

# A new morphological hierarchical representation of images

Application to text segmentation in document and natural images

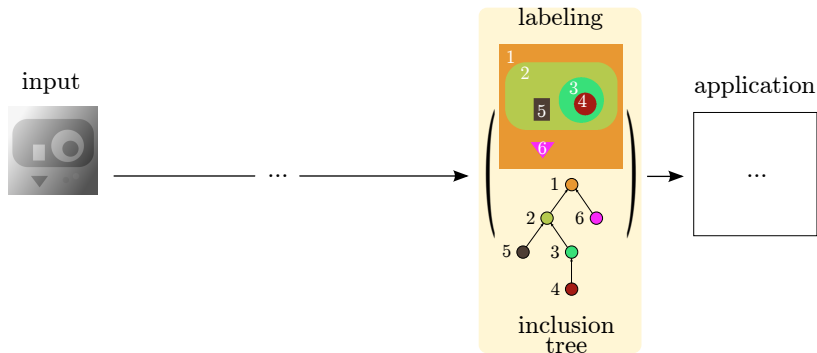
Thierry Géraud   Lê Duy Huynh   Yongchao Xu

EPITA Research and Development Laboratory (LRDE), France

`firstname.lastname@lrde.epita.fr`



GT GeoDis 2016, Massilia



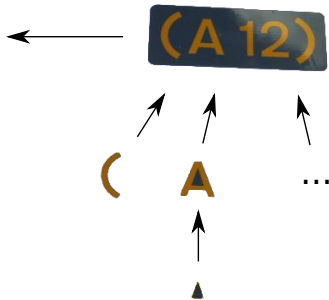
## Objective:

to compute a (useful / easy to use) hierarchical representation,  
which structures the image contents by inclusion

# Illustration



input

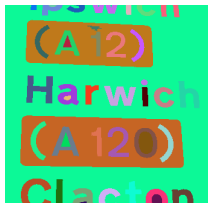


object inclusion

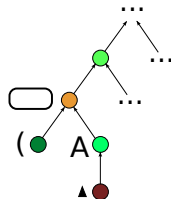
# Illustration



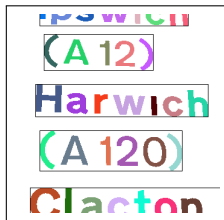
input



label image

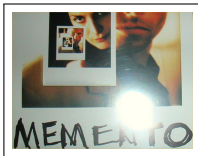


incl. tree



# Application #1: Text detection in natural images

Qualitative evaluation (from *Robust Reading* at ICDAR):



input

labeling (+ incl. tree)

result

Quantitative evaluation:

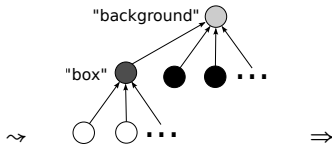
in [A Morphological Hierarchical Representation with Application to Text Segmentation in Natural Images](#) (submitted to ICPR 2016)

# Application #2: Self-dual binarization of doc images

Used in the competition *SmartDoc* at ICDAR 2015:



input image



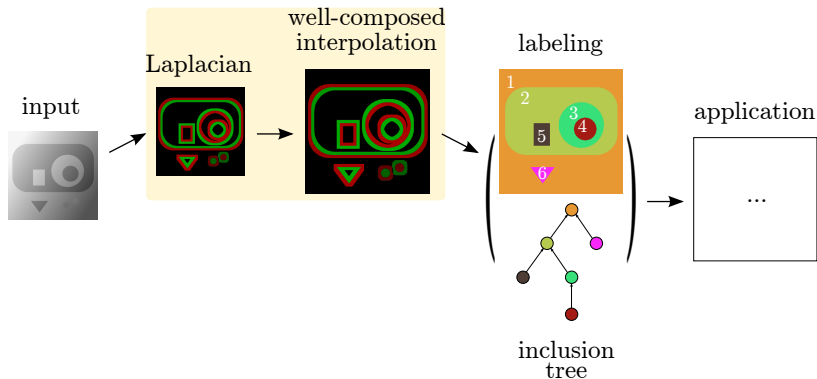
tree (+label image)



output

The point is:  
it is *not trivial* to get such inclusion structures

# Outline of the Proposed Solution



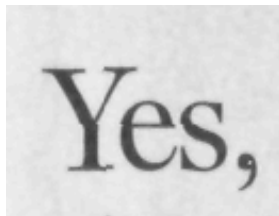
based on a Laplace operator *and* a well-composed interpolation

(so there's some topological considerations...)

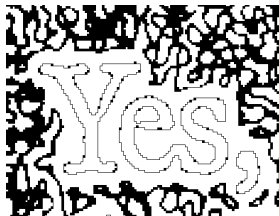


# About the Laplace Operator(s)

Zero-crossings of the Laplace operator  $\Delta u = u_{xx} + u_{yy}$ :

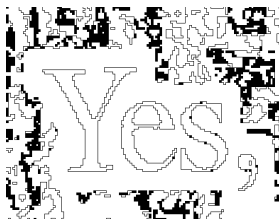


Original image



LoG  $17 \times 17$

Morphological version →

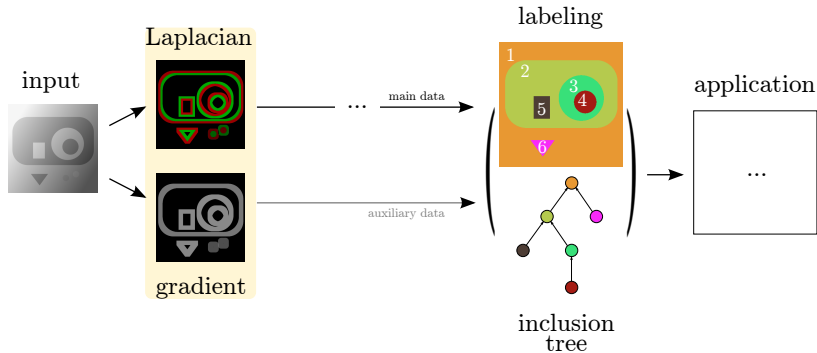


# Morphological Laplace Operator

Morphological Laplace Operator:  $\Delta_B = (\delta_B - \text{id}) - (\text{id} - \varepsilon_B)$

- few sensitive to
  - contrast changes
  - uneven illumination
  - the structuring element  $B$
- zeros stick to data :-) but...
  - we have “spurious” zeros (Pb #1)
  - there is no “trivial” inclusion (Pb #2) :-)

# Solving Pb #1



we compute  $\nabla_B = \delta_B - \varepsilon_B$  to *ignore* spurious  $\Delta_B = 0$   
(not detailed here)

## Rationale:

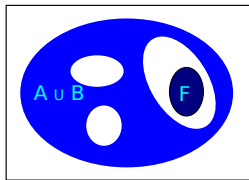
- we are interested in zero-crossings of the Laplace operator
- we want to get a partition of the image (the label image)
- we want the regions to arrange into an inclusion tree
- we do not want to favor a given contrast
- last we want to be fast...

~> *we want the 0-level lines given by the “tree of shapes” of  $\text{sign}(\Delta_B u)$*

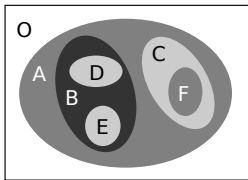
# Level Sets and Shapes

Given an image  $u : \mathcal{D} \subset \mathbb{R}^2 \rightarrow \mathbb{R}$

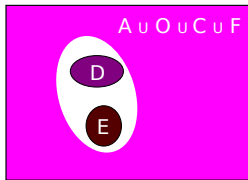
- lower level sets:  $\forall \lambda, [u < \lambda] = \{x \in \mathcal{D} \mid u(x) < \lambda\}$
- upper level sets:  $\forall \lambda, [u \geq \lambda] = \{x \in \mathcal{D} \mid u(x) \geq \lambda\}$



a lower level set



$u$



a upper level set

with the cavity-fill-in operator  $\text{Sat}$ :

- lower shapes:  $\mathcal{S}_{<}(u) = \{\text{Sat}(\Gamma); \Gamma \in \mathcal{CC}([u < \lambda])\}_\lambda$
- upper shapes:  $\mathcal{S}_{\geq}(u) = \{\text{Sat}(\Gamma); \Gamma \in \mathcal{CC}([u \geq \lambda])\}_\lambda$

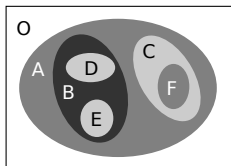
# Tree of Shapes and Tree of Lines

We have an inclusion *tree of shapes*:

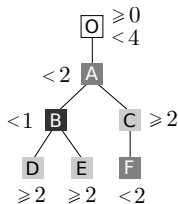
- $\mathfrak{S}(u) = \mathcal{S}_{<}(u) \cup \mathcal{S}_{\geq}(u)$ ,

and we have the contours of shapes:

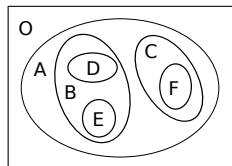
- $\{\partial\Gamma; \Gamma \in \mathfrak{S}(u)\}$  called *level lines*.



an image  $u$



its tree of shapes  $\mathfrak{S}(u)$

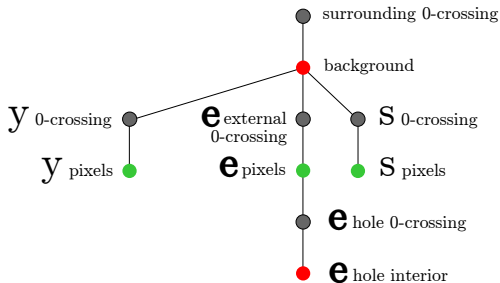


its level lines

The blue pill: so we have an inclusion *tree of level lines*...



$\Delta_{\square}(u)$  colorized



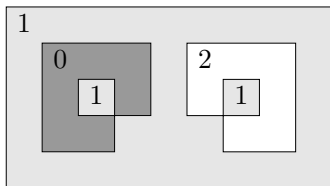
$\mathfrak{S}(\text{sign}(\Delta_{\square}(u)))$

# Tree of Shapes and Tree of Lines

The red pill: in the discrete settings we face some difficulties...

1	1	1	1	1	1	1	1	1
1	0	0	0	1	2	2	2	1
1	0	1	0	1	2	1	2	1
1	0	0	1	1	1	2	2	1
1	1	1	1	1	1	1	1	1

an image



its level lines

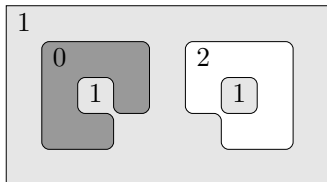
what is the inclusion of shapes, i.e., of level lines?  
consider the left inner square at 1, is it inside the 0s?



A solution in the *continuous setting*:

1	1	1	1	1	1	1	1	1
1	0	0	0	1	2	2	2	1
1	0	1	0	1	2	1	2	1
1	0	0	1	1	1	2	2	1
1	1	1	1	1	1	1	1	1

an image



its level lines

considering an *upper semi-continuous* function

level lines are Jordan curves and an inclusion tree exists

# Tree of Shapes and Tree of Lines

A solution in the *discrete setting*:

1	1	1	1	1	1	1	1	1
1	0	0	0	1	2	2	2	1
1	0	1	0	1	2	1	2	1
1	0	0	1	1	1	2	2	1
1	1	1	1	1	1	1	1	1

an image

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1
1	1	0	0	0	0	0	1	1	2	2	2	2	2	2	2	1
1	1	0	1	1	1	0	1	1	2	2	2	2	2	2	2	1
1	1	0	1	1	1	1	1	2	2	2	2	2	2	2	2	1
1	1	0	0	0	1	1	1	1	1	2	2	2	2	2	2	1
1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

its max-interpolated

It is *as if* we use c8 (resp. c4) for upper (resp. lower) shapes...

# Tree of Shapes and Tree of Lines

A solution in the *discrete setting*:

1	1	1	1	1	1	1	1	1
1	0	0	0	1	2	2	2	1
1	0	1	0	1	2	1	2	1
1	0	0	1	1	1	2	2	1
1	1	1	1	1	1	1	1	1

an image

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	1
1	1	0	0	0	0	0	1	1	2	2	2	2	2	2	2	2	2	2	1
1	1	0	1	1	1	0	1	1	2	2	2	2	2	2	2	2	2	2	1
1	1	0	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	1
1	1	0	0	0	1	1	1	1	1	2	2	2	2	2	2	2	2	2	1
1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

its max-interpolated

we do have Jordan curves and an inclusion tree *but* we are **not self-dual**  
we do **not** want to favor a particular contrast  $\rightsquigarrow$  solution discarded!

# A Self-Dual Digital Representation

A **self-dual** solution in the digital setting:

1	1	1	1	1	1	1	1	1
1	0	0	0	1	2	2	2	1
1	0	1	0	1	2	1	2	1
1	0	0	1	1	1	2	2	1
1	1	1	1	1	1	1	1	1

an image

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	0	0	0	0	1	1	1	2	2	2	2	1	1
1	1	0	1	1	1	0	1	1	1	2	1	1	1	2	1
1	1	0	1	1	1	0	1	1	1	2	1	1	1	2	1
1	1	0	1	1	1	1	1	1	1	1	1	1	1	2	1
1	1	0	0	0	1	1	1	1	1	1	1	2	2	2	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

a  $\mathfrak{F}$ -interpolated

interpolation obtained thanks to a front propagation method  $\mathfrak{F}$

↪ How to make  $nD$  functions WC in a self-dual way (ISMM 2015)

# About the $\mathfrak{F}$ -Interpolation

9	11	15
7	1	13
3	5	3

an image  $u$

9	9	11	11	15
8	8	8	8	13
7	7	1	8	13
7	7	5	8	8
3	5	5	5	3

the  $\mathfrak{F}$ -interpolation of  $u$

This  $n$ D well-composed interpolation is invariant by:

- contrast changes,
- classical rotations and symmetries,
- inversion (so it is self-dual).

...

# About the $\mathfrak{F}$ -Interpolation

9	11	15
7	1	13
3	5	3

an image  $u$

9	9	11	11	15
8	8	8	8	13
7	7	1	8	13
7	7	5	8	8
3	5	5	5	3

the  $\mathfrak{F}$ -interpolation of  $u$

Actually:

- this interpolation is a *digitally well-composed* image,
- it has a “purely self-dual” tree of shapes.  
[Links btw the morphological ToS and WC gray-level images \(ISMM 2015\)](#)
- its level sets are Jordan curves.



Original image



Morphological Laplacian  $\leadsto$  WC

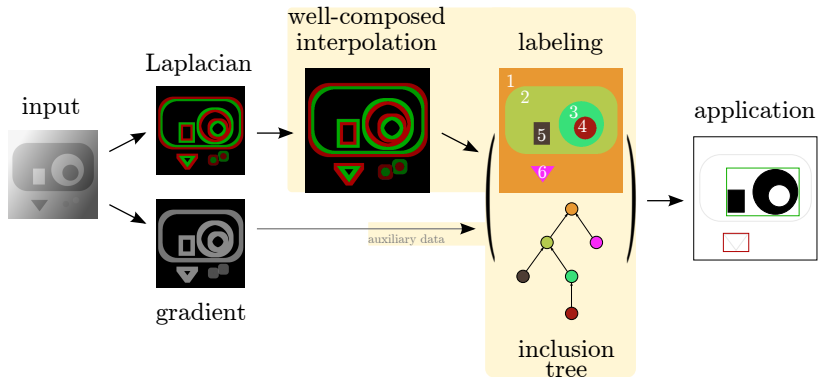


Hierarchical repr. (lab.+tree)



App. (here node selection and grouping)

# Computing the Hierarchical Structure



(Label image + tree) computation.



# Hierarchical Structure Computation

```
1 LABELING( $\Delta_{\square}^{wc}, \nabla_{\square}$ )
2
3 for all  $p$  do
4   label( $p$ )  $\leftarrow 0$ 
5   isContour( $p$ )  $\leftarrow$  false
6    $\ell \leftarrow 0$ ;
7
8 for all  $p$  do
9   if label( $p$ )  $\neq 0$  then continue
10  CONTOURIZE(parent*,  $\ell^*$ , isContour*,  $p$ ,  $\nabla_{\square}$ ) // * means input/output
11  label( $p$ )  $\leftarrow \ell$ 
12  Q.push( $p$ )
13  while not Q.is_empty() do
14     $q \leftarrow$  Q.pop()
15    for all  $n \in \mathcal{N}(q)$  do
16      if label( $n$ ) = 0 and  $\Delta_{\square}^{wc}(p) \times \Delta_{\square}^{wc}(n) \geq 0$  then
17        label( $n$ )  $\leftarrow \ell$ 
18        Q.push( $n$ )
19      else
20        isContour( $n$ )  $\leftarrow$  true
21
22 return (label, parent)
```

This is a simple front propagation blob labeling (yellowish) algorithm!  
with contour elimination and inclusion tree (parent) computation

Properties:

- linear time complexity,
- parallelizable,
- no  $\times 4$  factor: we can *emulate* the  $\mathfrak{F}$ -interpolation!
- easy to code.

- A hierarchical representation:
  - data simplification,
  - easy to use.
- Emphasis put on inclusion:
  - useful for some applications.
- From theory to practical results:
  - self-duality of cross-sections in digital topology...
  - some effective results.

- T. Géraud, E. Carlinet, S. Crozet, and L. Najman, “A quasi-linear algorithm to compute the tree of shapes of  $n$ -D images,” in *Proc of ISMM*, vol. 7883 of *LNCS*, pp. 98–110, Springer, 2013. [\[PDF\]](#)
- S. Crozet and T. Géraud, “A first parallel algorithm to compute the morphological tree of shapes of  $n$ D images,” in *Proc of ICIP*, pp. 2933–2937, 2014. [\[PDF\]](#)
- N. Boutry, T. Géraud, and L. Najman, “On making  $n$ D images well-composed by a self-dual local interpolation,” in *Proc of DGCI*, vol. 8668 of *LNCS*, pp. 320–331, Springer, 2014. [\[PDF\]](#)
- T. Géraud, E. Carlinet, S. Crozet, “Self-duality and digital topology: Links between the morphological tree of shapes and well-composed gray-level images,” in *Proc. of ISMM*, vol. 9082 of *LNCS*, pp. 573–584, Springer, 2015. [\[PDF\]](#)
- N. Boutry, T. Géraud, and L. Najman, “How to make  $n$ D functions well-composed in a self-dual way,” in *Proc. of ISMM*, vol. 9082 of *LNCS*, pp. 561–572, Springer, 2015. [\[PDF\]](#)
- L.D. Huynh, Y. Xu, T. Géraud, “A morphological hierarchical representation with application to text segmentation in natural images,” *Submitted to ICPR*, 2016. [\[PDF\]](#)