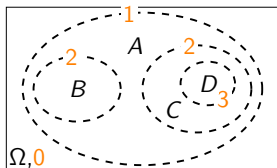
→ ToS of u = Maxtree of ω_{CV}



(a) u and its level lines.



(b) The ToS of u and ρ_{CV} (orange).



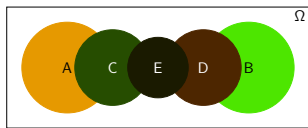
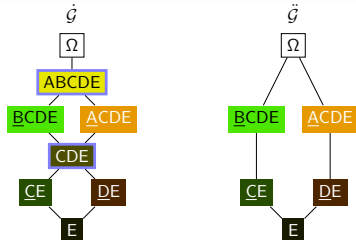
(c) The level lines of ω_{CV} .

Conclusion. Use the depth attribute on \mathcal{G} and reconstruct.

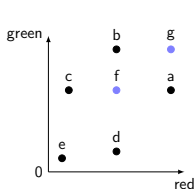
$\omega_{CV}(x)$ stands for:

The number of marginal level lines (that are nested) along the path from the border to the deepest shape that contains x .

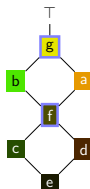
Differences with “Shape” component-graphs

Image u 

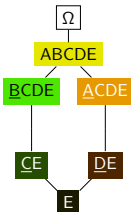
“Shape” Component graphs



Lattice of the values

 T_{red}  T_{green} 

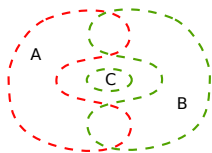
Graph of Shapes



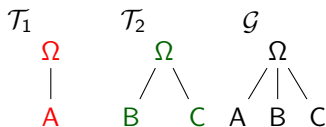
The graph of shapes

E. Carlinet, T. Géraud

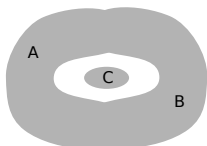
On the need of the saturation



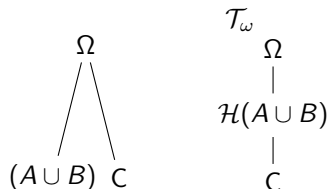
(a) Original.



(b) Marginal ToS and GoS.



(c) ω map.



(d) Maxtree of ω
(w/o cavity filling).

(e) Final Color ToS
(with cavity filling).

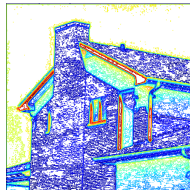
Effect of noise



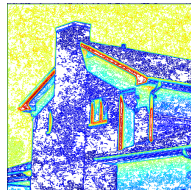
(a) House



(b) House (red channel) + Gaussian Noise ($\sigma = 20$, green channel)



(c) Level lines of the tos of (a). Level lines: 24k, avg. depth: 37, max. depth: 124.



(d) Level lines of the ctos of (b). Level lines: 48k, avg. depth: 48, max. depth: 127.

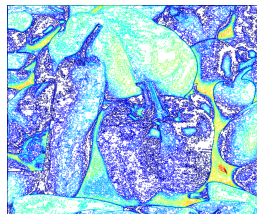
Effect of the dynamic



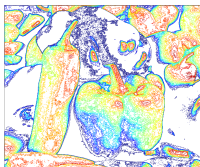
(a) Peppers (only red/green)



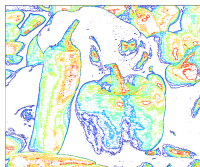
(b) Peppers (only red/green) with green sub-quant. to 10 levels



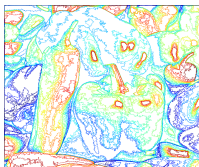
(c) Level lines of the red channel of (a) and (b)



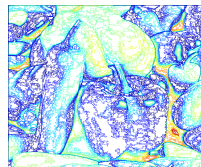
(d) Level lines of the green channel (a)



(e) Level lines of the green channel (b)



(f) Level lines of the ctos of (a)



(g) Level lines of the ctos of (b)