Morphological Filtering in Shape Spaces : Applications Using Tree-Based Image Representations

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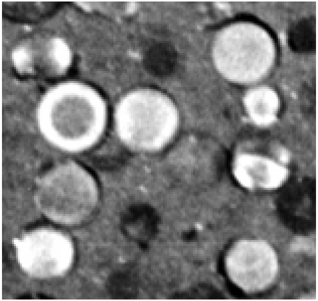
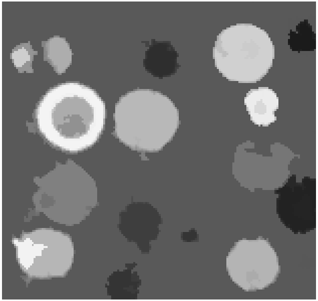
Motivation



Input image.



Result.

Question

How to obtain such a result?

Outline

Outline

Outline

Outline

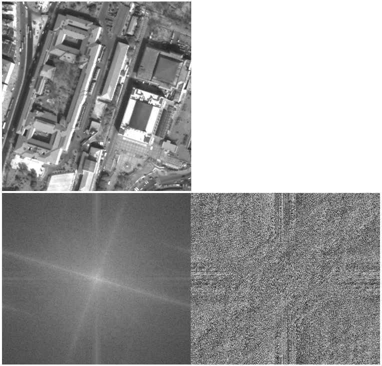
Outline

Outline

Image representations

Decomposition into primitive or fundamental elements that can be more easily interpreted:

- Functional decompositions;
- Multiresolution decompositions;
- Multi-scale representations;
- Threshold decompositions;
- Hierarchical representations.



Amplitude Phase

Image representations

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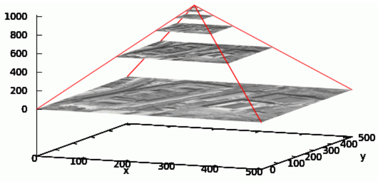

Image representations

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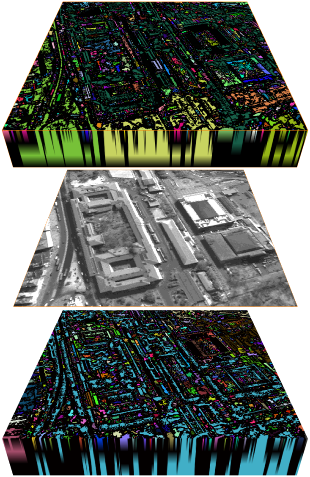

Image representations

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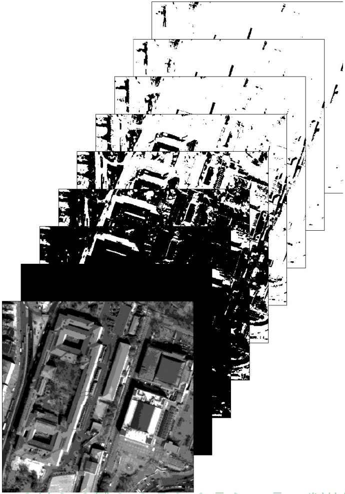

Image representations

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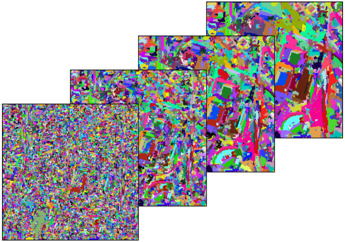


Image representations

Decomposition into primitive or fundamental elements that can be more easily interpreted:

- Functional decompositions;
- Multiresolution decompositions;
- Multi-scale representations;
- Threshold decompositions;
- Hierarchical representations.

Not mutually exclusive.
Properties inherited from those of underlying operations.
Choice driven by the application needs.

Connected operators

What's connected operators ?

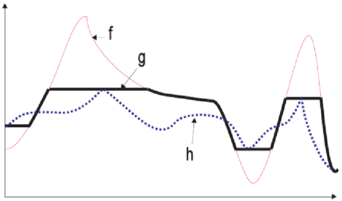
Filtering tools that merge flat zones

Properties

- No new contours
- Keep contours' position

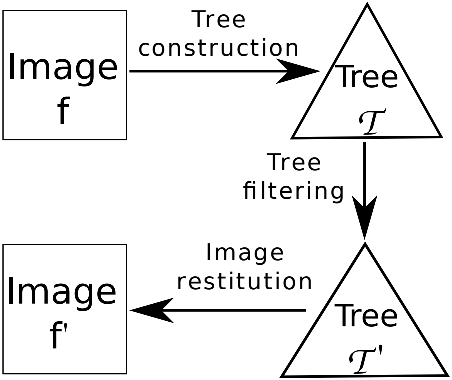
An example : Levelings

Remove details
Preserve the $\leq$ and $\geq$ order

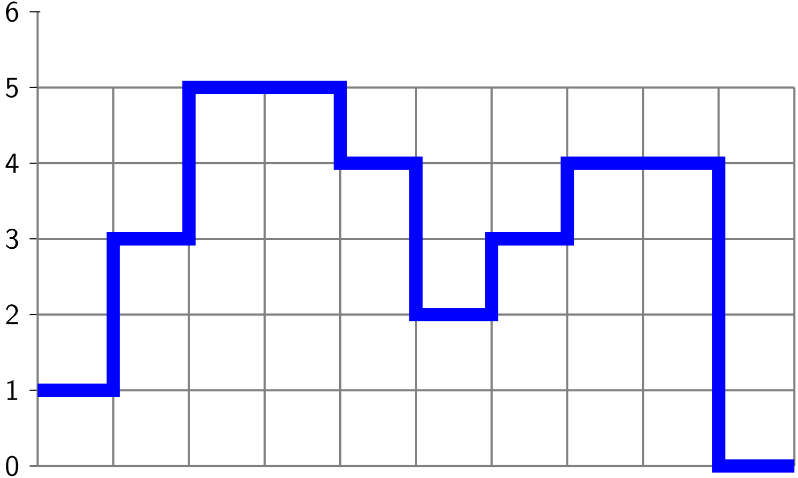


Leveling with marker.
f : input
h : marker
g : result

One popular implementation [Salembier & Wilkinson, SPM, 2009]

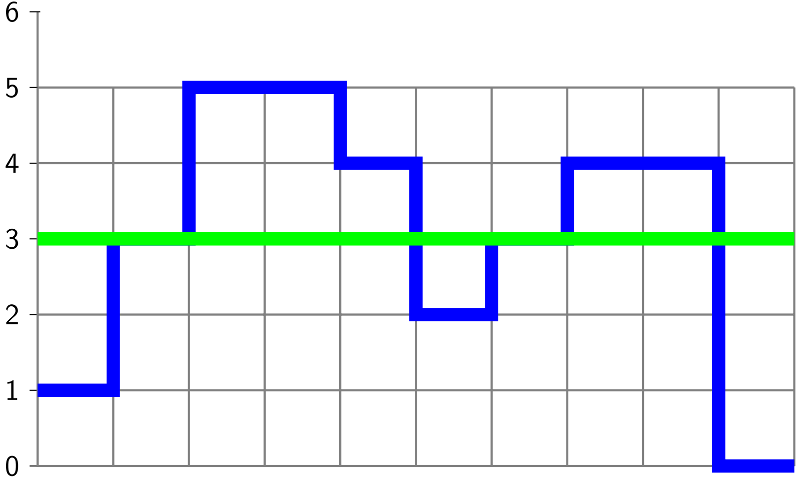


Level sets and components



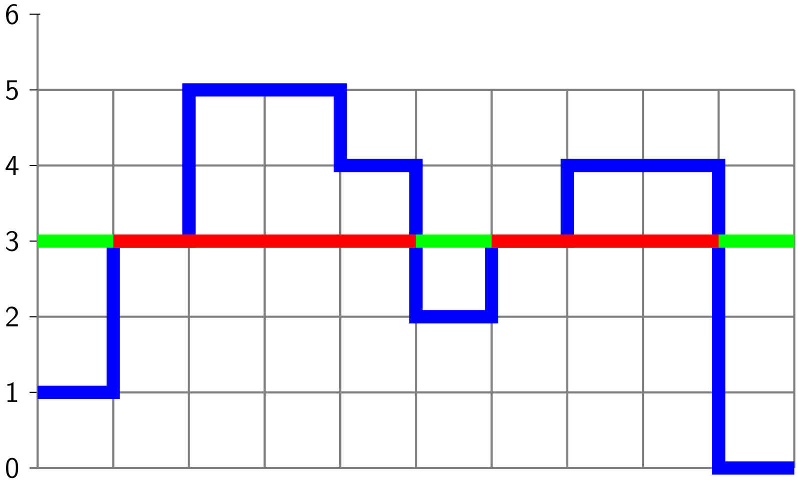
$y = F(x)$

Level sets and components



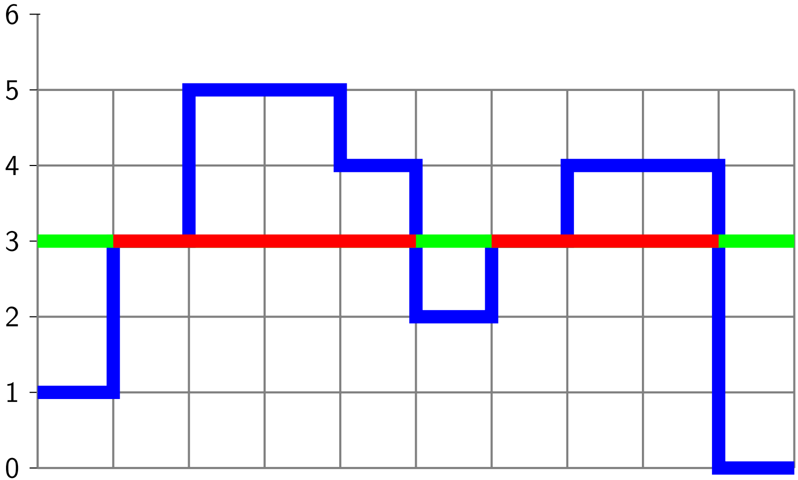
$F(x) \geq 3$

Level sets and components



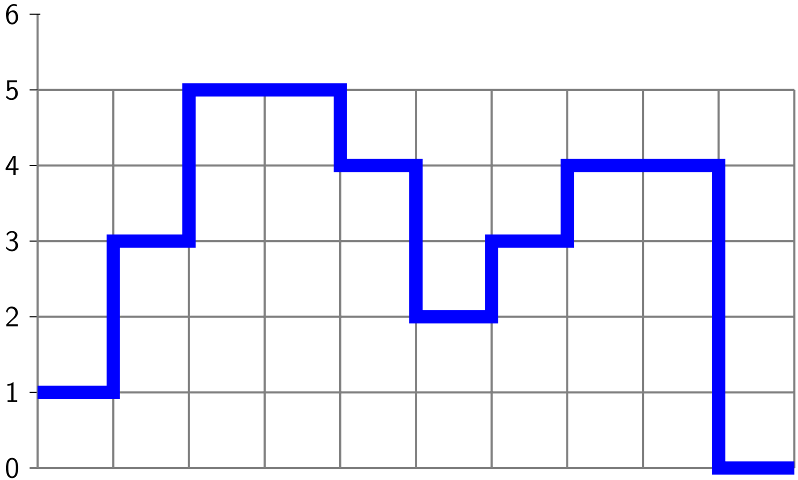
$$F_3 = \{x \mid F(x) \geq 3\}$$

Level sets and components

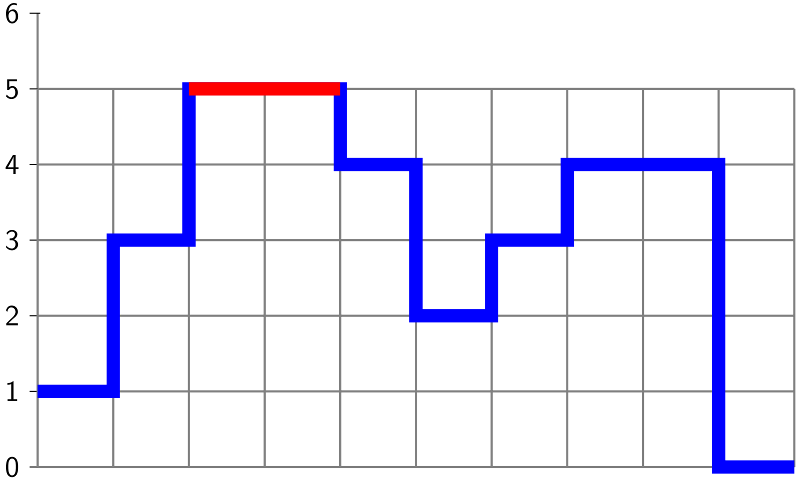


$$F_k = \{x \mid F(x) \geq k\}$$

(Max) component tree

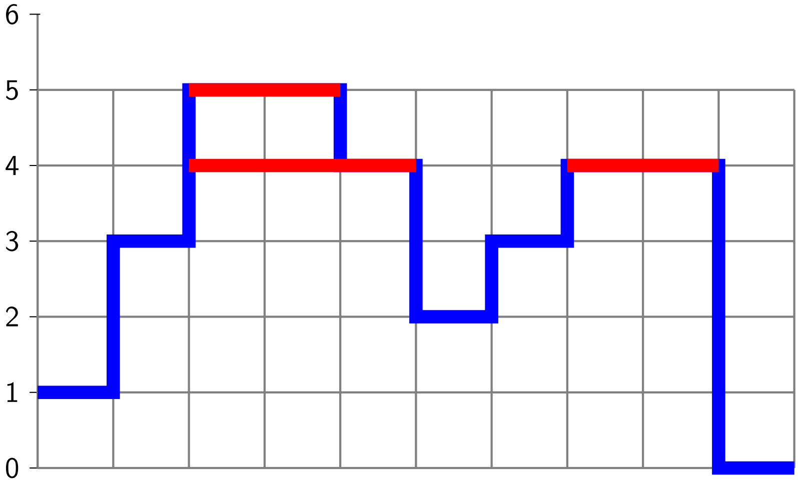


(Max) component tree



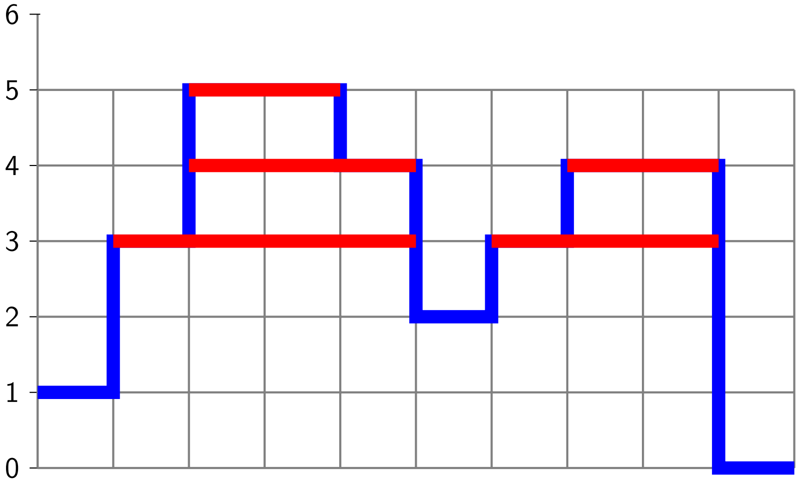
$$F_5 = \{x \mid F(x) \geq 5\}$$

(Max) component tree



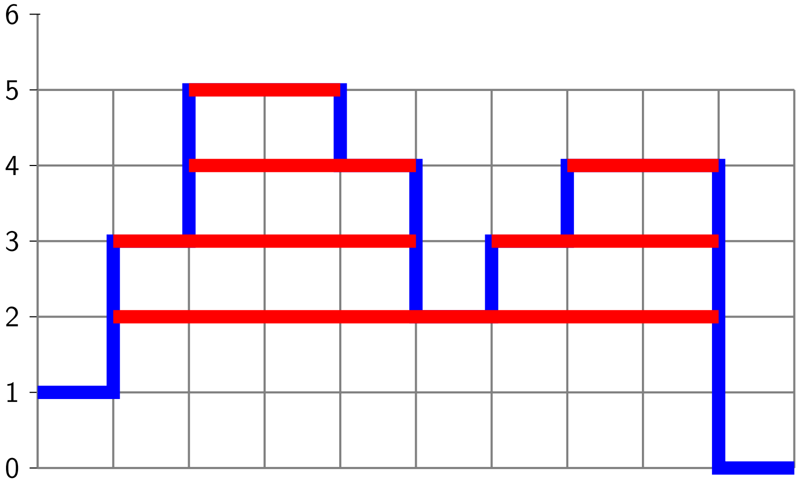
$$F_4 = \{x \mid F(x) \geq 4\}$$

(Max) component tree



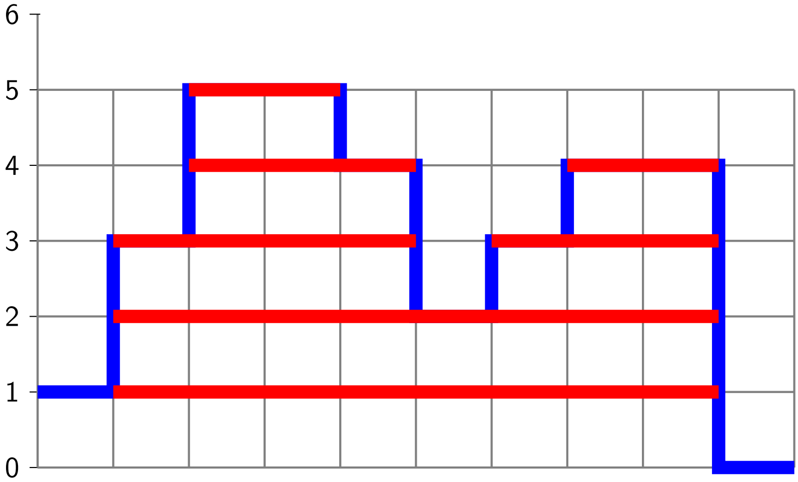
$$F_3 = \{x \mid F(x) \geq 3\}$$

(Max) component tree



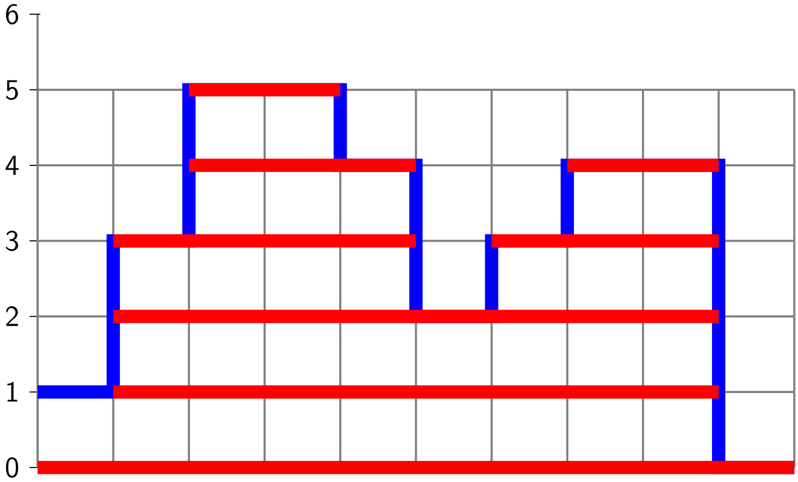
$$F_2 = \{x \mid F(x) \geq 2\}$$

(Max) component tree



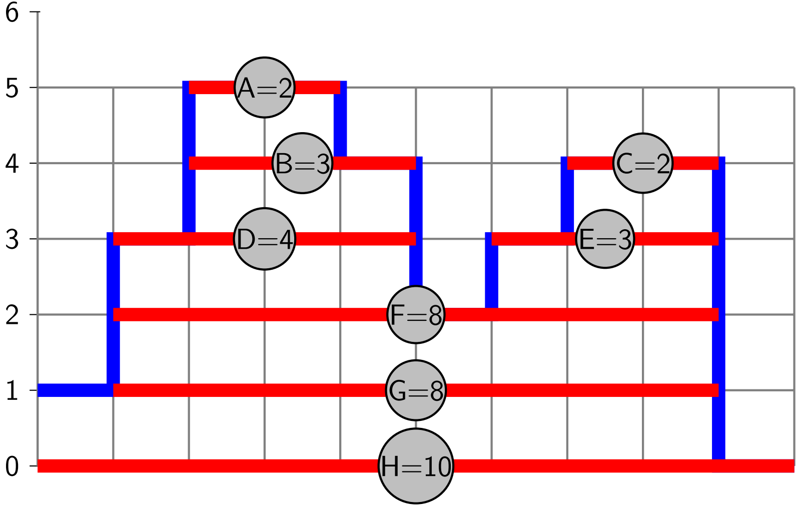
$$F_1 = \{x \mid F(x) \geq 1\}$$

(Max) component tree

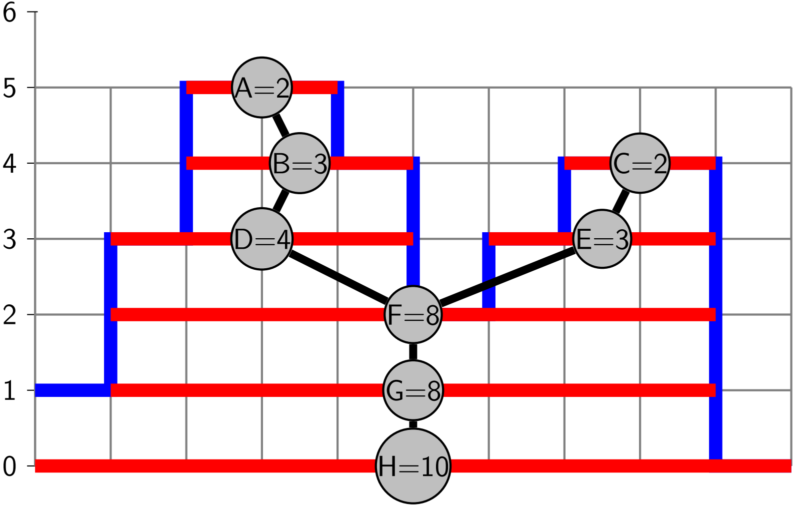


$$F_0 = \{x \mid F(x) \geq 0\}$$

(Max) component tree

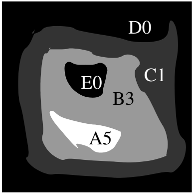


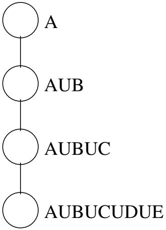
(Max) component tree



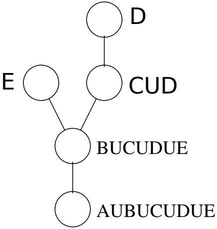
Components + inclusion relationship = component tree

Min-tree, max-tree and tree of shapes [Monasse, ITIP, 2000]

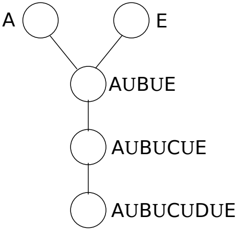




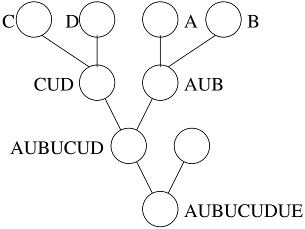
Max–tree



Min–tree

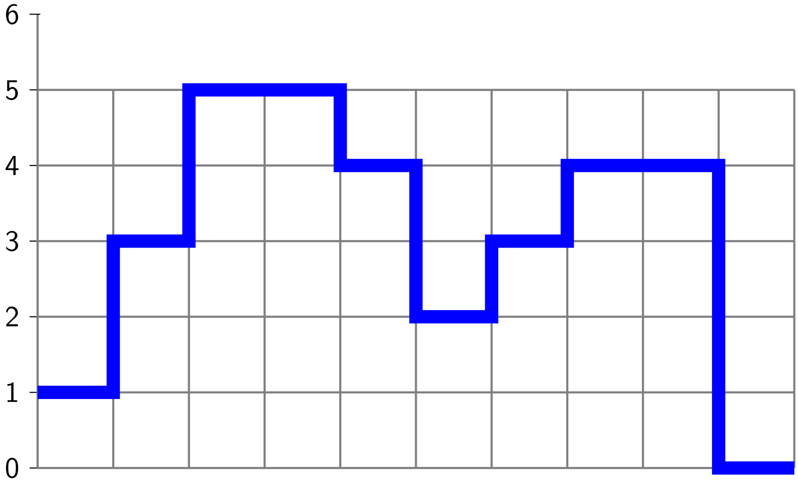


Tree of shapes

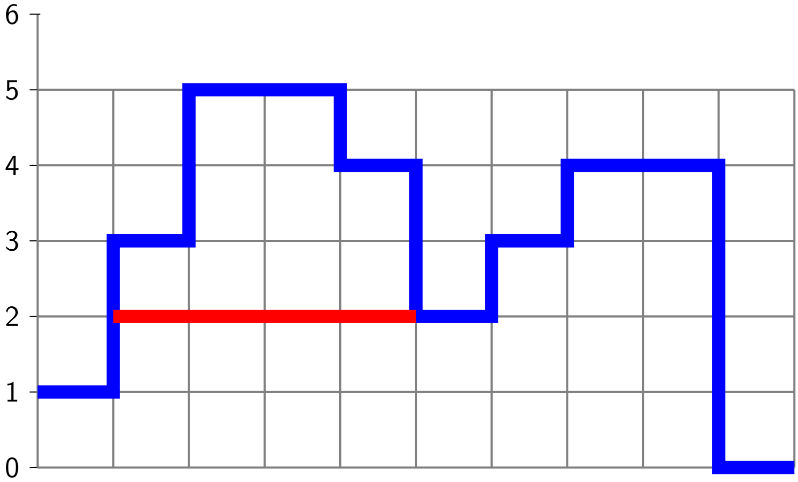


Binary Partition Tree

Attributes

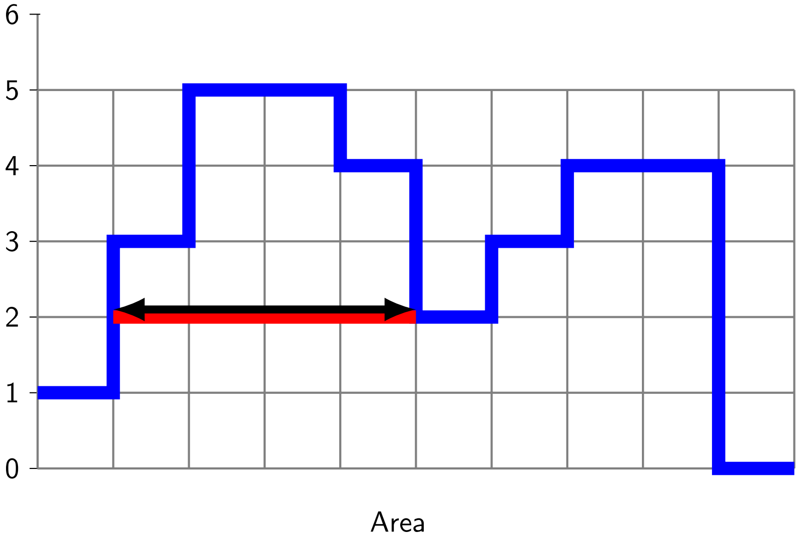


Attributes

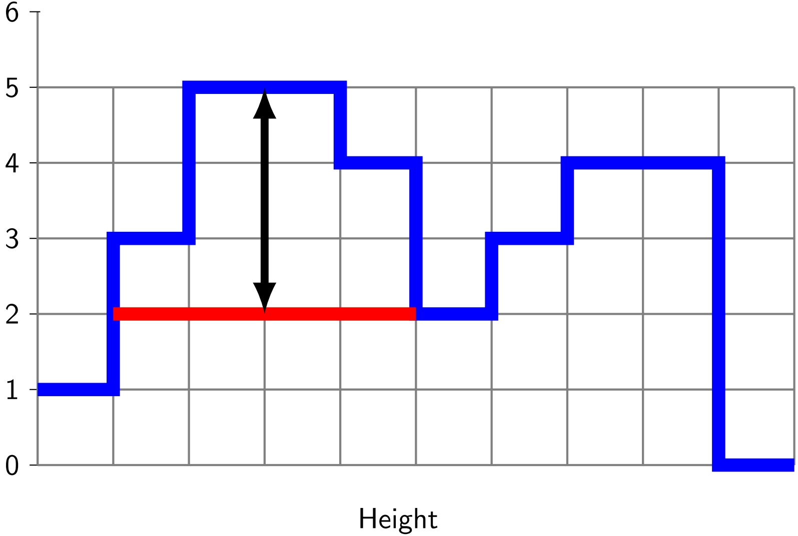


A connected component

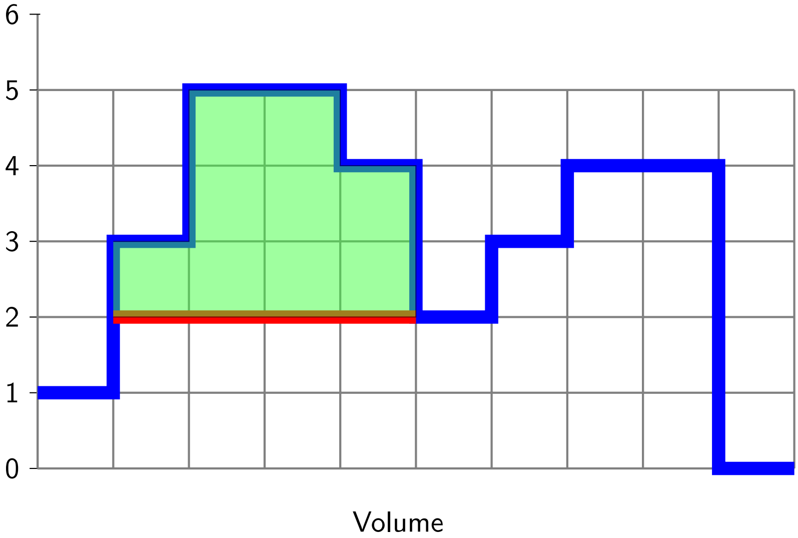
Attributes



Attributes



Attributes



Attributes

Increasing attributes

Increasing attributes : $A \subseteq B \Rightarrow \mathcal{A}(A) \leq \mathcal{A}(B)$
Examples : Area, height, volume.

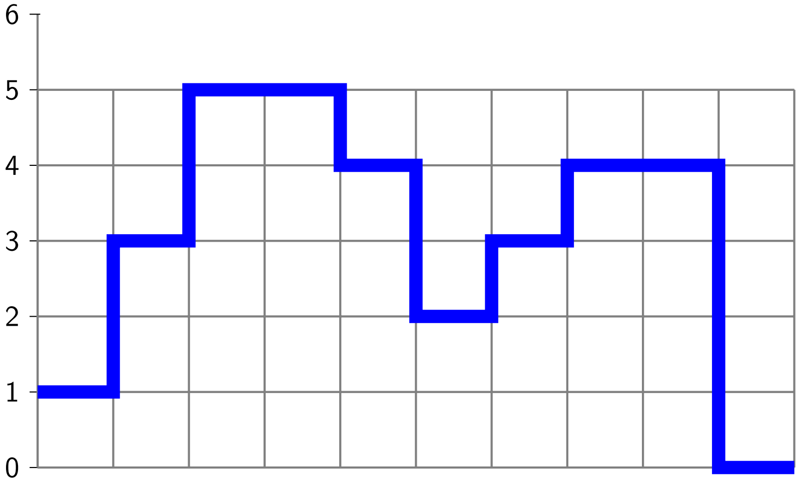
Non-increasing attributes

Shape attributes.

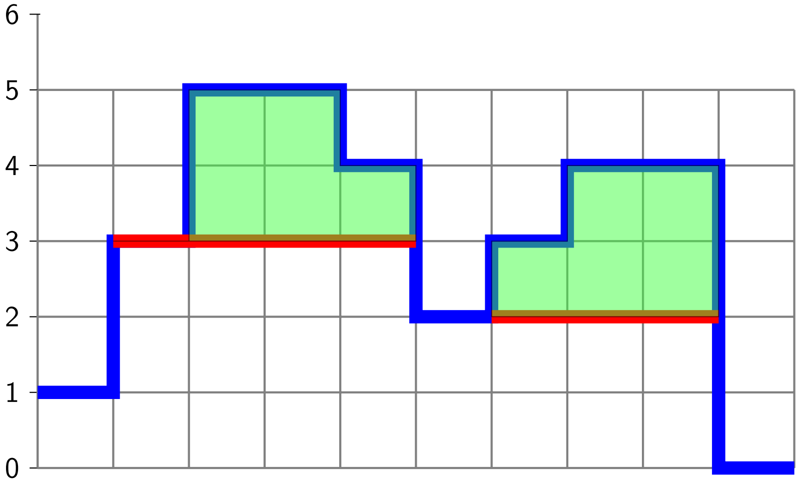
- I/A^2 minimum for a round object
- Circularity : $1 - L_{min}/L_{max}$
- Elongation : L_{max}/L_{min}

L_{min} and L_{max} : Length of the two main axes of the best fitting ellipse

Filtering with increasing attributes

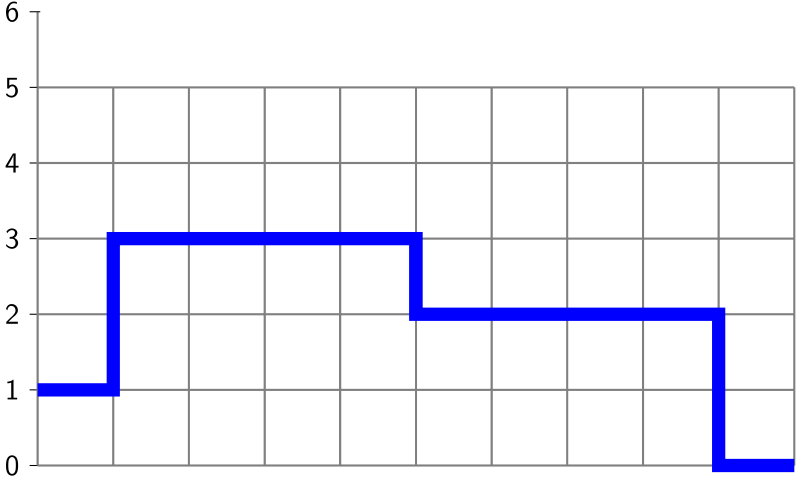


Filtering with increasing attributes



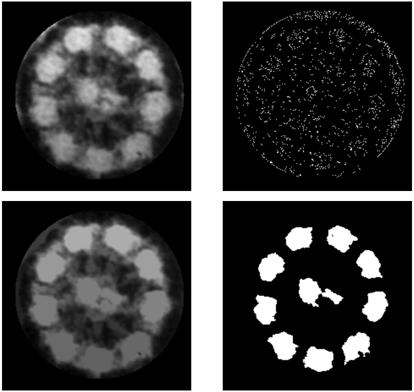
Volume ≤ 5

Filtering with increasing attributes



Filtered function

Application: Filtering with increasing attribute



Question

- *Increasing criterion (here, volume)*
- *How to process non-increasing criteria?*

Filtering with increasing attributes

Pruning the trees

$\mathcal{A} \uparrow$, Pruning the leaves = Attribute thresholding

Non-increasing attributes

How to process the filtering?

Filtering with non-increasing attributes [Salembier & Wilkinson, SPM, 2009]

Pruning strategies

- *Min*
- *Max*
- *Viterbi*

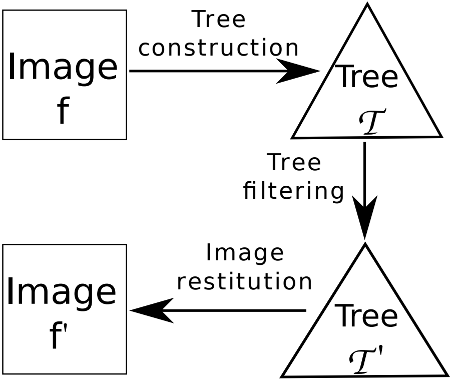
Remove the sub-tree rooted in the node

Attribute thresholding strategies

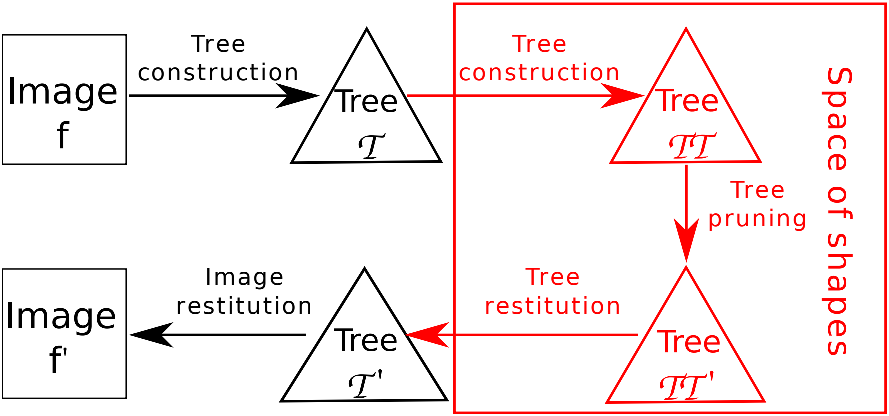
- *Direct*
- *Subtractive*

Remove the nodes under the threshold

Our proposed framework

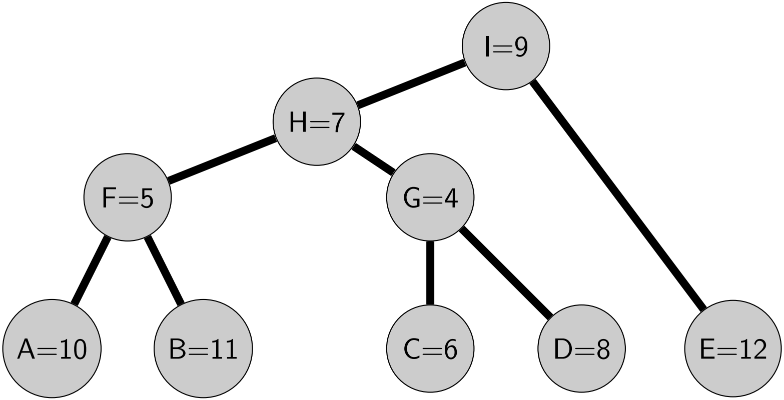


Our proposed framework [Xu & Géraud & Najman, ICPR, 2012]



Outline

Construction of second tree representation



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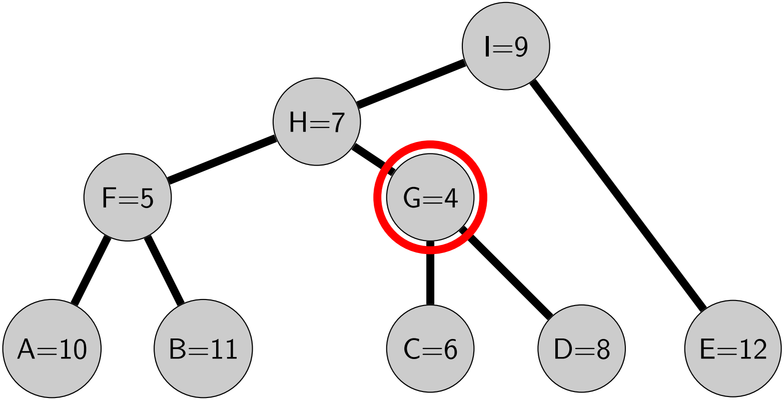
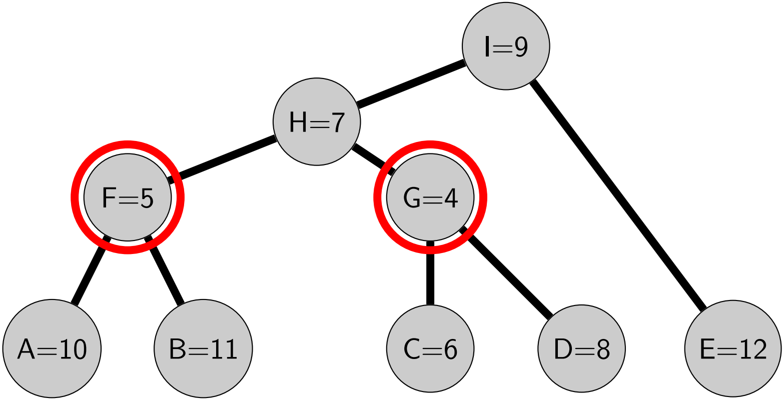


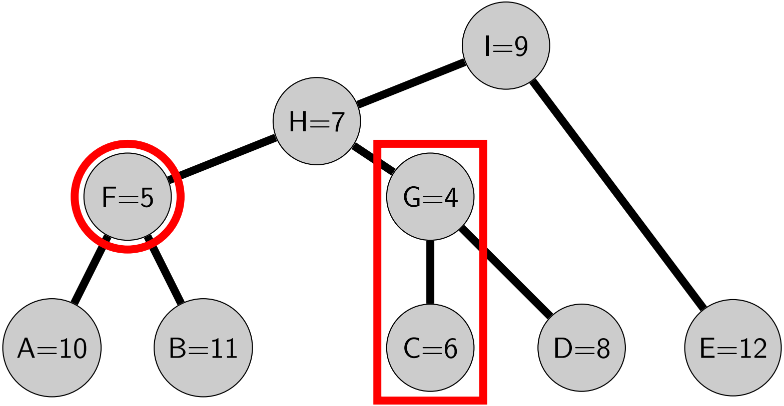
Level $\{x|A(x) \leq 4\}$

Construction of second tree representation



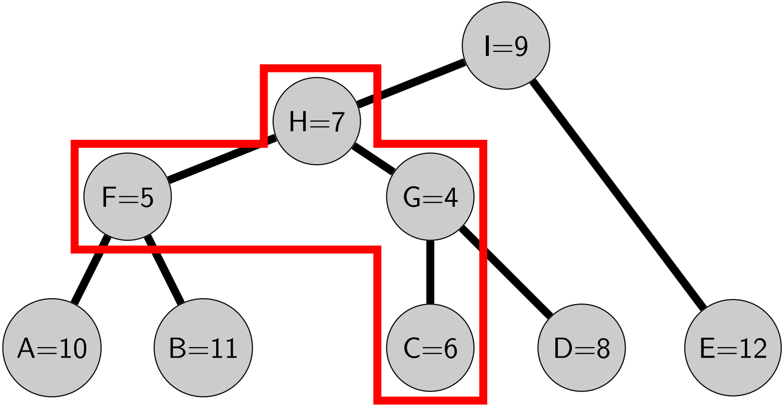
Level $\{x | A(x) \leq 5\}$

Construction of second tree representation



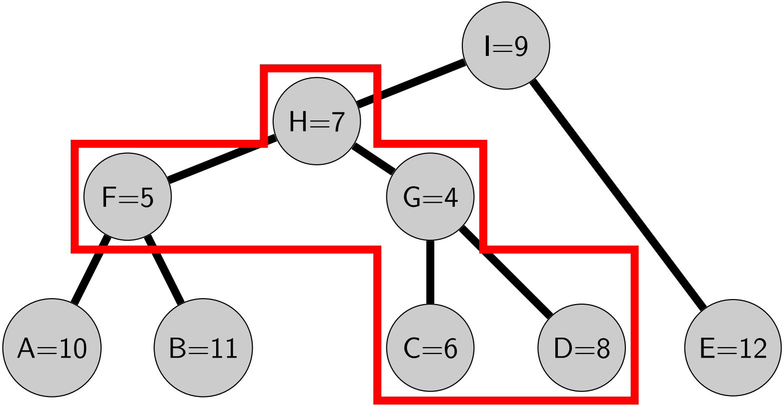
Level $\{x|A(x) \leq 6\}$

Construction of second tree representation



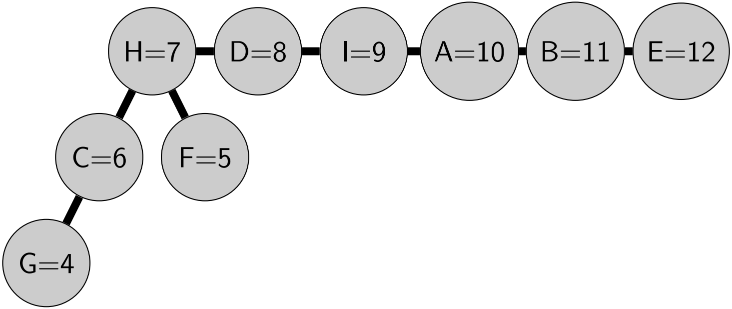
Level $\{x|A(x) \leq 7\}$

Construction of second tree representation



Level $\{x|A(x) \leq 8\}$

Min-tree of a tree-based image representation



Important idea

Computing a Min-Tree on a node-weighted graph instead of a matrix image
Easy thanks to Olena [Levillain & Géraud & Najman, ICIP, 2010], the generic image processing platform `http://olena.lrde.epita.fr`

Encompassing classical attribute filtering strategies

Increasing attribute $\mathcal{A}$

$\mathcal{T}\mathcal{T} = \mathcal{T}$

No need to check if the attribute is increasing or not

Attribute thresholding for non-increasing $\mathcal{A}$

$\mathcal{A}\mathcal{A} = \mathcal{A}$

$\mathcal{A}\mathcal{A}$ is the current level of $\mathcal{T}\mathcal{T}$

Pruning $\mathcal{T}\mathcal{T} =$ Attribute thresholding.

Shape-based levelings and Morphological shapings

Shape-based levelings

$\mathcal{T}$: Min-tree or Max-tree

The order $\leq$ and $\geq$ are preserved $\Rightarrow$ f' is a leveling of f.

$\Rightarrow$ **Shape-based levelings** NEW!

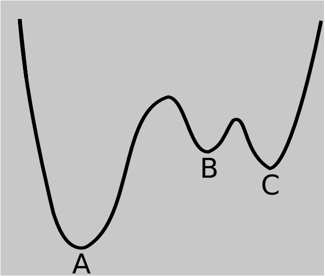
Morphological shapings

$\mathcal{T}$: Tree of shapes

The order $\leq$ and $\geq$ no more guaranteed, not levelings

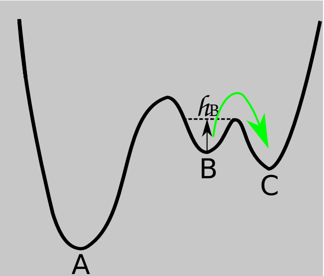
$\Rightarrow$ **Self-dual morphological shapings** NEW!

Extinction-based morphological shapings



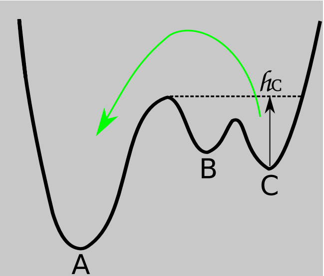
Given a strict order for the set of minima : $A \prec C \prec B$

Extinction-based morphological shapings



B merges with C

Extinction-based morphological shapings



C merges with A

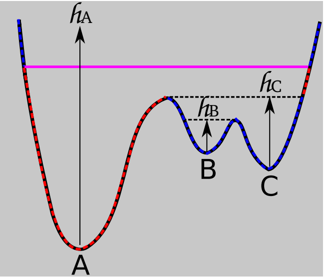
Extinction-based morphological shapings

Strategy

Preserve the **blobs of minima** whose extinction value $>$ a given value

Advantage

Only the shapes being meaningful enough compared with their context are preserved.



Extinction value of three minima.

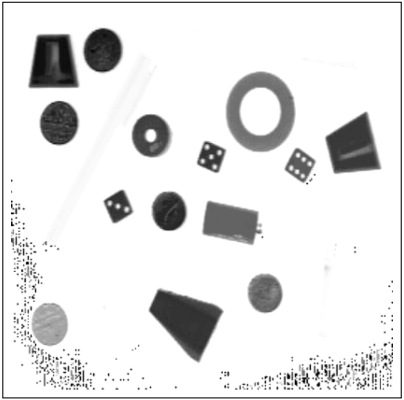
Outline

Shape-based levelings

Input image

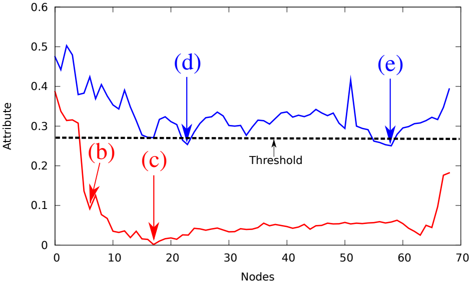
Round objects based leveling

Shape-based levelings

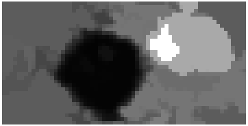


Difference of input image and the shape-based leveling

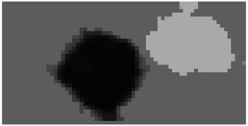
Morphological shapings



Evolution of circularity on two branches.

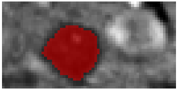


Thresholding.

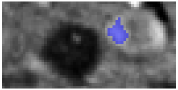


Our shaping.

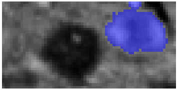
(b)



(c)

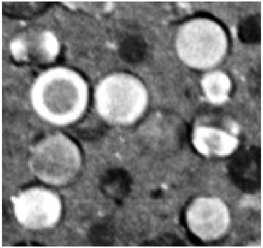


(d)

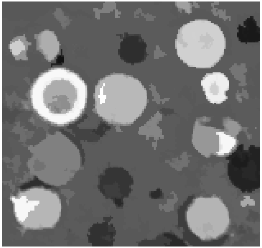


(e)

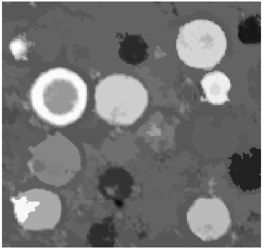
Morphological shapings



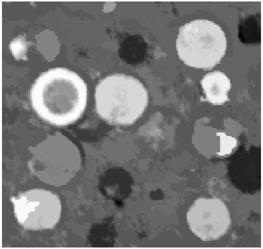
Input image.



Shaping based on $\mathcal{A}$

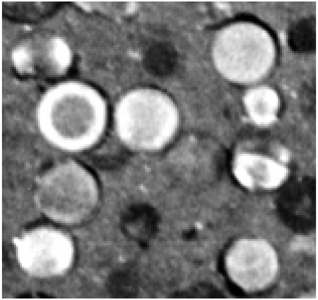


Low threshold of $\mathcal{A}$.

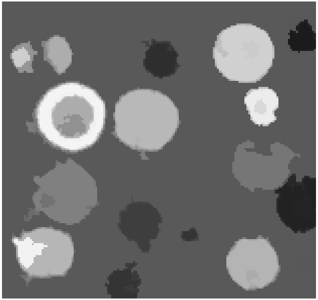


Higher threshold of $\mathcal{A}$.

Morphological shapings



Input image.



Our shaping 2.

Using a combination of attributes $\mathcal{A}$

Context-based estimator for object detection

[Xu & Géraud & Najman, ICIP, 2012]

Important idea

$$E(u, \partial\tau) = E_{int}(u, \partial\tau) + E_{ext}(u, \partial\tau) + E_{con}(u, \partial\tau)$$

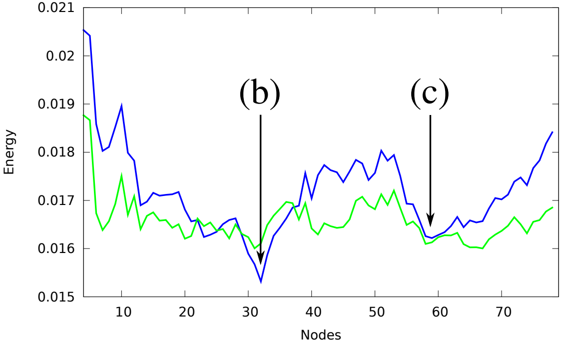
$$V(u, \mathcal{R}) = \sum_{p \in \mathcal{R}} \big(u(p) - \overline{u}(\mathcal{R})\big)^2$$

$$E_{ext}(u, \partial\tau) = \frac{V\big(u,\, \mathcal{R}^{\varepsilon}_{in}(\partial\tau)\big) + V\big(u,\, \mathcal{R}^{\varepsilon}_{out}(\partial\tau)\big)}{V\big(u,\, \mathcal{R}^{\varepsilon}_{in}(\partial\tau) \cup \mathcal{R}^{\varepsilon}_{out}(\partial\tau)\big)}.$$

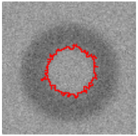
$$E_{int}(u, \partial\tau) = \sum_{e \in \partial\tau} |curv(u)(e)| \;/\; L(\partial\tau),$$

$$E_{con}(u, \partial\tau) = 1 \;/\; L(\partial\tau).$$

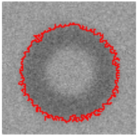
Context-based estimator for object detection



Energy in a branch of the tree;
blue : our energy; green : snake energy



(b)



(c)

Object detection principle

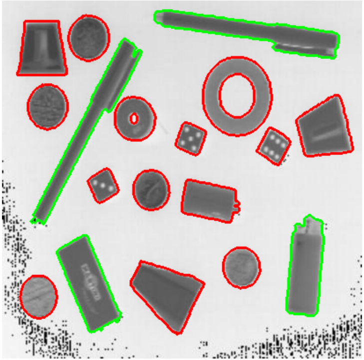
Significant minima $\Leftrightarrow$ Objects

Object detection results

Input image

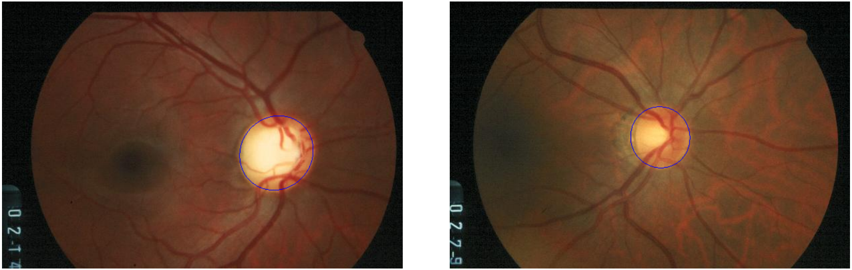
Objects detected

Object detection results



Objects detected using shape attribute
Red ones : circularity-based; Green ones : Inverse elongation-based

Object detection results

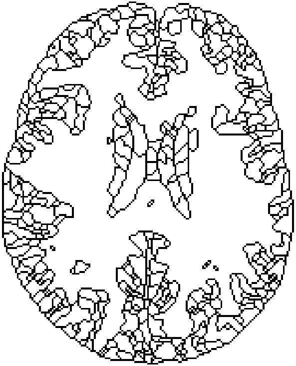


Object detected using designed shape attirbute
by combining roundness, area and position information

Outline

Saliency map

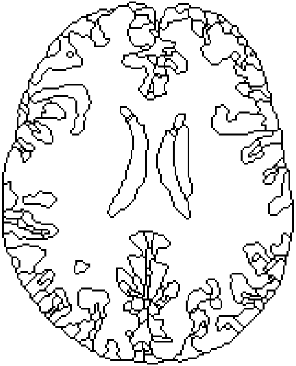
(a) Original image



(b) Some contours

Saliency map

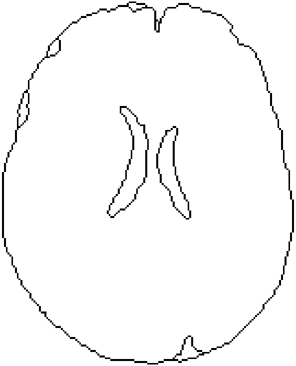
(a) Original image



(b) Some contours

Saliency map

(a) Original image

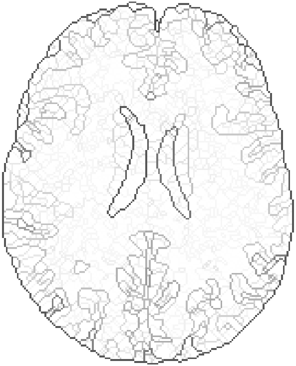


(b) Some contours

Saliency map

Stacking the contours gives a saliency map [Najman & Schmitt, PAMI, 1996]

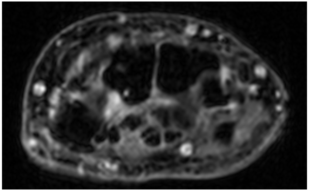
(a) Original image



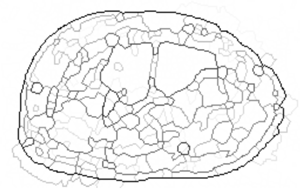
(b) A saliency map

Different representations

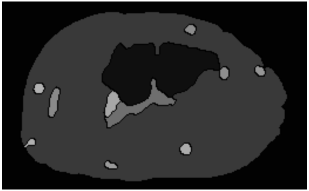
[L. Najman - JMIV - 2011] Mathematical definitions, equivalence between ultrametric watersheds, saliency maps and trees of segmentations



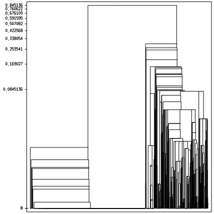
(a) Original image



(b) Ultrametric watershed



(c) One of the segmentations



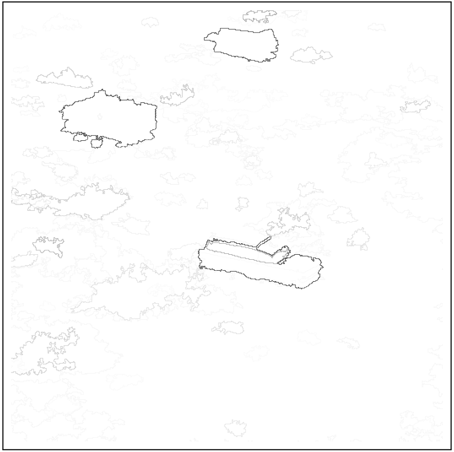
(d) Dendrogram

Saliency maps from shape-based filterings

Principle

Extinction value for minima $\Leftrightarrow$ Persistence of objects $\overset{\mathcal{W}}{\Rightarrow}$ Saliency maps.

$\mathcal{W}$: Weight the object contour with the maximum persistence of object that the contour belongs to.



Saliency map

Hierarchical simplification based on Mumford-Shah

Mumford-Shah energy with cartoon model

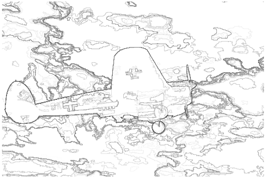
$$E_{\mathcal{T}} = \sum_{\partial\tau \in \mathcal{T}} \left(\sum_{p \in \mathcal{R}(\partial\tau)} \Big(u(p) - \overline{u}\big(\mathcal{R}(\partial\tau)\big)\Big)^2 + \nu L(\partial\tau) \right)$$

Attribute

ν measures the simplification level.

Hierarchical simplification based on Mumford-Shah

Original

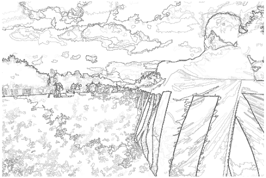


Saliency map

Simplified

Hierarchical simplification based on Mumford-Shah

Original



Saliency map

Simplified

Hierarchical simplification based on Mumford-Shah

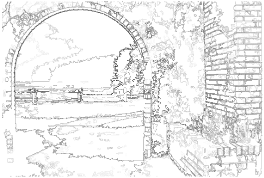
Original

Saliency map

Simplified

Hierarchical simplification based on Mumford-Shah

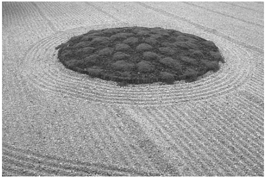
Original



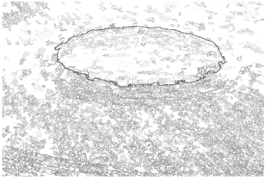
Saliency map

Simplified

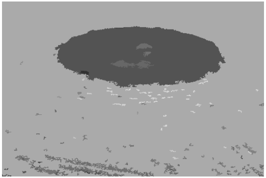
Hierarchical simplification based on Mumford-Shah



Original



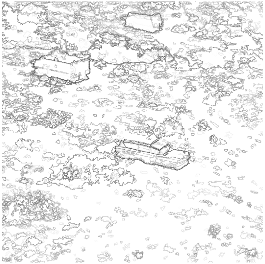
Saliency map



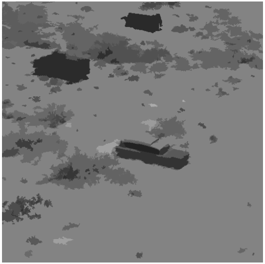
Simplified

Hierarchical simplification based on Mumford-Shah

Original



Saliency map



Simplified

Felzenswalb and Huttenlocher's algorithm [Felzenswalb & Huttenlocher], IJCV, 2004

1. Compute a minimum spanning tree (MST) of a dissimilarity
2. For each edge $\in$ MST linking two vertices x and y, in increasing order of their weights:
   (i) Find the region X that contains x.
   (ii) Find the region Y that contains y.
   (iii) Merge X and Y if

$$Diff(X, Y) < \min\{Int(X) + \frac{k}{|X|}, Int(Y) + \frac{k}{|Y|}\}$$

Question

Is k a scale parameter?

Causality principle

- A contour present at a scale k_1 should be present at any scale $k_2 < k_1$
- Not true with Felzenswalb and Huttenlocher's algorithm

Original

$k = 7500$ (8 regions)

$k = 9000$ (14 regions)

Application of our framework with attribute k

Answer

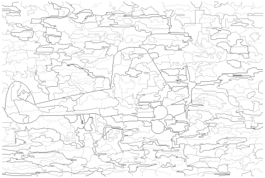
k is not a scale parameter.

Attribute from k

$$k = \max\Big\{\big(\mathit{Diff}(X,Y) - \mathit{Int}(X)\big) \times |X|, \big(\mathit{Diff}(X,Y) - \mathit{Int}(Y)\big) \times |Y|\Big\}$$

Saliency maps using the attribute k

Original



Saliency map

Segmentation(11 regions)

Saliency maps using the attribute k

Original

Saliency map

Segmentation(70 regions)

Saliency maps using the attribute k


Original


Saliency map


Segmentation(20 regions)

Outline

Conclusion

- Object filtering

1. Encompass the state of art
2. Shape-based levelings
3. Morphological shapings

- Object detection

1. Context-based estimator
2. Saliency map

Perspectives

- Attributes $\mathcal{A}$ and $\mathcal{AA}$
- Learning the attributes
- Strategies of dealing with second tree $\mathcal{TT}$
- Properties of the morphological shapings
- Saliency maps

Thanks for your attention !