Some Tree-Based Representations in Mathematical Morphology

T. Géraud

EPITA Research and Development Laboratory (LRDE), France

thierry.geraud@lrde.epita.fr

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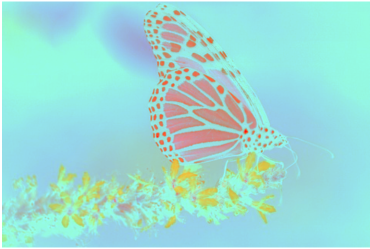
Forewords

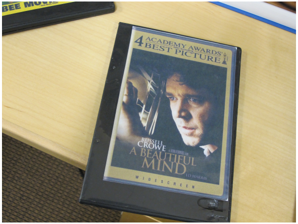


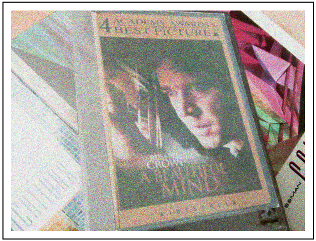
Which animal is it?

Forewords

How many signs are there?

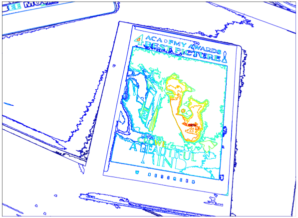
Forewords

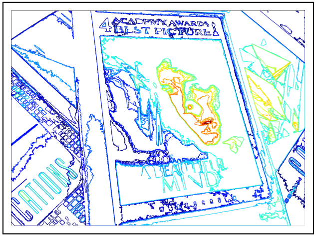




How to be robust to variations of the image contents?

Forewords





“stable” contours $\Leftrightarrow$ nice invariants

Forewords

Key ideas:

- we have to deal with
  - poor contrast
  - illumination changes
  - ...
- shapes matter a lot (colors are sometimes *not so* important)
- we want to ensure some strong "invariance" properties

$\rightsquigarrow$ mathematical morphology!

A Short History of Mathematical Morphology (MM)

- *Mid 60's*
  Invention of MM by Georges Matheron and Jean Serra in CMM, France.
- *From 70's to 80's*
  Extension to sets (binary images) to functions (gray-level images).
- *End of the 80's*
  MM on graphs is defined (~~structural elements~~ neighborhood).
- *1995*
  Connected operators appear...
- *Beginning 2YK's*
  First attempts to get MM works on color images.
- *Since then*
  Adaptive filtering, optimization-related MM, etc. but remaining very confidential...

Mathematical Morphology (MM)

Mathematical Morphology is:

- a mathematical framework
  $\rightsquigarrow$ some common properties are expected
- a large toolkit
  $\rightsquigarrow$ some tools are very simple and powerful
- a way of thinking...

Main idea

an image $f \equiv$ a landscape where $f(x, y)$ is the elevation at (x, y)

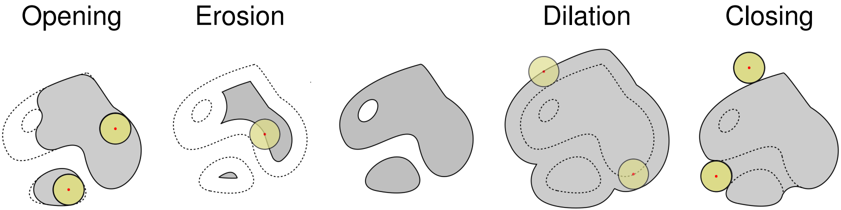
processing an image $\equiv$ modifying the landscape / function f

Mathematical Morphology (MM)

This is a landscape:

so we have paths, mountains, peaks, valleys, flat zones, level lines, crest lines, passes, catchment basins...

Basic operators



$\gamma_B(X) = \delta_B \circ \varepsilon_B(X)$ $\quad \varepsilon_B(X)$ $\quad X$ $\quad \delta_B(X)$ $\quad \phi_B(X) = \varepsilon_B \circ \delta_B(X)$

with B a structuring element / parameter

we have dual operators: $\delta_B(X) = \complement\, \varepsilon_B(\complement\, X)$ and $\phi_B(X) = \complement\, \gamma_B(\complement\, X)$

From sets to functions

From Mathematical Morphology on **sets**...

...we want to have Mathematical Morphology on **functions**

Threshold Decomposition Principle

Given a gray-level image / scalar function f.

Upper threshold set at level λ = binary image:

$$[\, f \geq \lambda \,] \;=\; \{\, x \in \Omega;\; f(x) \geq \lambda \,\} \;\in\; \mathcal{P}(\Omega)$$

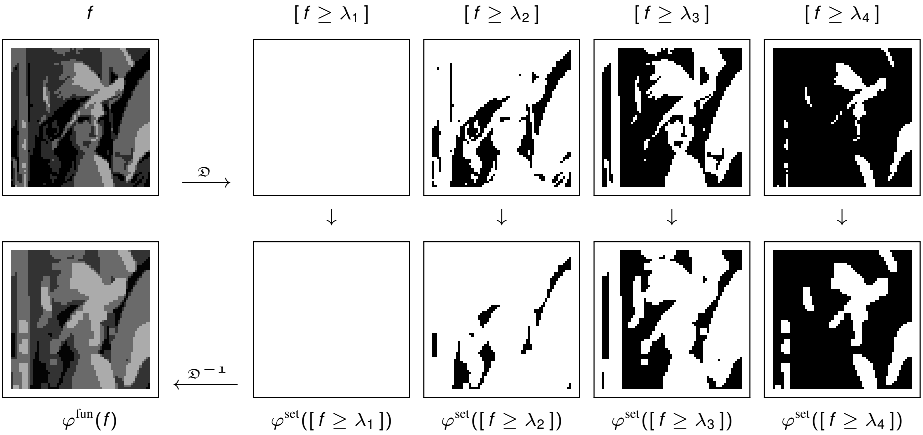
Decomposition:

$$f \;\xrightarrow{\;\mathfrak{D}\;}\; \{\, [\, f \geq \lambda \,] \,\}_\lambda$$

Extension from φ^{set} to φ^{fun} :

$$\{\, \varphi^{\text{set}}(\, [\, f \geq \lambda \,] \,) \,\}_\lambda \;\xrightarrow{\;\mathfrak{D}^{-1}\;}\; \varphi^{\text{fun}}(f)$$

Applying an Operator (the theoretical way)



with $\varphi = \delta_B$ in this example.

Contrast Change Invariance

With any contrast change g (non-decreasing function):

f $\quad$ $g \circ f$ $\quad$ ... $\quad$ $[\, g \circ f \geq \lambda'_i \,]$ $\quad$ ...

$\xrightarrow{g}$

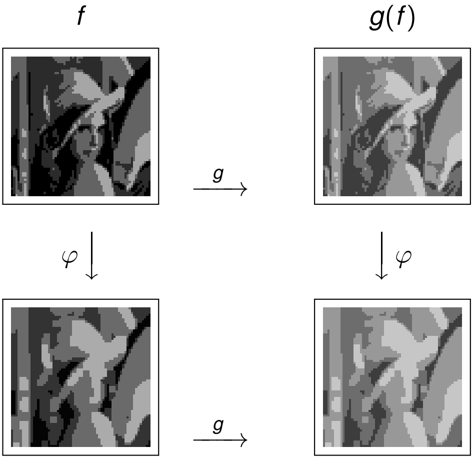
$\xrightarrow{\mathfrak{D}}$ $\quad$ ...

...

$[\, f \geq \lambda_i \,]$

$\lambda'_i = g(\lambda_i) \;\; \Rightarrow \;\; g \circ f$ and f have the same family of threshold sets!

Contrast Change Invariance



$\varphi(f)$

$$g \circ \varphi(f) = \varphi \circ g(f)$$

Contrast Change Invariance

Applying a morphological operator on these 3 images:

is equivalent (subject to g).

The right-most images are <u>not more difficult</u> to process than the left one!

Contrast does not matter for MM, meaning that:

- values does not matter but...
- ...only the ordering of values matters!

Contrast Inversion Invariance

If in addition φ is self-dual, that is $\varphi(f) = -\varphi(-f)$, we have:

$$\varphi \circ h = h \circ \varphi$$

with any *monotonic* change h.

Applying a *self-dual* morphological operator on these 3 images:

is equivalent (subject to h).

Local Illumination Change Invariance

Some operators φ behave the *same way* on:

and

they are invariant by any *local illumination* change ℓ:

$$\varphi \circ \ell = \ell \circ \varphi$$

Temporary conclusion

About Mathematical Morphology:

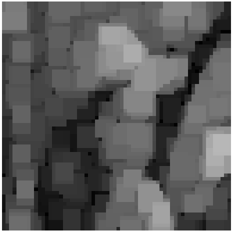
- many tools [...]
- sound mathematical foundations

  so we can understand what we have to do, and we can interpret the results we obtain
- important invariants for PR + CV

Operators deal with *(connected components of) threshold sets*:

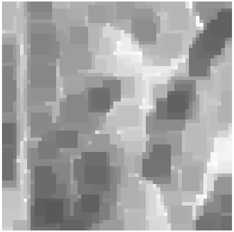
- so IP operators can do some PR tasks
- but MM with structuring elements shift contours :−(

An illustration

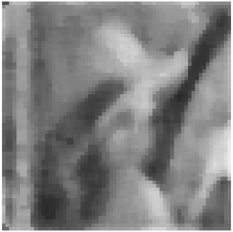


$\gamma_B(f)$

f



$\phi_B(f)$

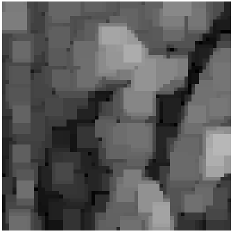


$\varphi_B^{\text{median}}(f)$

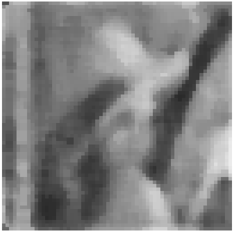
$\leftarrow$ self-dual $\rightarrow$

$(\gamma_B + \phi_B)/2$

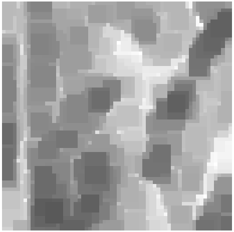
Pick your favorites...



$\gamma_B(f)$



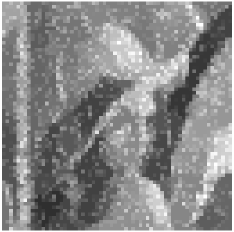
$\varphi_B^{\text{median}}(f)$



$\phi_B(f)$

$\gamma_{(\mathcal{A},\alpha)}(f)$

$\nu_{(\mathcal{A},\alpha)}(f)$



$\phi_{(\mathcal{A},\alpha)}(f)$

Morphological dual trees

When thresholding f at level λ, we get:

- an upper level set: $[f \geq \lambda] \;=\; \{\, x \in \Omega;\; f(x) \geq \lambda \,\}$
- a lower level set: $[f < \lambda] \;=\; \{\, x \in \Omega;\; f(x) < \lambda \,\}$

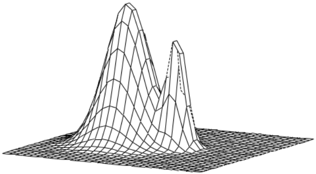
Considering the connected components ($\mathcal{CC}$) of threshold sets:

- **max-tree**: $\mathcal{T}_{\text{max}}(f) \;=\; \{\, \Gamma \in \mathcal{CC}(\,[f \geq \lambda]\,)\,\}_\lambda$
- **min-tree**: $\mathcal{T}_{\text{min}}(f) \;=\; \{\, \Gamma \in \mathcal{CC}(\,[f < \lambda]\,)\,\}_\lambda$

We have the duality: $\mathcal{T}_{\text{min}}(-f) \;=\; \mathcal{T}_{\text{max}}(f)$

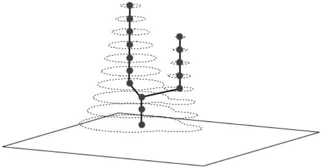
Morphological dual trees

Example with the max-tree:



f

$\equiv$

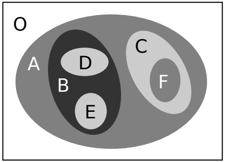


$\{ [f \geq \lambda] \}_\lambda$

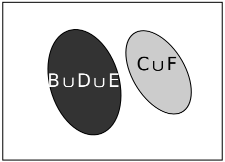
The morphological tree of shapes (ToS)

Using the cavity-fill-in operator (Sat):

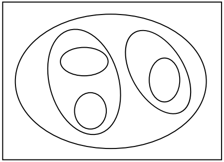
- **tree of shapes**: $\mathfrak{S}(f) = \{\, \mathrm{Sat}(\Gamma);\ \Gamma \in \mathcal{T}_{\mathrm{max}}(f) \cup \mathcal{T}_{\mathrm{min}}(f) \,\}$
- it is also the inclusion tree of level lines



f



two shapes of f



level lines of f

We have the self-duality: $\mathfrak{S}(-f) = \mathfrak{S}(f)$

Invariances

With g strictly increasing:

$$\mathcal{T}_{\max}(g \circ f) \; = \; \mathcal{T}_{\max}(f) \quad \text{and} \quad \mathcal{T}_{\min}(g \circ f) \; = \; \mathcal{T}_{\min}(f)$$

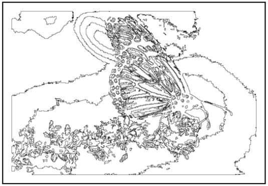
With h strictly monotonic:

$$\mathfrak{S}(h \circ f) \; = \; \mathfrak{S}(f)$$

With ℓ some local illumination changes:

$$\mathfrak{S}(\ell \circ f) \; = \; \mathfrak{S}(f)$$





Advertising

Having a tree structure is great:

- simple structure
- easy to browse and do some iterative / recursive computations
- easy to transform
- and very versatile

A **node** represents a **component** of the image

$\Rightarrow$ we are at **Pattern Recognition level**!

Connected Operators

Definition

A morphological operator φ is a *connected operator* iff:

$$\forall f,\ \forall x \mathcal{N} x',\ \ \varphi(f)(x) \neq \varphi(f)(x') \ \Rightarrow\ f(x) \neq f(x').$$

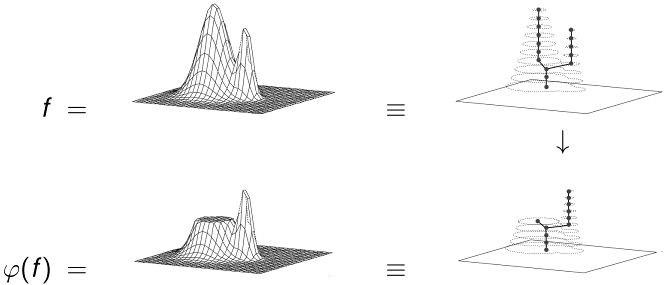
A very interesting class of operators:

- do not create new contours $\Rightarrow$ do not shift contours
- not based on structuring elements
- intuitive, powerfull, and efficient
- can be implemented as tree filtering

Connected Operators

A well-known family:

- remember that "MM operating = landscape modifying"
- tree-pruning $\rightsquigarrow$ connected operators



Connected Operators

Pruning is based on:

- an increasing attribute $\mathcal{A}$ = *effect* of the filtering
- and a threshold α = *strength* of the filtering
- nodes *n* such as $\mathcal{A}(n) < \alpha$ are removed

The *type* of filtering depends of the tree:

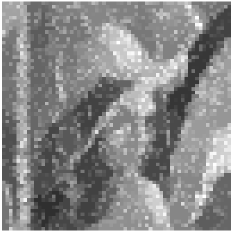
on $\mathcal{T}_{\max}(f)$

openings $\gamma_{(\mathcal{A},\alpha)}(f)$

on $\mathfrak{S}(f)$

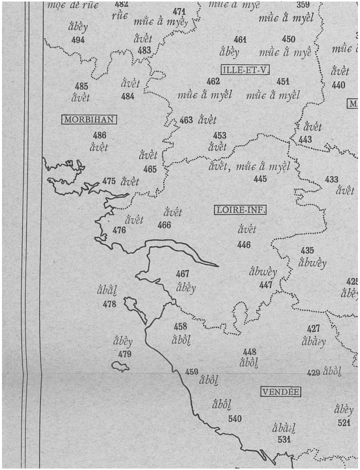
grain filters $\nu_{(\mathcal{A},\alpha)}(f)$

on $\mathcal{T}_{\min}(f)$



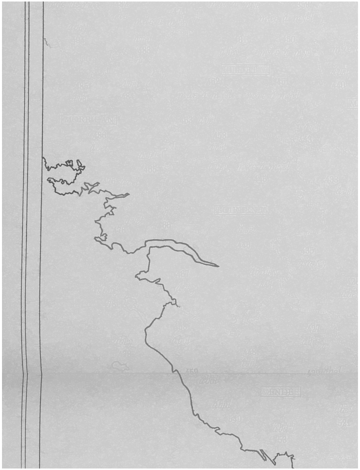
closings $\phi_{(\mathcal{A},\alpha)}(f)$

Illustrations



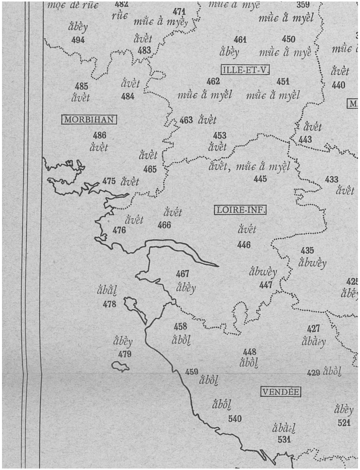
input

Illustrations



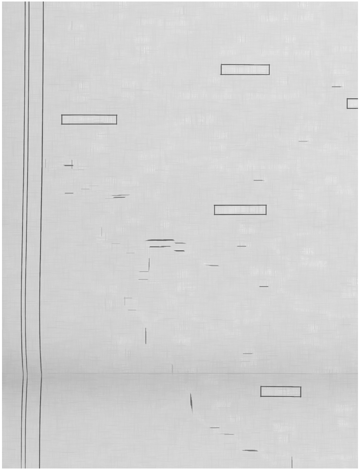
coast and frames

Illustrations



input

Illustrations



we apply $\min(\phi_{B_{\text{horizontal}}}, \phi_{B_{\text{vertical}}})$...

Illustrations

...and some connected operators

A glitch

For morphological operators to work:

getting threshold sets (and trees) means ordering pixel values.

A lot of data are **not** scalar, but multi-variate data:

- color images
- multi-modal medical images
- multi-spectral and hyper-spectral satellite images
- ...

A major issue is: how to get a *sensible* ordering for vectors?

Hum... red is greater than green, or is it the contrary?

ToS for multi-variate data

Consider

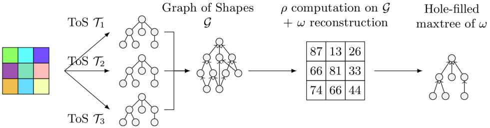
- a function having n scalar components: $\boldsymbol{f} = (f_1, \ldots, f_n)$
- several monotonic transforms ℓ_i to form: $\boldsymbol{\ell} = (\ell_1, \ldots, \ell_n)$
- the marginal composition: $\boldsymbol{\ell} \circ \boldsymbol{f} = (\ell_1 \circ f_1, \ldots, \ell_n \circ f_n)$

If we can compute a tree $\mathfrak{S}$ from multi-variate functions such that:

$$\mathfrak{S}(\boldsymbol{\ell} \circ \boldsymbol{f}) = \mathfrak{S}(\boldsymbol{f})$$

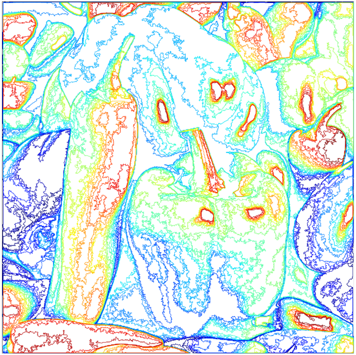
we have a “tree of shapes” for multi-variate data.

Actually we did it:



ToS for multi-variate data

$\boldsymbol{f}$



Level lines from $\mathfrak{S}(\boldsymbol{f})$

No ordering of colors is required! :-)

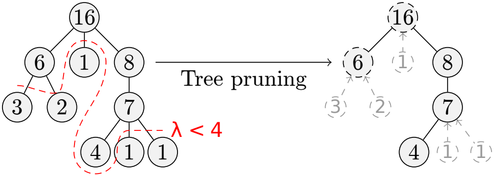
Transition

There are many morphological tools

among them we have trees

so let's see some applications...

App: Grain filter



1. Compute an increasing attribute $\mathcal{A}$ over the tree.
2. Threshold to prune, and reconstruct.

$\mathcal{A}$ can be: area, diameter, width and/or height...

App: Grain filter

FIRST

future. According to Dealogic, 87 SPACs have begun trading in the U.S. since the end of 2003, buying some well-known companies like smoothie purveyor Jamba Juice. Last year alone, 40 SPACs worth $3.4 billion were announced, up from $484 million two years earlier.

SPACs are essentially shell companies. They go public with little more to show investors than a management team and an agreement that the money raised will be used to fund an acquisition in a particular sector, such as retail, or in an emerging market like China. Ergo, a SPAC is a roll of the dice. "Imagine paying $50 to go to a Broadway show, and you have no idea what's behind the curtain," says lawyer Mitchell Littman of Littman Krooks LLP, who works with a lot of SPACs. "You are relying on the fortune and integrity of the management team." Once a deal is completed, the SPAC's managers (who typically receive 20% of the public shares as compensation) are free to sell their holdings, usually after a lockup period.

Today's SPAC boom harkens back to the flurry of "blank check" companies that sprang up in the 1980s. Those were widely discredited by a wave of scams in which fraudsters would take a shell company public, announce a merger, pump the stock, and then dump it before everyone realized that the hot target company was anything but. Investors have a lot more protection with SPACs, which must hold almost all the money in escrow until a deal is done.

From Charney's perspective, there is no downside to the arrangement. It lets him raise cash relatively quickly, remain the largest shareholder, and avoid having to sell other investors on his vision until after his company is already publicly traded. "I'm in a hurry to get the money," he says, citing his plan to expand beyond his current 145 stores. "What's great about the SPAC is, it's kind of in reverse. First you get publicly traded, then you do the compliance stuff. And if you brought in private equity, they might want to run the company in a more [cautious] manner," he says, referring to American Apparel's risqué image.

FIRST YOU GET PUBLICLY TRADED, THEN YOU DO COMPLIANCE.

Keeping control is particularly important for a guy who is the creative force behind the brand, as well as one whose flamboyant management style—there is one sexual-harassment suit outstanding against him by a former employee; two others have been dismissed—might scare off many investors. "He is very talented," says former J.C. Penney CEO Allen Questrom, a partner in private-equity firm Lee Equity Partners, "but he is a *meshugana* [read: nutty] type guy. It's very difficult to see him in a public sphere." (Charney, for his part, dismisses the talk about his reputation as "all tabloid.")

Charney's new bedfellows don't appear concerned, even if they're more likely to wear dark suits than American Apparel's colorful leggings. Endeavor's president, Jonathan Ledecky, is familiar with unusual financing vehicles; he founded U.S. Office Products, the once-hot roll-up that went bankrupt in 2001 (long after Ledecky cashed out). Boldfaced names on the board include Kerry Kennedy, Bobby's daughter, and Edward Mathias, a managing director at the Carlyle Group. On Jan. 31 the same group formed a new $250 million SPAC, Victory Acquisition, underwritten by Citi. The filing notes that it could compete directly with Endeavor. And as with Endeavor, once Victory goes public it will pay a Ledecky "affiliate" $7,500 monthly to run its Manhattan office.

Hedge funds are drawn to these investments. Since SPACs typically must announce a deal within 18 months of their IPOs or return the money raised, with interest, hedgies see them as safe places to park cash while potentially profiting from the price fluctuations between a deal's announcement and its close—a sort of arbitrage play. That may be why Steven Cohen's $12 billion hedge fund SAC Capital snapped up 8.7% of Endeavor's stock when the deal was announced.

Even if investors are SPAC-happy, not all regulators are: So far only the American Stock Exchange lists SPACs, and many bankers and lawyers say the SEC is deliberately taking its time vetting deals, hoping to slow down the pace of offerings. SPACs are probably best left to those, like Charney, who like to live life on the edge.

SPAC Attack!
The volume of special purpose acquisition companies going public in the U.S. is rising sharply.

Number of deals
Value of deals (in billions)

	2004	'05	'06
Number of deals	13	29	40
Value of deals (in billions)	$0.5	$2.1	$3.4

UNHEALTHY RETURN Since smoothie chain Jamba Juice went public in November via a SPAC, its shares have fallen 19%.

FORTUNE CHART / SOURCE: DEALOGIC; PHOTO: DAVID MCNEW—GETTY

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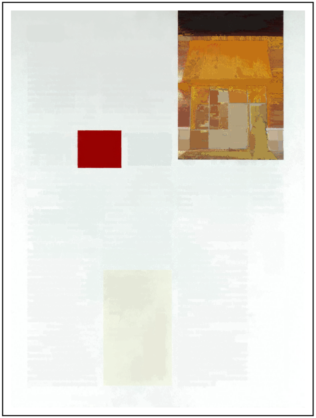


getting the yellowish low-contrasted box is possible, and with very precise contours

App: Grain filter

AFRICA

Emotional Intelligence

Soul City, South Africa's wildly popular soap opera, spreads public-health messages across the continent

By PETER HAWTHORNE JOHANNESBURG

It's 8:30 on a Tuesday night, and millions of South Africans are tuning in to find out if the white social worker, Karen, will be able to adopt AIDS orphan Benni—"People who love me always end up dying," the child says. No, it's not *Melrose Place* or *E.R.* or any of the other imported dramatic series that fill the country's airwaves. It's a show about South Africans themselves. *Soul City*, the local soap opera that began as a good deed, is now an award-winning, multi-media business internationally recognized for its role in Third World development. It is also one of the top three most widely watched progams on South African TV.

Soul City reflects the life and hard times of people in a typical South African urban black township. In fact, most of the series' location shots are taken in Alexandra, one of the oldest and most squalid of South Africa's black slums, just outside Johannesburg. And *Soul City's* clinic, where much of the drama is centered, is based on an actual clinic in Alexandra.

That medical theme is not surprising, since the idea for the show came from two young white South African doctors, Garth Japhet and Shereen Usdin, who had studied and worked in township hospitals and clinics, including in Alexandra. They envisioned a dramatic program that would highlight the social and health problems—particularly HIV-AIDS—in the townships. They convinced a long list of donors—which today include the European Union, Britain's Department for International Development, UNICEF, British Petroleum, the mobile-phone network MTN and the South African government's Department of Health—to put up the money, and *Soul City* was launched nationwide on the South African Broadcasting Corporation's TV service in 1994. "We realized that to be effective we had to go out at prime time, we had to keep it going and we had to be as good as or even better than any other production," says Japhet. Since then, *Soul City* has collected a bundle of prizes. On June 14, the show received a special award from Britain's One World Broadcasting Trust for its role in "illuminating the concerns of developing societies."

Because AIDS is a huge problem in South Africa—almost 10% of the population is infected with HIV—the subject is featured in every installment of the show. Other topics include violence against women, drunkenness, rape, child abuse, sexual conduct, occupational health, disability, setting up a business, even buying and selling a used car. "This is not just about reality in life, it's about real-life skills," says Japhet, who is executive director of the nongovernmental Institute for Health and Development Communications, which produces the show. *Soul City* also has a radio version broadcast daily in nine of South Africa's 11 official languages, as well as weekly newspaper columns and a series of educational booklets.

Characters speak in several of South Africa's official languages, and the series is subtitled in English. It has been shown in Zimbabwe, Zambia, Nigeria, Namibia, Malawi and Barbados and has been dubbed into French and Portuguese for showing in Ivory Coast, Mozambique and other countries. "This concept has got legs," says Japhet. "It moves."

It has also spun off *Soul Buddyz*, a show for younger viewers, which began a 26-episode run on SABC-TV last year. Much as *Soul City* does for adults, *Soul Buddyz* looks at AIDS and other problems as they affect the young. The show is already being sought by countries outside South Africa.

What makes *Soul City* so successful is not merely its authentic portrayal of township life but also its intimacy with viewers. Many of the actors live in the townships. "That sometimes presents a problem," says Lebo Ramafoko, producer of the latest series. "Our actors are seen as real people. They have to become the advocates and counselors they portray on the screen." Some of the cast members have been discovered through *Soul City's* annual "Search for a Star" initiative. Among them is Doreen Kotsedi-Pitse, a blind woman who will play the part of a radio talk-show coordinator. Kotsedi-Pitse lost her sight as a result of domestic violence, an issue frequently raised by *Soul City*. "It made us all aware how uncomfortably close we are to real life," says Ramafoko.

Soul City is affecting the way South Africans lead their real lives. A study sponsored by the E.U. says there is evidence that the program has played "a major role" in increasing public knowledge of HIV-AIDS and safer sexual behavior. Researchers say, for instance, that there is a "significant" association of the use of condoms with people who watch *Soul City*. The survey did not say whether Benni would find a happy new home. For more about that, tune in to next week's episode. ■

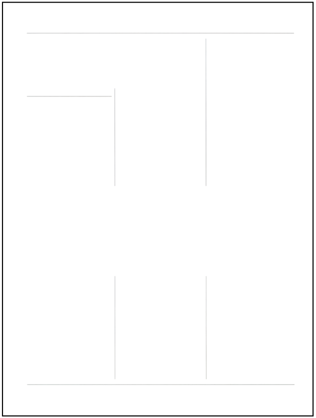
IT'S A WRAP: Actors on the set take a breather from filming after a successful take

HIGH DRAMA: Confronting prejudices toward HIV is one of Soul City's main objectives

EDUCATIONAL FOCUS: Benni tries his hand learning sign language at the township clinic

38

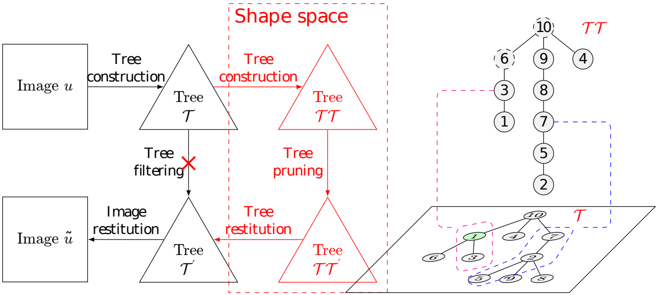
TIME, JULY 2, 2001



Left as an exercise so DIY... (this is filtering at PR level!)

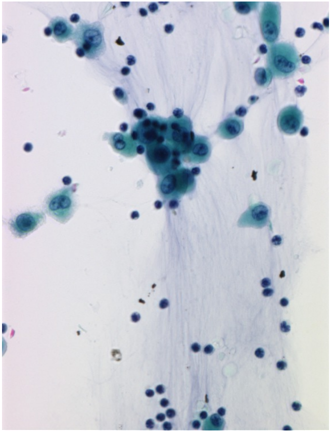
App: Shaping

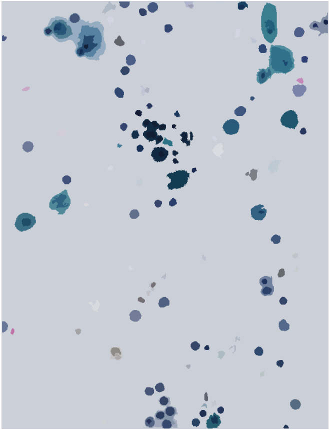
When $\mathcal{A}$ is not increasing:



it is no more a pruning.

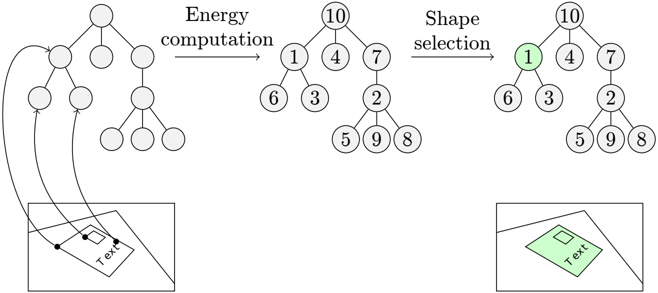
App: Shaping





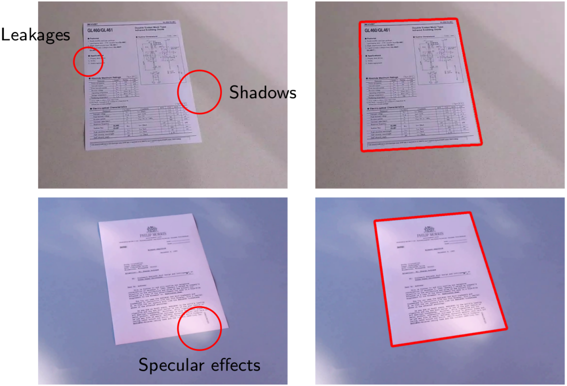
Nodes with low circularity are filtered out.

App: Object detection

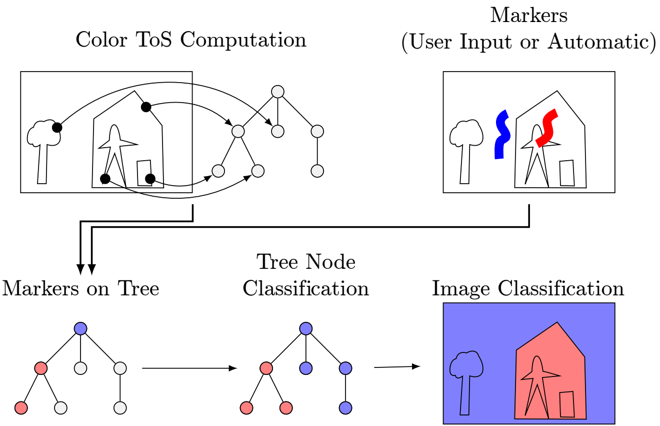


1. Valuate an energy adpated to the object(s) to detect
2. Retrieve the shape(s) with minimal energy

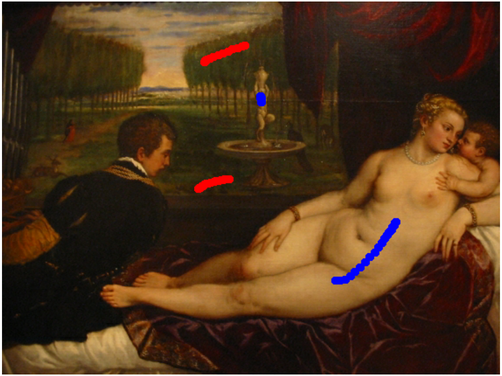
App: Object detection



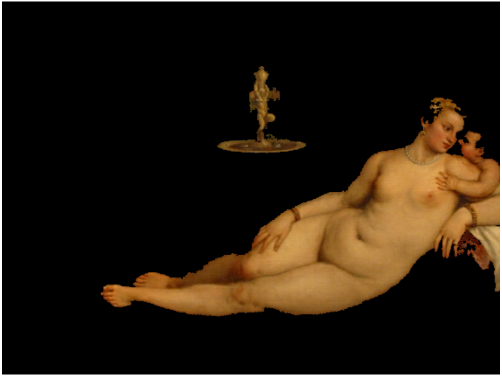
App: Object picking



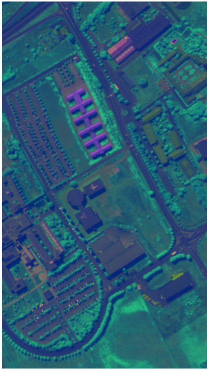
App: Object picking



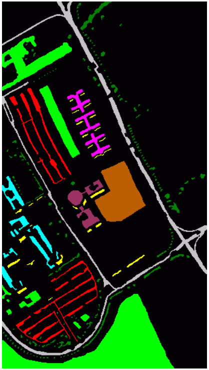
App: Object picking



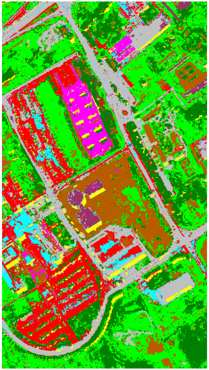
App: Object picking → classification



3 PCA components

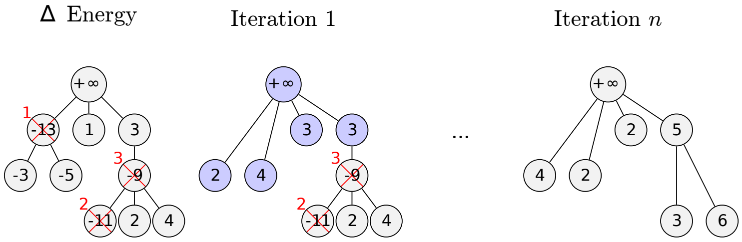


GT



result

App: Simplification / segmentation



1. Sort nodes by increasing meaningfulness
2. Compute ΔE on the tree
3. For each node
   - if removing it makes the energy decrease ($\Delta E < 0$)
     remove it and update the local Δ energy of its relatives
   - otherwise stop

App: Simplification / segmentation

based on the cartoon model of the Mumford-Shah functional

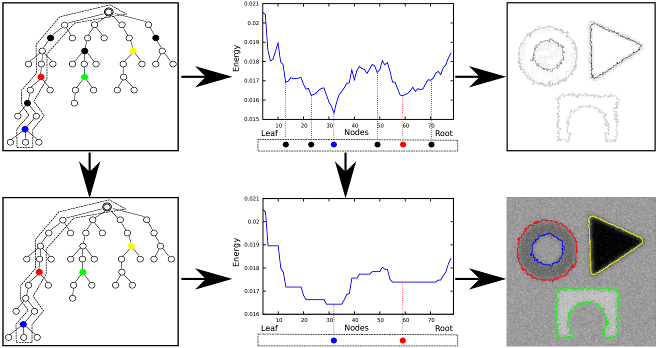
App: From a direct approach to a hierarchical one

For both filtering and segmenting $\rightsquigarrow$ we set a strength value

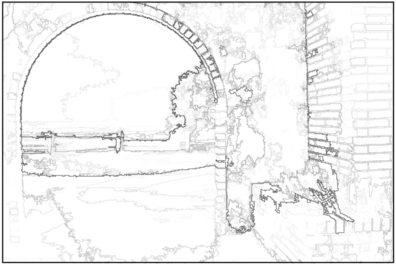
- we can analyze what happens when this strength is varying
- we are interesting in the *saliency* of components
(their persistence w.r.t. this strength)

App: From a direct approach to a hierarchical one

Illustration with ToS + contour meaningfulness energy + shaping:



App: Saliency map

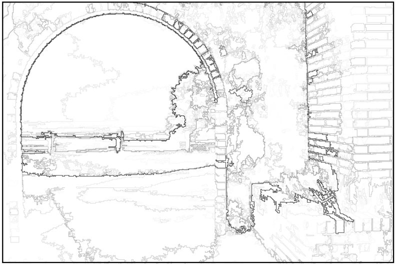


saliency map

original image

there is an underlying hierarchy of segmentations

App: Saliency map

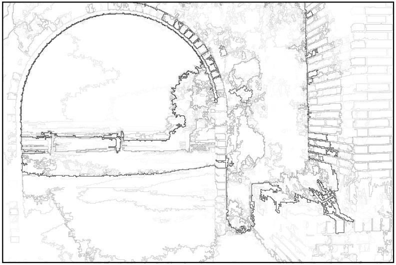


saliency map

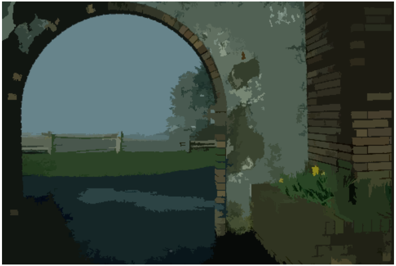
low threshold

there is an underlying hierarchy of segmentations

App: Saliency map



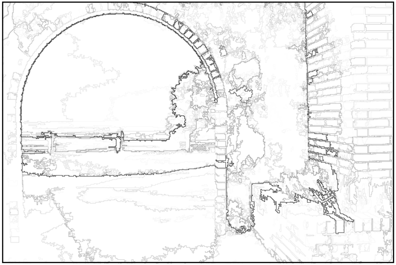
saliency map



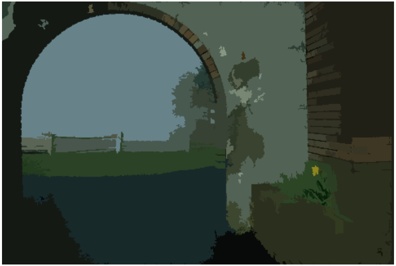
mid threshold

there is an underlying hierarchy of segmentations

App: Saliency map



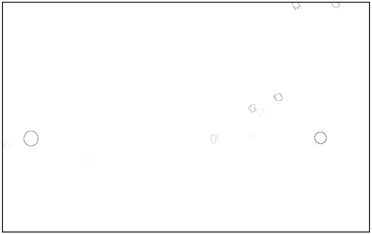
saliency map



high threshold

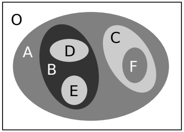
there is an underlying hierarchy of segmentations

App: Saliency map

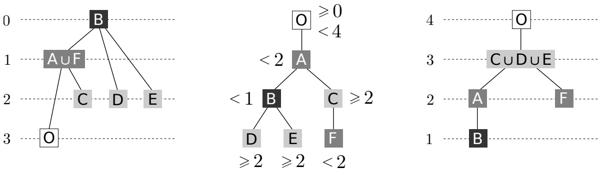


Conclusion

three ways to represent a landscape with component inclusion



f



$\mathcal{T}_{\max}(f)$ $\mathfrak{S}(f)$ $\mathcal{T}_{\min}(f)$

from which we can derive many simple applications

Conclusion

Pros:

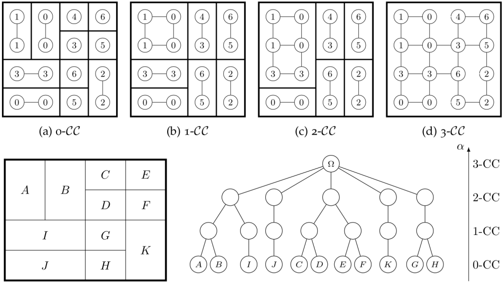
- tree = very convenient structure
- at pattern recognition level
- important invariants of MM

Cons:

- sometimes, object contours $\approx$ only *parts* of level lines

Conclusion

Actually there exist some other morphological hierarchies...



(e) Finest partition of flat zones (left) and the associated α-tree (right).

hierarchy of quasi-flat zones

Conclusion

Actually there exist some other morphological hierarchies...

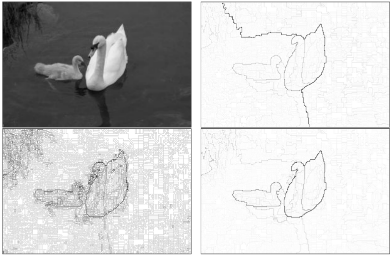


Figure 16: Hierarchies of watershed. Top left: original, top right: flooding by dynamics, bottom left: flooding by area, bottom right: flooding by volume. Hierarchies are represented through their saliency map.

hierarchy of minima dynamics / watersheds

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The end

Thanks for your attention; any questions?