Un algorithme de complexité linéaire pour le calcul de l'arbre des formes

E. Carlinet, S. Crozet, T. Géraud

firstname.lastname@lrde.epita.fr

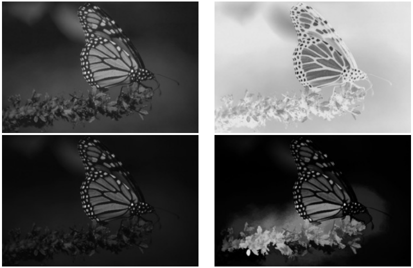
EPITA Research & Development Laboratory (LRDE)

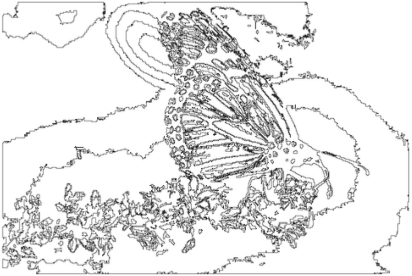
2018/06/27 — RFIAP — Marne-la-Vallée, France

Forewords

What is the Tree of Shapes (def 1)?

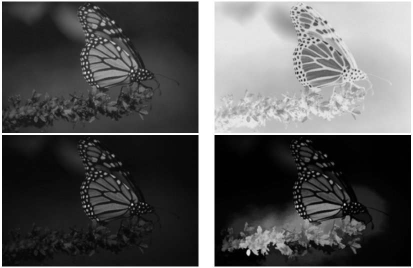
- Definition 1: The tree of inclusion of the image level lines

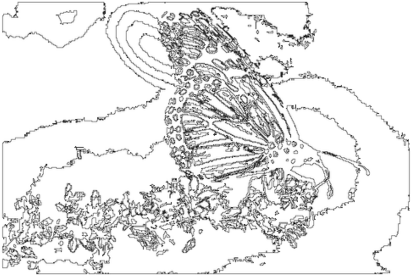




What is the Tree of Shapes (def 1)?

- Definition 1: The tree of inclusion of the image level lines





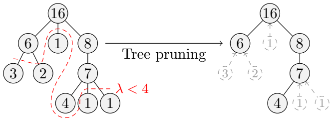
The organization of the level lines is:

- invariant by a *global* contrast change (increasing function)
- invariant by gray-level inversion
- robust to *local* changes of illumination

Applications

From *general* **simple** filters. . .

- Grain filter (= tree pruning)

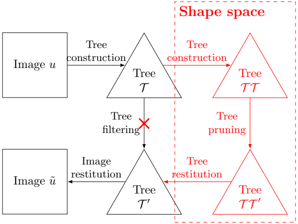


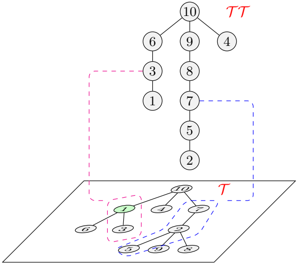
This is a connected operator: no contours are shifted (some contours are removed, based on the component size).

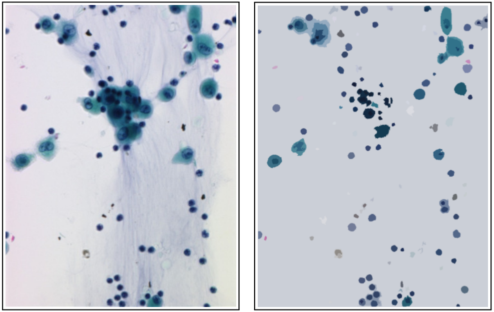
Applications

From *general* **simple** filters, or *general* **advanced** filters. . .

- Shapings (= filetring in the space of shapes)





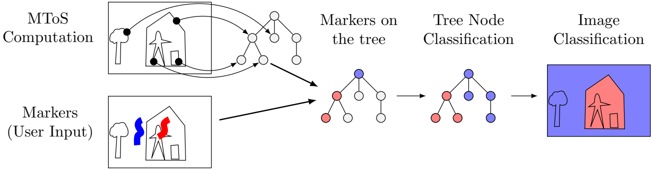


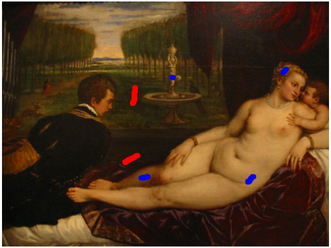
Xu, Géraud, and Najman. *Connected filtering on tree-based shape-spaces.* PAMI 2016

Applications

From *general* **simple** filters, or *general* **advanced** filters, to **app-specific** methods. . .

- Interactive segmentation



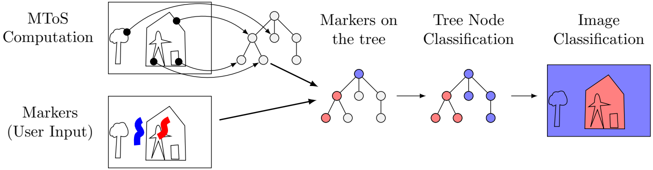


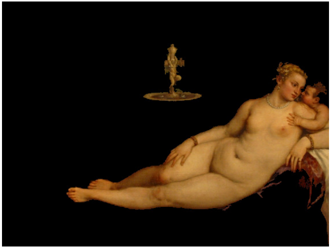
Carlinet and Géraud. *Morphological object picking based on the color ToS.* IPTA 2015

Applications

From *general* **simple** filters, or *general* **advanced** filters, to **app-specific** methods. . .

- Interactive segmentation



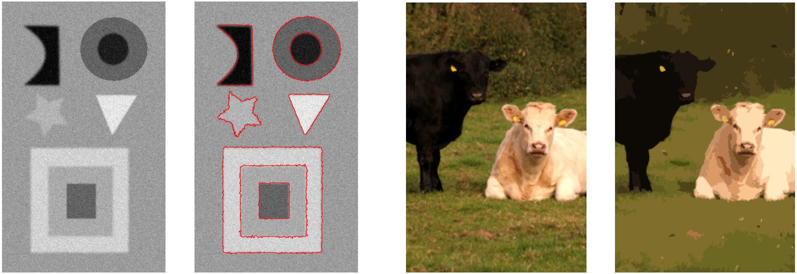


Carlinet and Géraud. *Morphological object picking based on the color ToS.* IPTA 2015

Applications

From *general* **simple** filters, or *general* **advanced** filters, to **app-specific** methods, passing by **general-purpose** CV methods

- Object detection / Image simplification

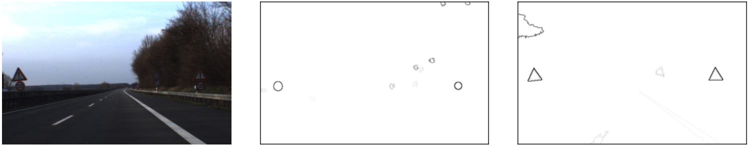


Xu, Géraud, and Najman. *Context-based energy estimator: Application to object segmentation on the ToS*. ICIP 2012

Applications

From *general* **simple** filters, or *general* **advanced** filters, to **app-specific** methods, passing by **general-purpose** CV methods

- Hierarchy of Segmentations



Xu, Carlinet, Géraud, and Najman. *Hierarchical segmentation using tree-based shape spaces.* PAMI 2017

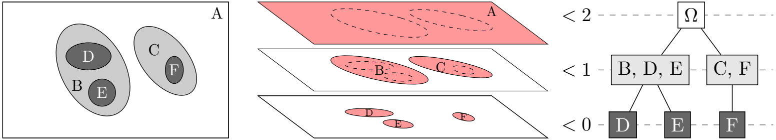
What is the Tree of Shapes (def 2)?

- **Definition 2**:
  The tree of inclusion $\mathcal{T}$ of the hole-filled connected components

What is the Tree of Shapes (def 2)?

- **Definition 2**:
  The tree of inclusion $\mathcal{T}$ of the hole-filled connected components
- Lower level sets and min-tree

$$[u < \lambda] = \{x \in X \mid u(x) < \lambda\}$$
$$\mathcal{T}_<(u) = \{\Gamma \in \mathcal{CC}([u < \lambda])\}$$



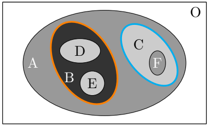
(a) u (b) $[u < \lambda]$ (c) $\mathcal{T}_<(u)$

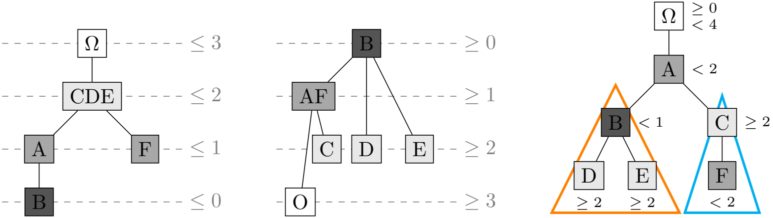
Upper level sets and max-tree defined dually.

What is the Tree of Shapes (def 2 cont.)?

- **Definition 2**:
  The tree of inclusion $\mathcal{T}$ of the hole-filled connected components from min and max-trees:

$$\mathcal{T} = Sat(\mathcal{T}_{<}(u)) \cup Sat(\mathcal{T}_{>}(u))$$





(a) Min-Tree (b) Max-Tree (c) ToS

Computing the ToS

How to

Actually, few algorithms. . .

1. The FLLT: Monasse & Guichard, *Fast Computation of a Contrast Invariant Image Representation.* TIP 2000.
2. The FLST: Caselles & Monasse, *Geometric Description of Images as Topographic Maps.* LNCS 1984, 2009.
3. Song, *A Topdown Algorithm for Computation of Level Lines.* TIP 2007.
4. Géraud et al, *A Quasi-Linear Algorithm to Compute the Tree of Shapes of nD Images.* ISMM 2013.

How to

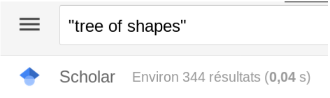
Actually, few algorithms...

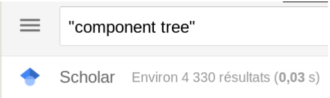
1. The FLLT: Monasse & Guichard, *Fast Computation of a Contrast Invariant Image Representation.* TIP 2000.
2. The FLST: Caselles & Monasse, *Geometric Description of Images as Topographic Maps.* LNCS 1984, 2009.
3. Song, *A Topdown Algorithm for Computation of Level Lines.* TIP 2007.
4. Géraud et al, *A Quasi-Linear Algorithm to Compute the Tree of Shapes of nD Images.* ISMM 2013.

... but many issues:

- (1,2,3) *worst* time complexity is $O(N^2)$
- (1,2,3) are hard to implement
- (2,3) are limited to 2D images
  and (1) might be untractable in 3D...
- (4) is quasi-linear but has a high constant multiplier

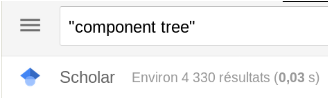
The idea





- There is much more literature about min/maxtrees
- Thus, more research about efficient algorithms:
  - Moschini et al. *A hybrid shared-memory parallel max-tree algorithm for extreme dynamic-range images.* PAMI 2018.
  - Götz et al. *Parallel computation of component trees on distributed memory machines.* TPDS 2018.

The idea



- There is much more literature about min/maxtrees
- Thus, more research about efficient algorithms:
  - Moschini et al. *A hybrid shared-memory parallel max-tree algorithm for extreme dynamic-range images.* PAMI 2018.
  - Götz et al. *Parallel computation of component trees on distributed memory machines.* TPDS 2018.

Idea

- Turn the ToS computation into a max-tree computation

A new ToS algorithm

ToS algorithm

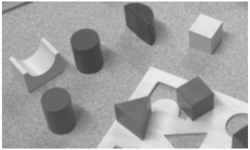
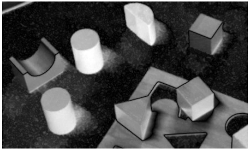
A two steps algorithm:

1. Turn the image into a *depth* map
2. Compute the max-tree of the *depth* map

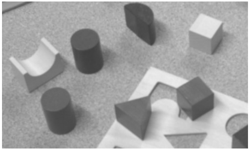
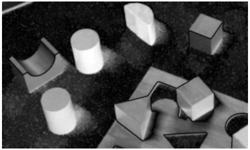
Algorithm properties

- $O(n)$ for low-quantized data and non-degenerated cases
- Worst cases:
  - *quasi*-linear for low quantized data
  - $O(n \log n)$ for high-quantized data

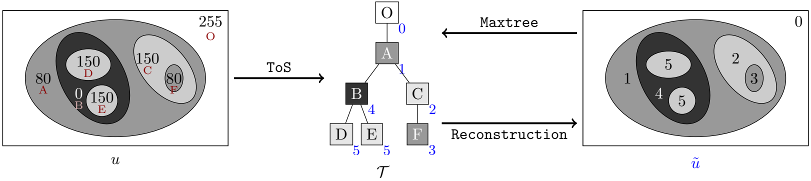
Step #2: max-tree computation

`ToS(`  `) = maxtree(`  `)`

Step #2: max-tree computation

`ToS(`  `) = maxtree(`  `)`

Explanations



Q: how to get $\tilde{u}$ without $\mathcal{T}$?

Step #1: turning the image into a *depth* map

Slight modification of a step from Géraud *et al.*, ISMM 2013:

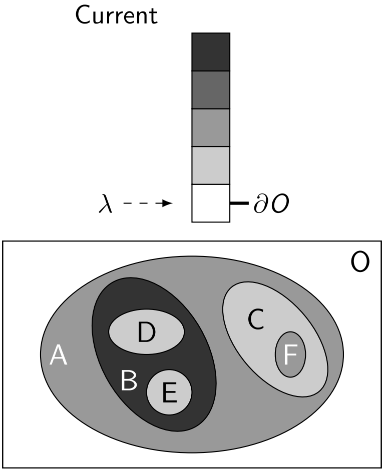
- Sort the pixels in the *descending tree order*

---

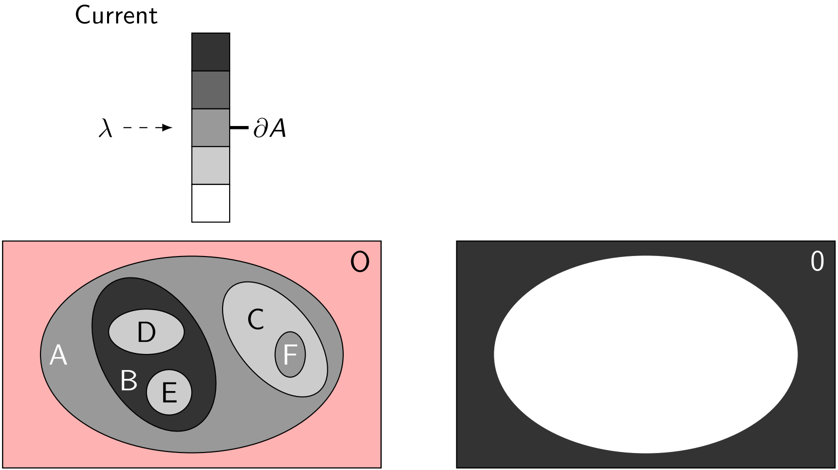
sorting the pixels means progress **continuously**
both in *image space*[1] and in *value space*[2]
(starting from the image boundary, i.e., the root node)

[1]through a spatially consistent growing
[2]jumping from a gray level to the *next* one (either upper or lower)

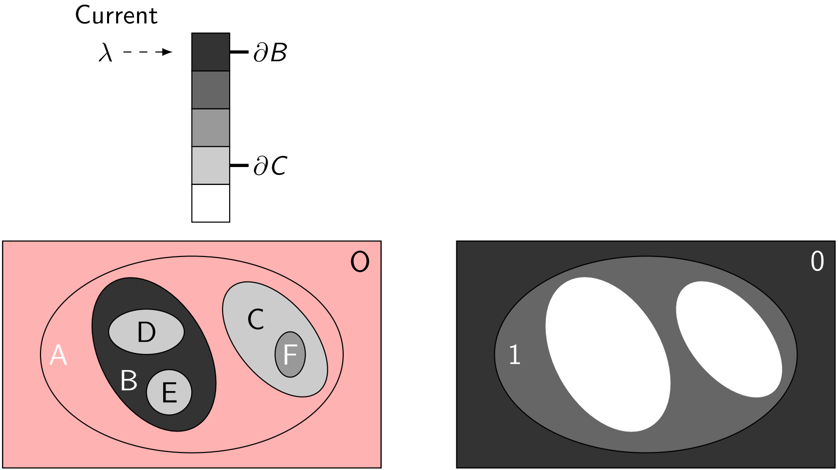
Step #1: turning the image into a *depth* map



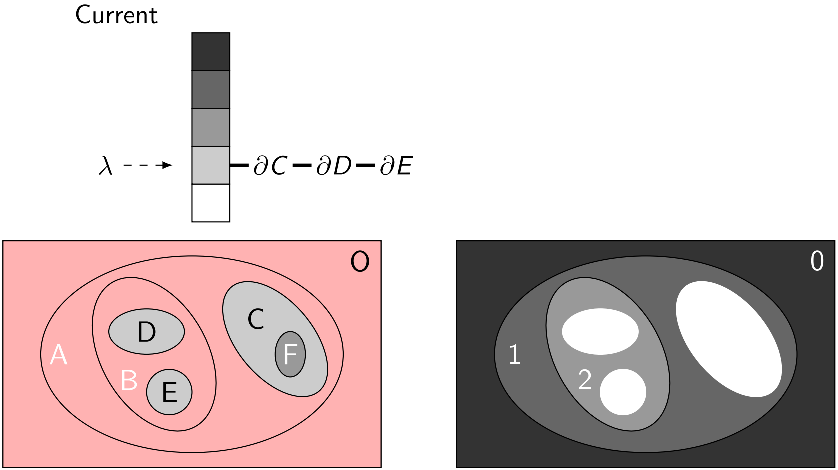
Step #1: turning the image into a *depth* map



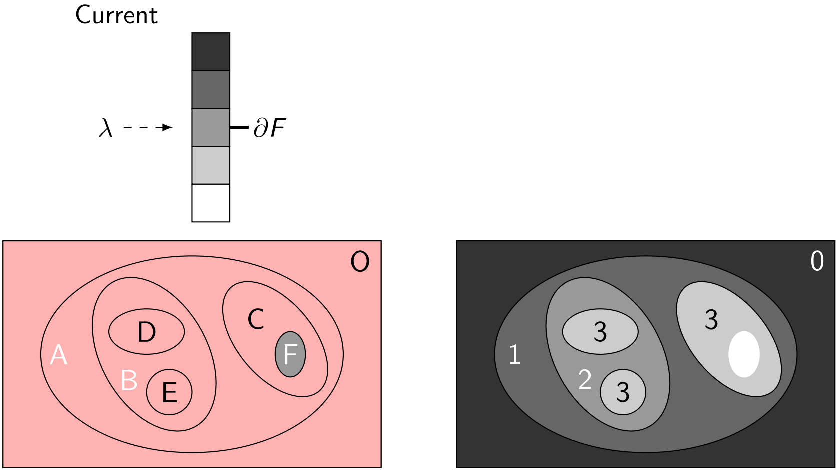
Step #1: turning the image into a *depth* map



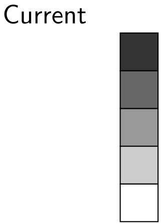
Step #1: turning the image into a *depth* map

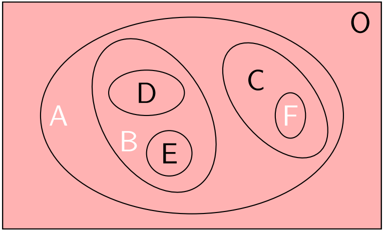


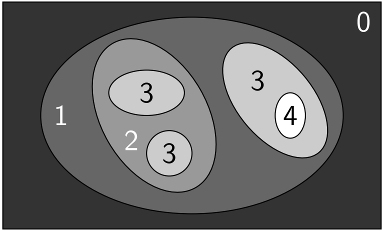
Step #1: turning the image into a *depth* map



Step #1: turning the image into a *depth* map

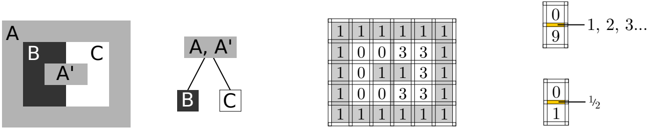






Problem(s) & solution(s)

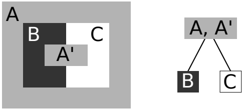
Problem #1

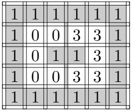


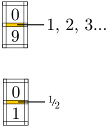
We need to pass between pixels and with intermediate values

Problem(s) & solution(s)

Problem #1







We need to pass between pixels and with intermediate values

Solution #1 (not presented in this talk)

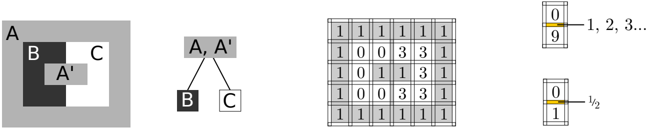
Interval-valued set on the Khalimsky grid (see *Géraud et al.*, ISMM 2013)

⇒ Problem #2

It requires to *double twice* the size of the image: x16 in 2D, x64 in 3D

Problem(s) & solution(s)

Problem #1



We need to pass between pixels and with intermediate values

Solution #1 (**not presented in this talk**)

Interval-valued set on the Khalimsky grid (see *Géraud et al.*, ISMM 2013)

⇒ Problem #2

It requires to *double twice* the size of the image: x16 in 2D, x64 in 3D

Solution to the problem #2 of the pb #1's soluce (**in the paper**)

A way to reduce memory space usage: *only* x4 in 2D, x8 in 3D

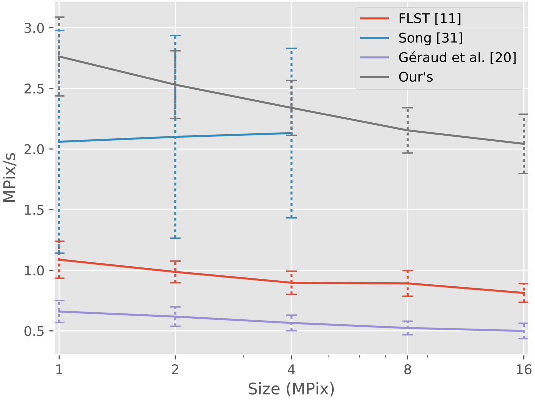
Results

Performances

Protocol

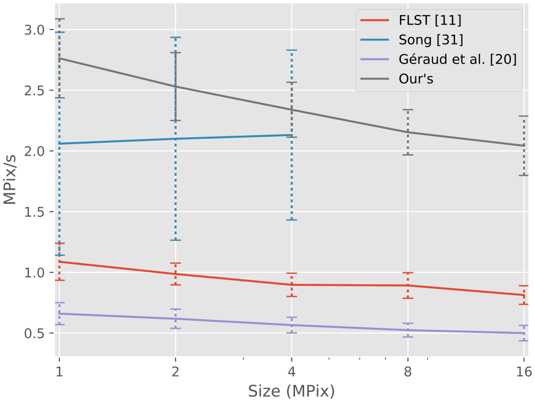
- Competitors:
  - Song, 2007
  - Géraud et al., 2013
  - *FLST*: Caselles & Monasse, 2009
- 20-MPix natural images (cropped from 1M to 16M)
- Intel Core i7 7500U, 2.7Ghz, 8Gb of RAM

Performances



- Most algorithms are (quite) linear in practice ($O(n^2)$ worst case not reached)

Performances



- Our algorithm is:
  - 4X faster than *Géraud et al.*
  - 2.5X faster than the *FLST*
  - more stable than *Song*'s

Conclusion

What we have proposed:

- an idea: turning the ToS into a MaxTree computation
- an optimization (not presented here)

Why?

To use any *blasting* maxtree algorithm you need, e.g.

- for distributed sytems
- for parallel shared-memory systems
- for embedded systems (i.e. low memory constraints)

Conclusion

What we have proposed:

- an idea: turning the ToS into a MaxTree computation
- an optimization (not presented here)

Why?

To use any *blasting* maxtree algorithm you need, e.g.

- for distributed sytems
- for parallel shared-memory systems
- for embedded systems (i.e. low memory constraints)

What's next?

- Improving the *step 1* (in terms of efficiency/concurrency. . .)
- Optimization generalized to n-D (described only in 2D for now)

Reproductible Research

- Implemented with our library

`https://gitlab.lrde.epita.fr/olena/pylene`

- Source code available

`https://gitlab.lrde.epita.fr/olena/pylene-apps`

Thanks for your attention. Any questions?