Regular Model Checking of Epistemic Logic

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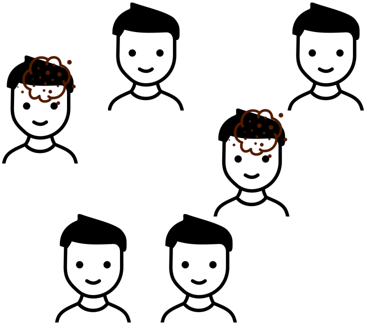
2 MAX PLANCK INSTITUTE FOR SOFTWARE SYSTEMS

joint work with Anthony W. Lin[1,2], Felix Thoma[1]

Muddy Children Puzzle (Littlewood, 1953)



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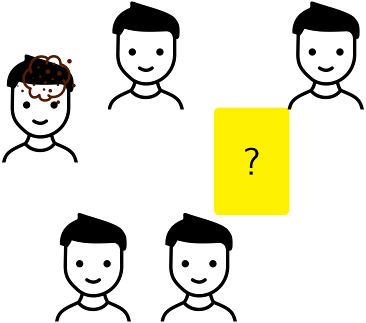

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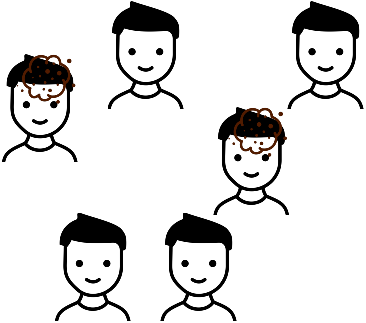

Father: at least one of you is muddy!

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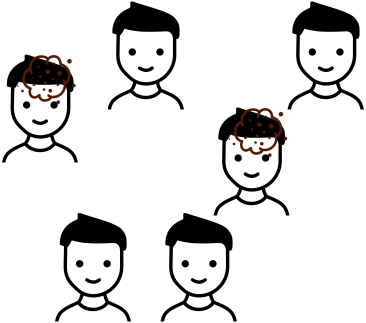

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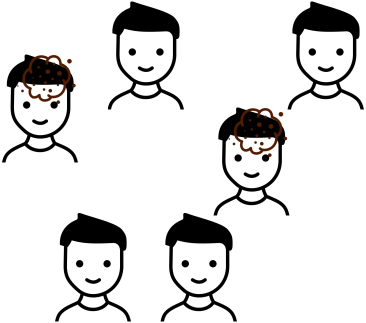

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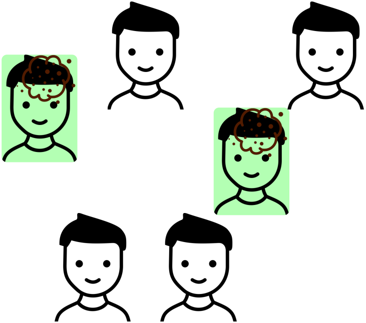

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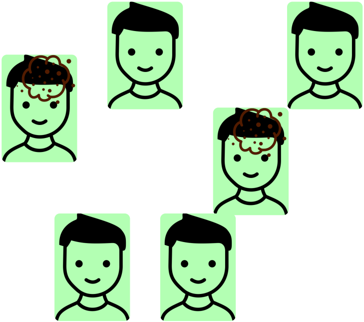

Father: at least one of you is muddy!

Children: ???

Father: indeed, no one knows.

Muddy children: ah, yes we know we're muddy.

Clean children: ah, yes we know we're clean.

Setting

- **Common Knowledge** framework
- Communication primitive: **Public Announcement**
- **Model checking** problem: given some model $\mathcal{M}$ and some specification φ, decide whether $\mathcal{M} \models \varphi$;
- **Parameterized** problem: the number of agents (children) is **not fixed**.

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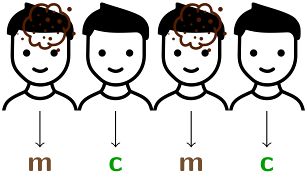
Outline

1. **Parameterized Public Announcement Logic** on **Regular Structures**;
2. **Active Learning** of **Iterated Public Announcement**
3. **Extensions**

1: Parameterized Public Announcement Logic (Modal Logic)

Indistinguishability Relation



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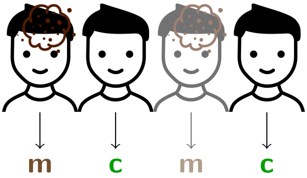

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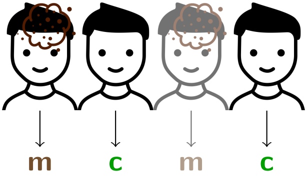
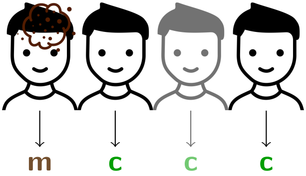


2 might think:



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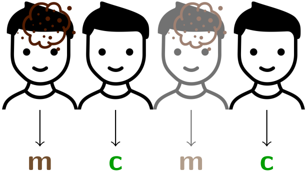
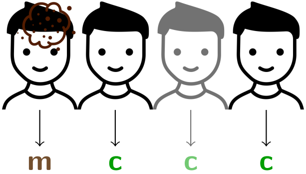


2 might think: $\overset{2}{\sim}$



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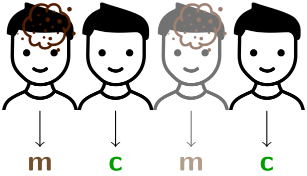

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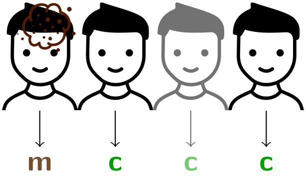
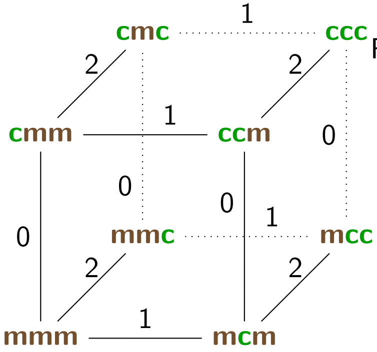


(S5): For every i, $\overset{i}{\sim}$ is an equivalence relation.

Kripke Structures

Case with 3 children



From a given state s,

Kripke Structures

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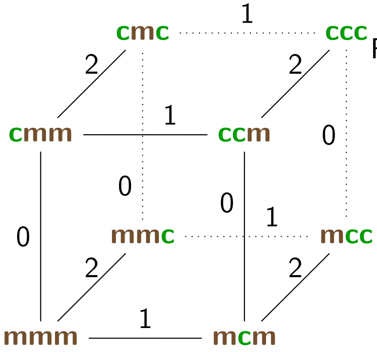

From a given state s,

- If $s \overset{i}{\sim} t$, i *may think* we are in t.

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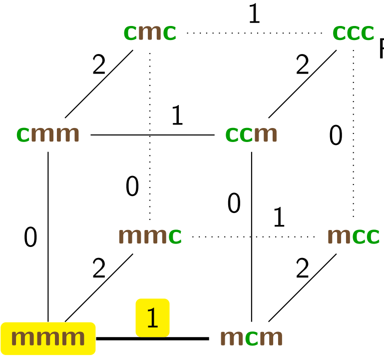

From **mmm**, agent 1 knows that third letter is **m**

From a given state s,

- If $s \overset{i}{\sim} t$, i *may think* we are in t.
- If for all t such that $s \overset{i}{\sim} t$, t satisfies some property φ, then i *knows* that φ holds (from s).

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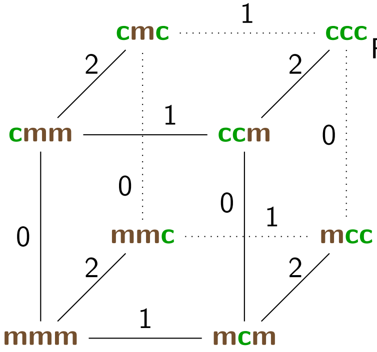

From a given state s,

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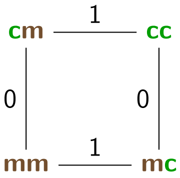
- Any agent knows the structure of the graph: *common knowledge*.

Parameterized Verification of a property φ



Parameterized Verification of a property φ

$$\begin{tikzcd} \mathbf{cm} \arrow[r, dash, "1"] \arrow[d, dash, "0"'] & \mathbf{cc} \arrow[d, dash, "0"'] \\ \mathbf{mm} \arrow[r, dash, "1"] & \mathbf{mc} \end{tikzcd} \models \varphi?$$

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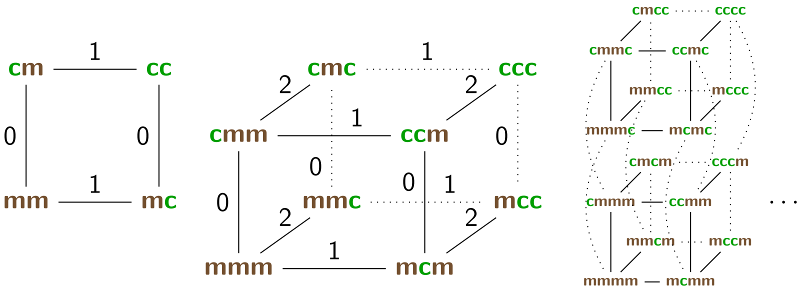

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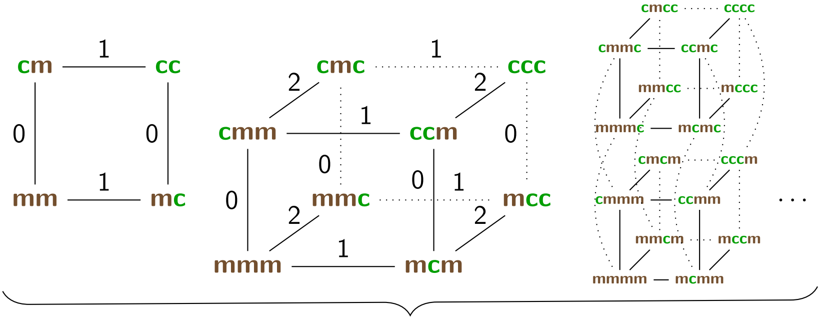

$\models \varphi$?

$\hookrightarrow$ **Infinite** collection of systems

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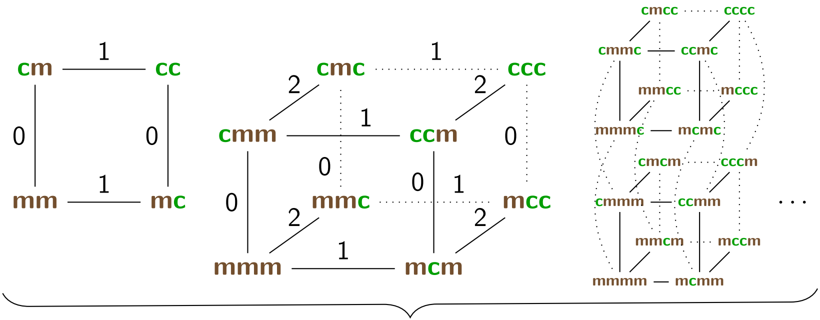

$\models \varphi$? (for **all** states of **all** instances)

$\hookrightarrow$ **Infinite** collection of systems

PAL: Public Announcement Logic (Plaza, 07)

$\varphi ::= p \mid \top \mid \varphi \wedge \varphi \mid \neg \varphi \mid K_a \varphi \mid \langle \varphi ! \rangle \psi$

Where:
$p \in AP$ is an atomic proposition;
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PPAL: Parameterized Public Announcement Logic

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$$\varphi ::= p_{\mathbf{i}} \mid \top \mid \varphi \wedge \varphi \mid \neg \varphi \mid K_{\mathbf{i}}\varphi \mid \langle \varphi ! \rangle \psi \mid \exists \mathbf{i} : \varphi \mid \mathbf{i} = \mathbf{0} \mid \mathbf{i} \% \mathbf{k} = \mathbf{0} \mid \mathbf{i} = \mathbf{j} + \mathbf{a}$$

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$p \in AP$ is an atomic proposition;
$a \in \mathbb{N}$ is a constant;
i, j are index variables, for **agents** and **atomic propositions**. Modal logic, also similar to wS1S. Now combined with Public Announcements.

Semantics of a PPAL formula φ

$$
[\![\varphi]\!] \left(\begin{tikzcd}
 & \mathbf{cmc} \arrow[rr, dotted, dash, "1"] \arrow[dd, dotted, dash, "0"'] & & \mathbf{ccc} \arrow[dd, dotted, dash, "0"'] \\
\mathbf{cmm} \arrow[ur, dash, "2"] \arrow[rr, dash, "1"] \arrow[dd, dash, "0"'] & & \mathbf{ccm} \arrow[ur, dash, "2"] \arrow[dd, dash, "0"'] & \\
 & \mathbf{mmc} \arrow[rr, dotted, dash, "1"] & & \mathbf{mcc} \\
\mathbf{mmm} \arrow[ur, dash, "2"] \arrow[rr, dash, "1"] & & \mathbf{mcm} \arrow[ur, dash, "2"] &
\end{tikzcd} \right) = \{ s \mid s \models \varphi \}
$$

Semantics of a PPAL formula φ

"All children are clean"

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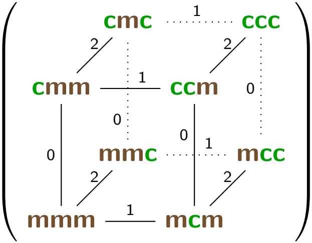

$= \{s \mid s \vDash \varphi\}$

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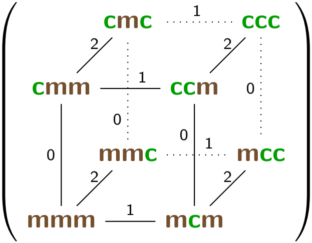
$[\![\forall i : \neg \mathbf{m}_i]\!]$



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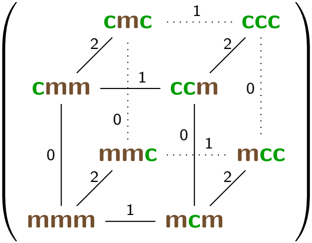
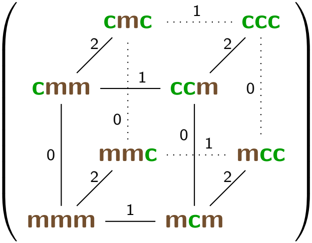


$= \{\mathbf{ccc}\}$

Semantics of a PPAL formula φ

"After announcing there is at least one muddy child,
$\underbrace{\text{all the muddy children know they're muddy''}}_{\varphi \equiv \forall j : \mathbf{m}_j \rightarrow K_j \mathbf{m}_j}$

$[\![\langle\ \exists i : \mathbf{m}_i\ !\rangle\varphi]\!]$



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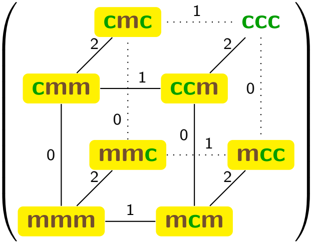
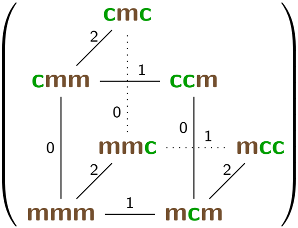
$[\![\langle \boxed{\exists i : \mathbf{m}_i} ! \rangle \varphi]\!]$ 

$= [\![\varphi]\!]$ 

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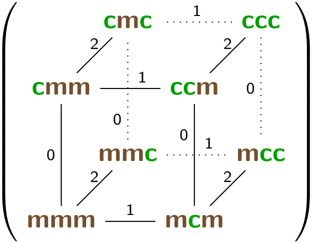

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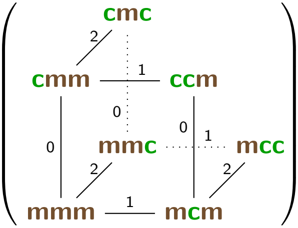
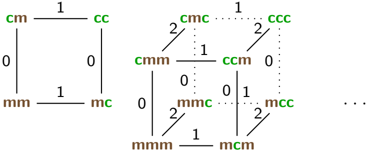


$= \{\mathbf{cmc}, \mathbf{mcc}, \mathbf{ccm}\}$

Semantics of φ on a **paramaterized** system

$$[\![\langle \exists i : \mathbf{m}_i! \rangle \forall j : \mathbf{m}_j \to K_j \mathbf{m}_j]\!] \left(\quad \cdots \right) =$$



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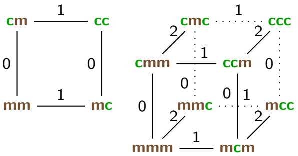

$$\left\{ \begin{matrix} \mathbf{mc}, \\ \mathbf{cm}, \end{matrix} \qquad \begin{matrix} \mathbf{mcc}, \\ \mathbf{cmc}, \\ \mathbf{ccm}, \end{matrix} \qquad \begin{matrix} \mathbf{mccc}, \\ \mathbf{cmcc}, \\ \mathbf{ccmc}, \\ \mathbf{cccm}, \end{matrix} \ldots \right\}$$

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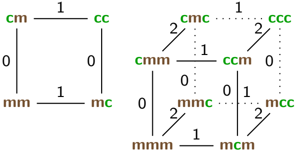

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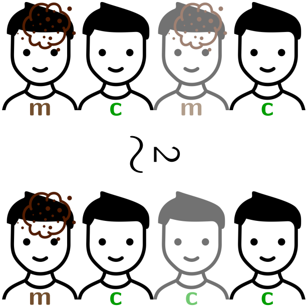
$$\{\mathbf{c}\}^* \cdot \{\mathbf{m}\} \cdot \{\mathbf{c}\}^*$$

Regular Encoding



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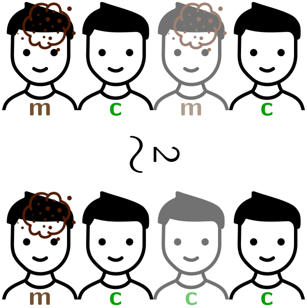
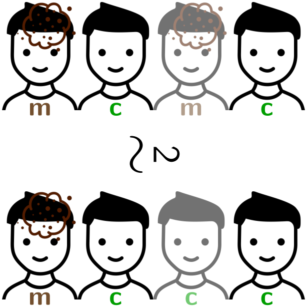


Encoded as

m	c	m	c
0	0	1	0
m	c	c	c

Regular Encoding



Encoded as

m	**c**	**m**	**c**
0	0	1	0
m	**c**	**c**	**c**

$$\in \begin{pmatrix} \{\mathbf{m}, \mathbf{c}\} \\ \times \\ \{0, 1\} \\ \times \\ \{\mathbf{m}, \mathbf{c}\} \end{pmatrix}^*$$

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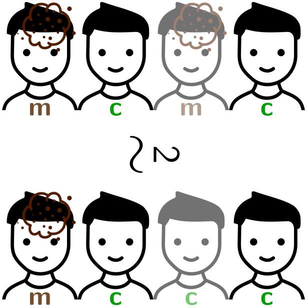
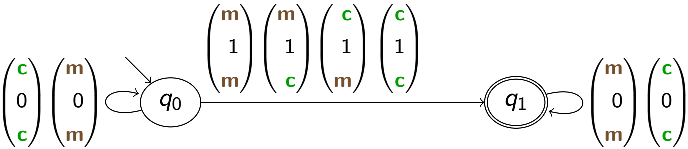


Encoded as $\begin{array}{|c|c|c|c|} \hline \mathbf{m} & \mathbf{c} & \mathbf{m} & \mathbf{c} \\ 0 & 0 & 1 & 0 \\ \mathbf{m} & \mathbf{c} & \mathbf{c} & \mathbf{c} \\ \hline \end{array} \in \begin{pmatrix} \{\mathbf{m}, \mathbf{c}\} \\ \times \\ \{0, 1\} \\ \times \\ \{\mathbf{m}, \mathbf{c}\} \end{pmatrix}^*$



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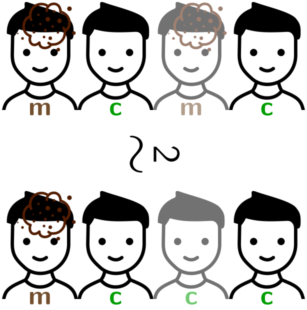

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0	0	1	0
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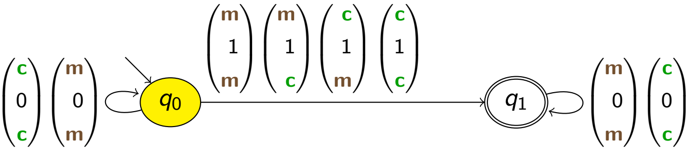

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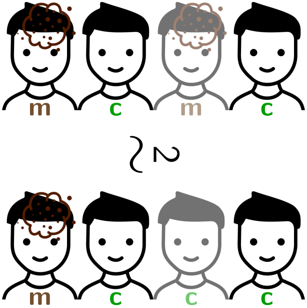

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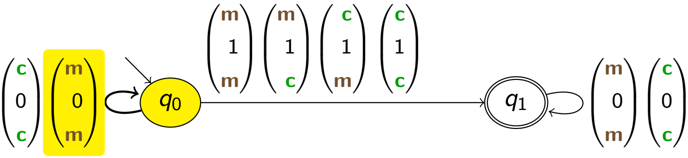

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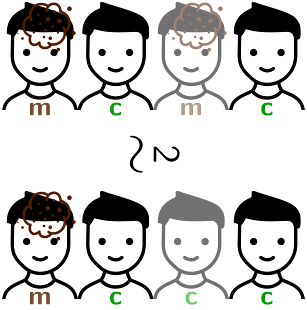

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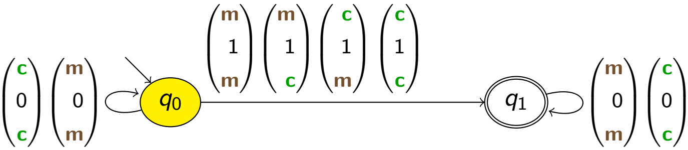

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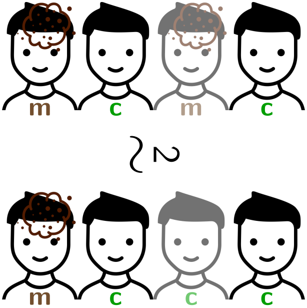
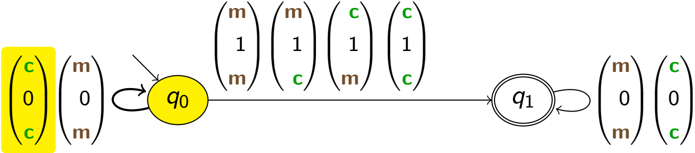


Encoded as

m	c	m	c
0	0	1	0
m	c	c	c

$$\in \begin{pmatrix} \{\mathbf{m}, \mathbf{c}\} \\ \times \\ \{0,1\} \\ \times \\ \{\mathbf{m}, \mathbf{c}\} \end{pmatrix}^*$$



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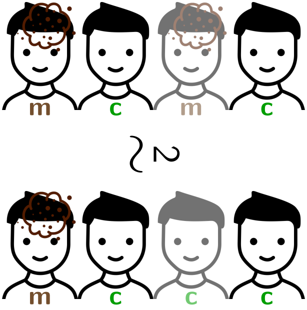

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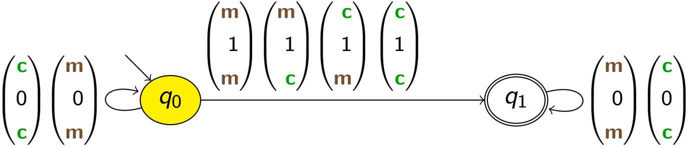

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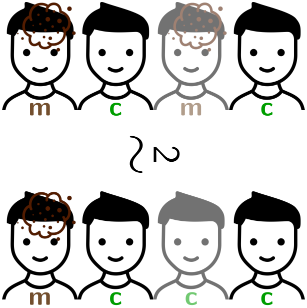

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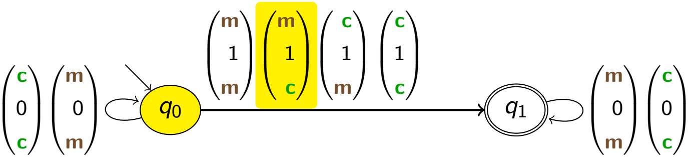

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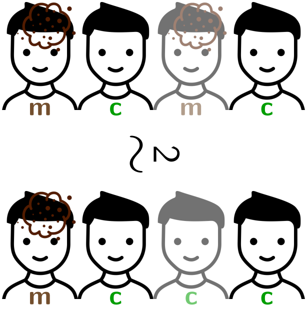

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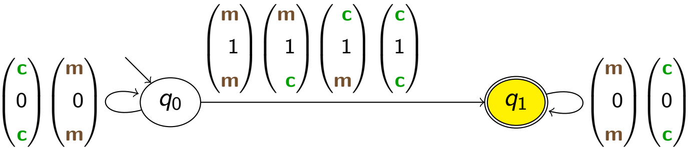

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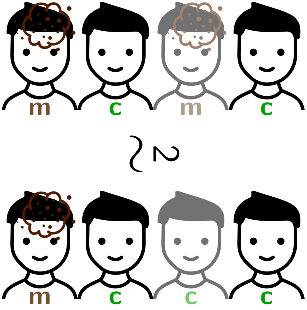

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$$\begin{tikzcd} q_0 \arrow[loop left, "\begin{pmatrix} \mathbf{c} \\ 0 \\ \mathbf{c} \end{pmatrix} \begin{pmatrix} \mathbf{m} \\ 0 \\ \mathbf{m} \end{pmatrix}"] \arrow[rrr, "\begin{pmatrix} \mathbf{m} \\ 1 \\ \mathbf{m} \end{pmatrix} \begin{pmatrix} \mathbf{m} \\ 1 \\ \mathbf{c} \end{pmatrix} \begin{pmatrix} \mathbf{c} \\ 1 \\ \mathbf{m} \end{pmatrix} \begin{pmatrix} \mathbf{c} \\ 1 \\ \mathbf{c} \end{pmatrix}"] & & & q_1 \arrow[loop right, "\begin{pmatrix} \mathbf{m} \\ 0 \\ \mathbf{m} \end{pmatrix} \boxed{\begin{pmatrix} \mathbf{c} \\ 0 \\ \mathbf{c} \end{pmatrix}}"] \end{tikzcd}$$

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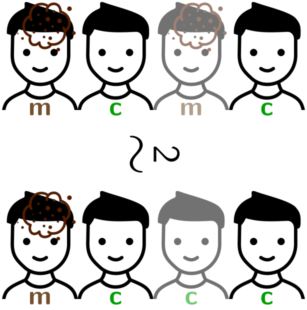

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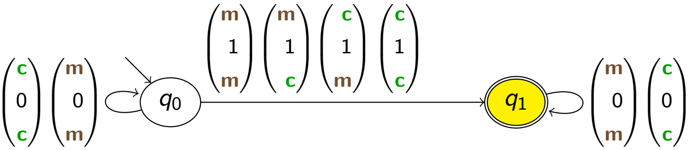

Definition (**Regular** Kripke structure)

$\mathcal{M} = (S, \Sigma, AP, \sim, L)$ where:

- Σ finite alphabet;
- AP finite set of atomic propositions;
- $S \subseteq \Sigma^*$;
- For all $0 \leq i < |s|$, $L_i(s) \subseteq AP$;
- $\sim$ is encoded as a **length-preserving** transducer:

$$T_{\mathcal{M}} = \left\{ s \otimes \left(\underbrace{0 \ldots 0}_{i} \cdot 1 \cdot \underbrace{0 \ldots 0}_{|s|-i-1} \right) \otimes t \;\middle|\; s \overset{i}{\sim} t \right\}$$

Contribution: Regular Semantics of a PPAL formula

$$\left\{ \begin{array}{|c|}\hline w_1 \\ \hline 0^{i-1}10^{n-i-1} \\ \hline w_2 \\ \hline \end{array} \;\middle|\; w_1 \overset{i}{\sim} w_2 \right\} \xrightarrow{[\![\varphi]\!]} \left\{ \boxed{\;w\;} \;\middle|\; \mathcal{M}, w \models \varphi \right\}$$

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Theorem

If $\mathcal{M}$ is a regular Kripke structure, then $[\![\varphi]\!](\mathcal{M})$ is a **regular language***.*
Moreover, the transformation is **effective***.*

PPAL model checking is **decidable**.

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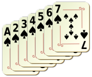
PPAL model checking is **decidable**.

Application Verify the parameterized solution of van Ditmarsch (2003) for 3+x+1 cards.

Another Example: Russian card problem, with $3 + 3 + 1$

cards: 

Alice's goal:
- Making Bob aware of her hand;
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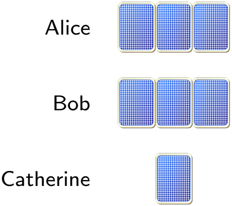

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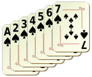
cards: 

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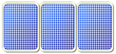
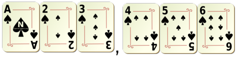
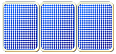
Alice  "I have one of these hands:" 

Bob 

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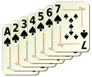
cards: 

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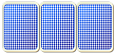
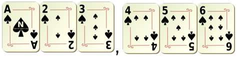
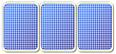
Alice  "I have one of these hands:" 

Bob 

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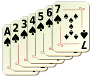
cards: 

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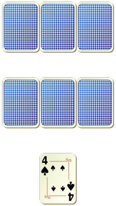
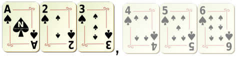


"I have one of these hands:" 

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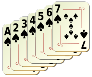
cards: 

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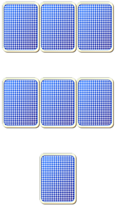

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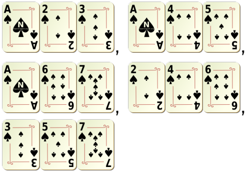

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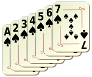

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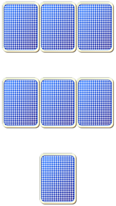

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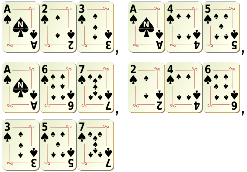

2: Iterated Announcement

- Safety Regular Model Checking
- Active Learning approach
- Learning Disappearance Relation

How many announcements? Iterated Announcement

How many announcements before a (sub)formula holds?
Muddy Children Example:
One muddy child $\rightarrow$ 1 PA;

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k muddy children require **k** announcements to conclude on their state.

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No **fixed** PPAL formula
$PPAL^* = PPAL +$ **iterated announcement**:

$$\langle\varphi!\rangle^*\psi \equiv \exists k \in \mathbb{N} : \underbrace{\langle\varphi!\rangle \ldots \langle\varphi!\rangle}_{k \text{ times}} \psi$$

Safety Regular Model Checking

Let us first consider a more classical problem:

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Definition (Safety Analysis)

Given $\mathcal{M} = (S, \Sigma, AP, \sim, L)$ a regular Kripke structure and two regular sets $Init, Bad \in \mathrm{Reg}(\Sigma)$.
Is the system *Safe*? That is to say:

$$\forall w \in Init, \forall w' \in \Sigma^*, w \sim^* w' \quad \Rightarrow w' \notin Bad$$

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In other words: **Decide** whether $(Init \otimes Bad) \cap T_{\sim^*} = \emptyset$, where $T_{\sim^*}$ is the transducer of the **transisitive closure** of $\sim$.
Decidability:

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Decidability:**No**

Safety Analysis Strategies

Some techniques:

- Regular Model Checking **Using Inference** of Regular Languages (Habermehl and Vojnar, 04)
- Parameterized verification through **view abstraction** (Abdulla, Haziza, and Holik, 15)
- Regular Model Checking using **Widening Techniques** (Touili, 01)
- Regular Model Checking Using **Solver Technologies** and **Automata Learning** (Neider and Jansen, 13)

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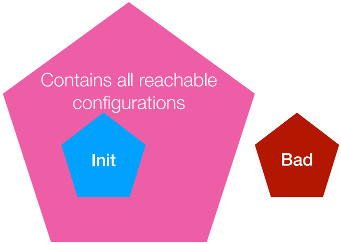
Most of these are based on finding a **regular invariant**:

Definition

$I \in \mathrm{Reg}(\Sigma)$ such that

1. $\mathit{Init} \subseteq I$;
2. $I \cap \mathit{Bad} = \emptyset$;
3. $\mathit{Post}(I) \subseteq I$



Active Learning: the 20 questions game example

"Think of a character, object or animal, and let me ask you questions."

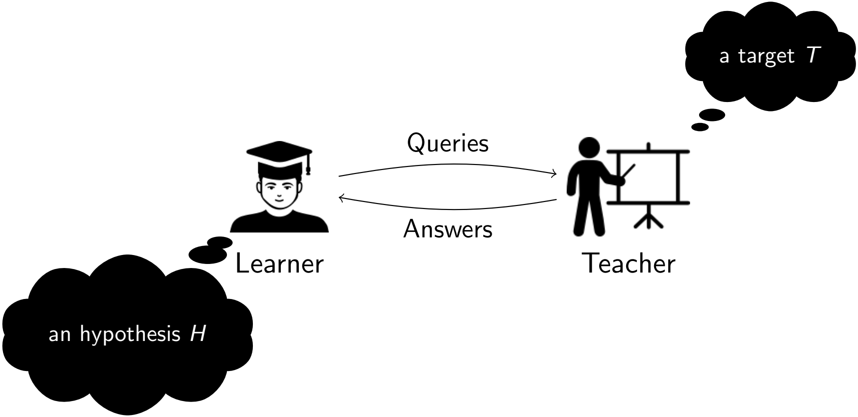
The **learner** asks arbitrary questions

The **teacher** answers by `YES`/`NO`/`MAYBE`.

`https://akinator.com`

`https://en.wikipedia.org/wiki/Twenty_questions`

Active Machine Learning



The **Concept class** $\mathcal{C}$ describes all possible values of $H \in \mathcal{C}$;
Goal for Learner: find $H = T$ or at least $H \sim= T$;
Active learning: the learner chooses the questions.

Regular Language Active Learning (Angluin's L*, 87)

For regular machine learning, the concept to learn is a **finite automaton** $\mathcal{H}$:

- **concept class** $\mathcal{C}$ is the set of all finite automata over Σ
- The **target** is a language $L \subseteq \Sigma^*$.

$$\textbf{Goal}: \mathcal{L}(\mathcal{H}) = L$$

Two types of queries:

- **M**embership queries: "Does $w \in L$?" for some given $w \in \Sigma^*$
  Answer: `YES` or `NO`;
- **EQ**uivalence queries: "Is $\mathcal{L}(\mathcal{H}) = L$?" for some given DFA $\mathcal{H}$
  Answer: `YES` or `NO` and a **counterexample** $w \in \mathcal{L}(\mathcal{H}) \Delta L$

Symmetric Difference of A **and** B: $A \Delta B := A \backslash B \cup B \backslash A$.

Example of a Learner: learnlib

Java Library for active learning of regular languages:
`https://learnlib.de/`

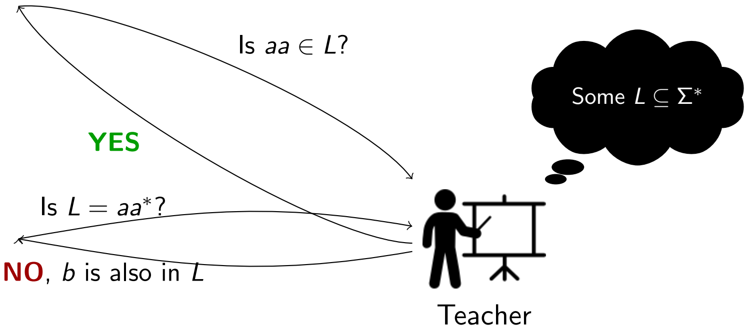
github.com/LearnLib/learnlib/wiki/Instantiating-a-simple-learning-setup

This whole procedure can be implemented as follows:

```java
DefaultQuery<Input, Word<String>> counterexample = null;
do {
    if (counterexample == null) {
        learner.startLearning();
    } else {
        boolean refined = learner.refineHypothesis(counterexample);
        if (!refined) {
            System.err.println("No refinement effected by counterexample!");
        }
    }

    counterexample = eqoracle.findCounterExample(learner.getHypothesisModel(), alphabet);

} while (counterexample != null);

// from here on learner.getHypothesisModel() will provide an accurate model
```

The do-while loop will be executed as long as counterexamples are discovered by the equivalence oracle. Once the loop terminates the hypothesis model provided by the learner is guaranteed to be an exact representation of the target system if the equivalence oracle is guaranteed to find any behavioral mismatches between the hypothesis and the target system (which is the case in this example).

Implementation of a Teacher



A teacher provides **Oracles** (here: a **M** oracle and a **EQ** oracle).

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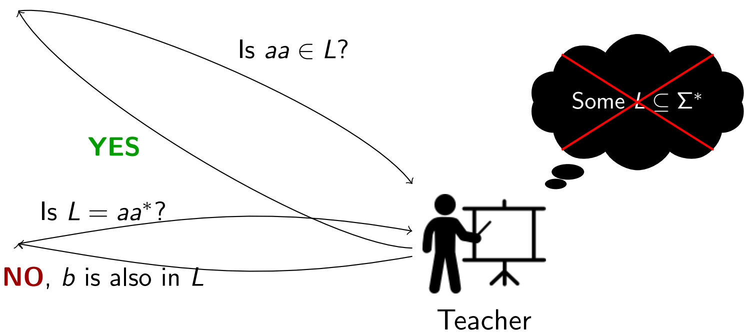

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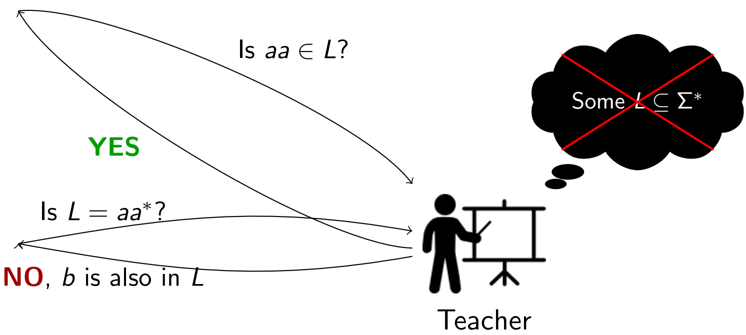

A teacher provides **Oracles** (here: a **M** oracle and a **EQ** oracle).

- In practice, the teacher may **not know the target** L, this is fine as long as he **can answers the queries**.
- The target *might* not be **regular**. In this case, the learner will **never manage** to find a suitable automaton $\mathcal{H}$.

A safety teacher

A regular invariant is defined as first-order properties.

Definition (Regular Invariant)

$I \in \mathrm{Reg}(\Sigma)$ such that

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- **M**embership: Given w, check whether $\exists w' \in Init : w' \sim^* w$.
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Uncovered here: there might be more than one invariant, so there is some slack in the oracle's answer (for membership queries, and for counterexample in (3)).

How many announcements? Iterated Announcement

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Theorem

Model checking of a regular Kripke structure against:

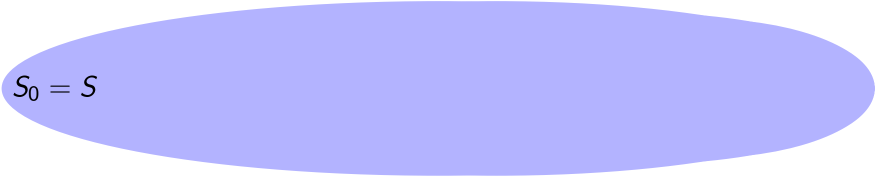
- *a PPAL formula is* **decidable***;*
- *a $PPAL^*$ formula is* **undecidable***.*

We design a **semi-decision** procedure.

Disappearance relation for φ

$$s \preceq t \quad \text{if, and only if,} \quad \forall k,\ s \in S_k \Rightarrow t \in S_k$$

where S_k state space left after k announcements $\langle \varphi ! \rangle$.



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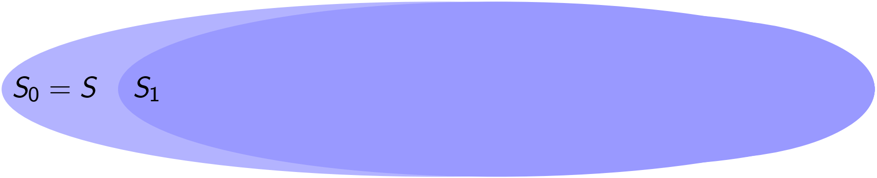

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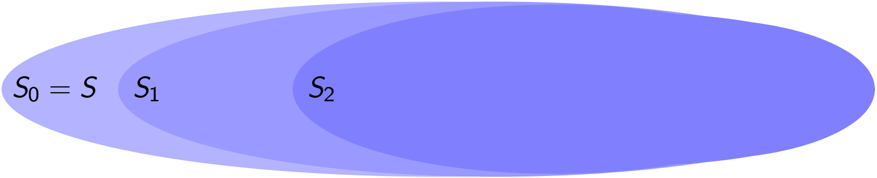

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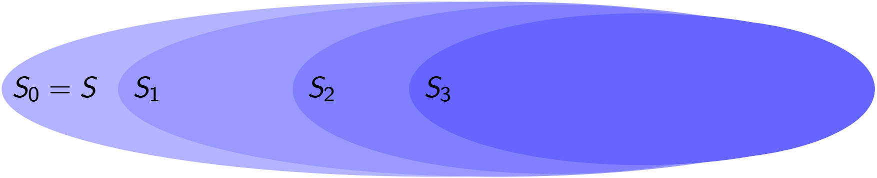

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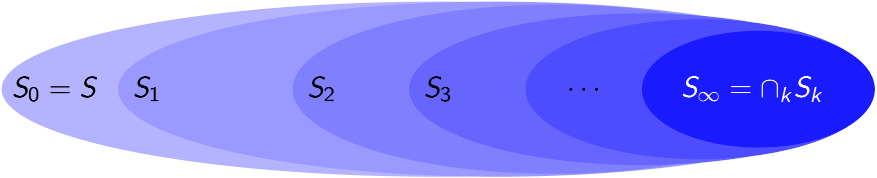

Abstracting away from the iteration

Claim: $\mathcal{M}, s \vDash \langle\varphi!\rangle^*\psi$ if, and only if,

$$\underbrace{\exists t \notin S_\infty \wedge t \preceq s \wedge}_{\exists k \in \mathbb{N}} \quad \mathcal{M}_{|\underbrace{\{u \mid t \preceq u\}}_{S_k}}, s \vDash \psi$$

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Consequence: if $\mathcal{M}$ and $L_\preceq = \left\{ \begin{array}{|c|}\hline s \\ \hline t \\ \hline \end{array} \;\middle|\; s \preceq t \right\}$ are **regular**,

then $[\![\langle\varphi!\rangle^*\psi]\!](\mathcal{M})$ is **effectively regular**.

Contribution: Learning $\preceq$

Theorem

Given some PPAL formula φ, and $\preceq$ its disappearance relation. The learning procedure terminates and returns $L_{\preceq}$ if, and only if, $L_{\preceq}$ is **regular**.

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We run L* algorithm (Angluin, 87) by implementing:

1. A **membership oracle**: given $s, t \in S$, $s \overset{?}{\preceq} t$;
2. An **equivalence oracle**: given L' regular, does $L' = L_{\preceq}$ and if not, provide **counterexample** $w \in L \backslash L_{\preceq} \cup L_{\preceq} \backslash L$.

Theorem (Unique Characterization)

For $R \subseteq S \times S$, $R = \preceq$, *iff:*

1. **Reflexive**
   and
2. **Transitive**
   and
3. **Total**
   and
4. **Some "F" property**

Theorem (Unique Characterization)

For $R \subseteq S \times S$, $R = \preceq$, *iff:*

1. $\forall s : (s, s) \in R$
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2. $\forall s_1, s_2, s_3 : (s_1, s_2) \in R \wedge (s_2, s_3) \in R \rightarrow (s_1, s_3) \in R$
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FO properties over R

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$\rightarrow$ **Regular Language Queries**

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4. $\forall s \begin{cases} \textit{either } (1)\ \forall t : (s,t) \in R \to (t,s) \notin R \leftrightarrow t \in F(sR\cdot) \\ \quad\ \textit{or } (2)\ \forall t : (s,t) \in R \to (t,s) \in R \land t \in F(sR\cdot) \end{cases}$

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   $\rightarrow$ Effective and Uniformly Regular

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3: Applications and extensions

Muddy Children

"After **arbitrary** but **finitely** many rounds, all the muddy children know they're muddy."

$$\varphi = \langle \exists i : \mathbf{m}_i ! \rangle \langle \underbrace{\exists i : \mathbf{m}_i \wedge \neg K_i \mathbf{m}_i}_{\psi} ! \rangle^* \bot$$

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Counter measure: restrict to states $s \in \{\mathbf{m}\}^* \cdot \{\mathbf{c}\}^*$.

Soundness: the model and the formula are stable by permutation.

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$$\varphi = \boxed{\langle \forall i, \mathbf{m}_{i+1} \to \mathbf{m}_i ! \rangle} \langle \exists i : \mathbf{m}_i ! \rangle \langle \underbrace{\exists i : \mathbf{m}_i \wedge \neg K_i \mathbf{m}_i}_{\psi} ! \rangle^* \bot$$

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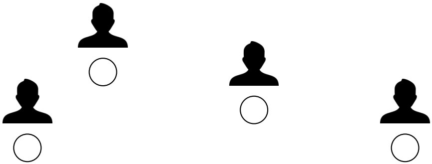
Dining Cryptographer algorithm



- Every cryptographer i has a private boolean p_i.
- Goal: Decide whether $\sum_i p_i > 0$ without disclosing the p_i's

Algorithm:

- For each i, sample a boolean c_i **shared** between i and $i + 1\%N$.
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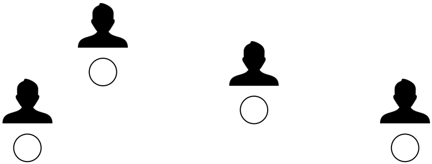

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Simplifications: non-probabilistic setting, sequential announcements.

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How to **mechanize** these announcements, to verify the following properties?

Formalization

Private Variables: $(p_i)_{i \in [1;N]} \in \{0,1\}^N$
Goal1: "Everyone knows whether someone paid"

$$\forall i, (K_i \exists j : p_j) \vee (K_i \forall j : \neg p_j)$$

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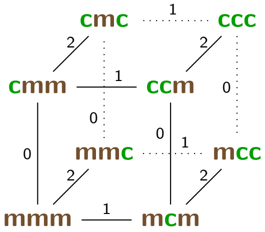
"sampling random variables": $(c_i)_{i \in [1;N]}$
Announcement of agent i: **result** of computation
$r_i = c_i \oplus c_{i+1\%N} \oplus p_i$
Agents compute $\bigoplus_i r_i = \bigoplus_i p_i$

Public Announcement Whether

We introduce the new construct $\langle\varphi!!\rangle\psi$.
"After announcing **whether** there is at least one muddy child, φ"

$[\![\langle\ \exists i : \mathbf{m}_i\ !!\rangle\varphi]\!]$ (

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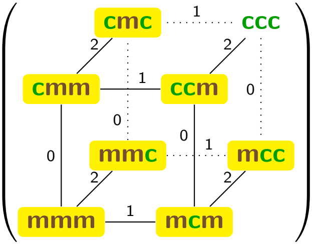

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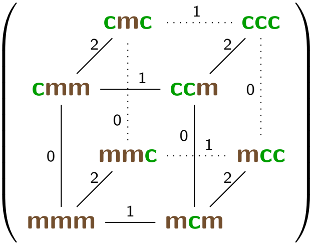

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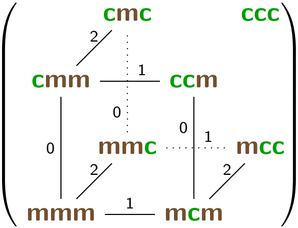

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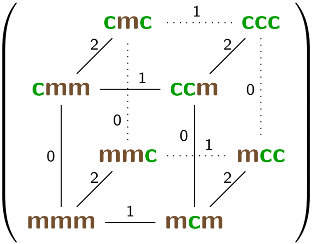

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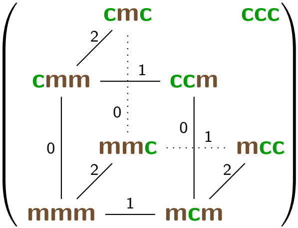

Good news

- Still regular: $[\![\langle \varphi !! \rangle \psi]\!](\mathcal{M}) = [\![\psi]\!](\ldots)$
- $\langle \varphi !! \rangle^* \psi$ can be computed with a disappearance relation **on pair of states**:

$$L_{\preceq} \subseteq (\Sigma' \times \Sigma')^* \quad \text{where } \Sigma' = \Sigma \times \{0,1\} \times \Sigma$$

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or one single formula $\varphi(i)$:arameterized by $i \in Agt$.

Theorem (Addressed in Felix Thoma's BA thesis)

There exists φ' without free variables, such that

$$\langle c_0 \oplus c_1 \oplus p_0!!\rangle\langle c_1 \oplus c_2 \oplus p_1!!\rangle \dots \langle c_N \oplus c_0 \oplus p_0!!\rangle\varphi_{correct} \equiv \langle\varphi'!!\rangle^*\varphi_{correct}$$

Key ideas:

- Track the announcements already been made, by *evaluating the current common knowledge*.
- The same announcement should be made from all the states, **at the same time**.
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- The same announcement should be made from all the states, **at the same time**.
- Solution 1: common knowledge operator $\sim^*$ (similar to safety analysis).
- Solution 2: introducing $All(\varphi)$ operator, whose semantics is regular.

Summary

Regular Model Checking approach to

Knowledge Reasoning over Parameterized Systems

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Learning of $\preceq$'s **regular** encoding

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Regular Models

Future work:

- **Dynamic** Epistemic Logic (more systematic way to *dining cryptographer*, stochastic behaviours).
- **Planning**: how to synthesize announcements (card protocols).
- **Mechanize** the symmetry reductions (*Parikh images*).
- More succint models (expressing fixed point in *MONA*).

Thanks for your attention