Emptiness of Powerset Büchi Automata using Inclusions Tests

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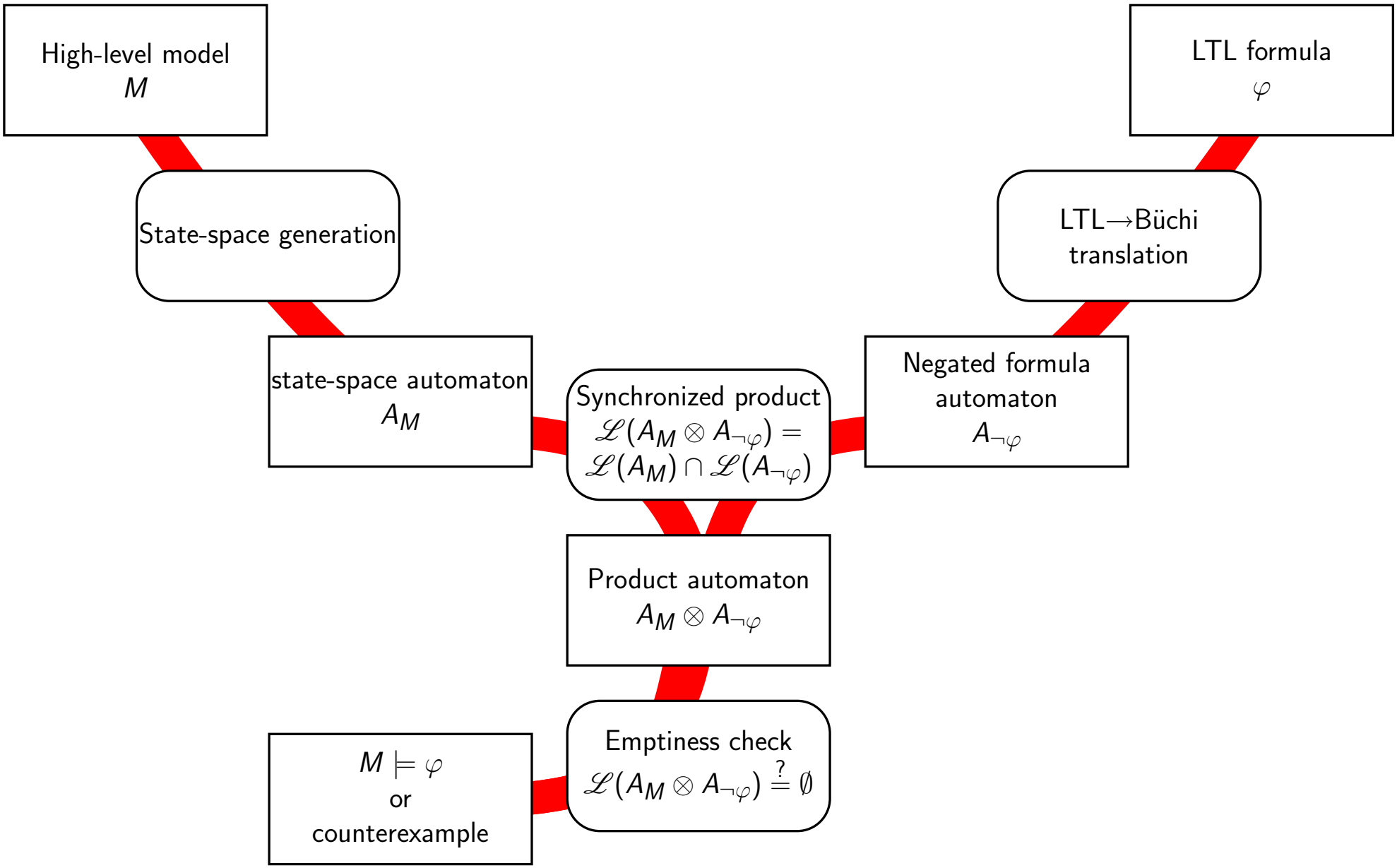
Automata-Theoretic Approach to Model Checking



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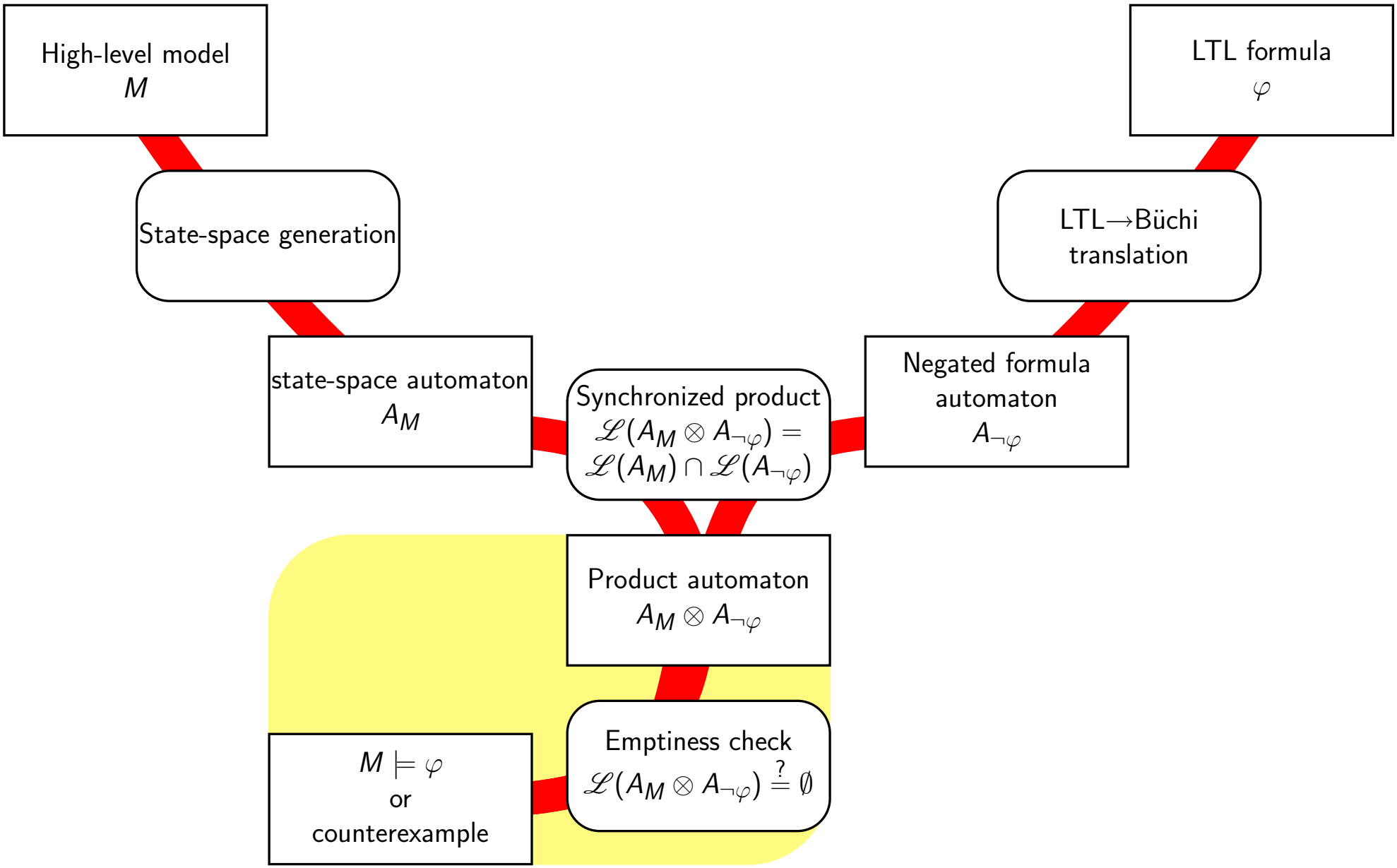
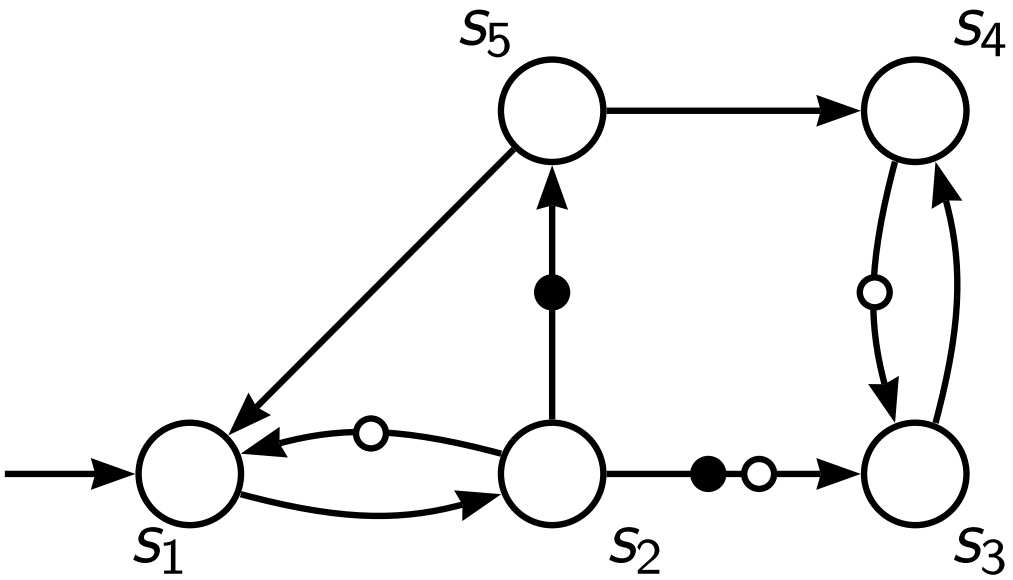


Emptiness Checks for Generalized Büchi Automata

Transition-based Generalized Büchi Automata

An infinite run of this automaton is accepting if it visits a transition from each accepting condition ($\bullet, \circ, \ldots$) infinitely often.



Emptiness Check = Does an automaton have no accepting run?
$\Longrightarrow$ Search for an accepting cycle reachable from the initial state.

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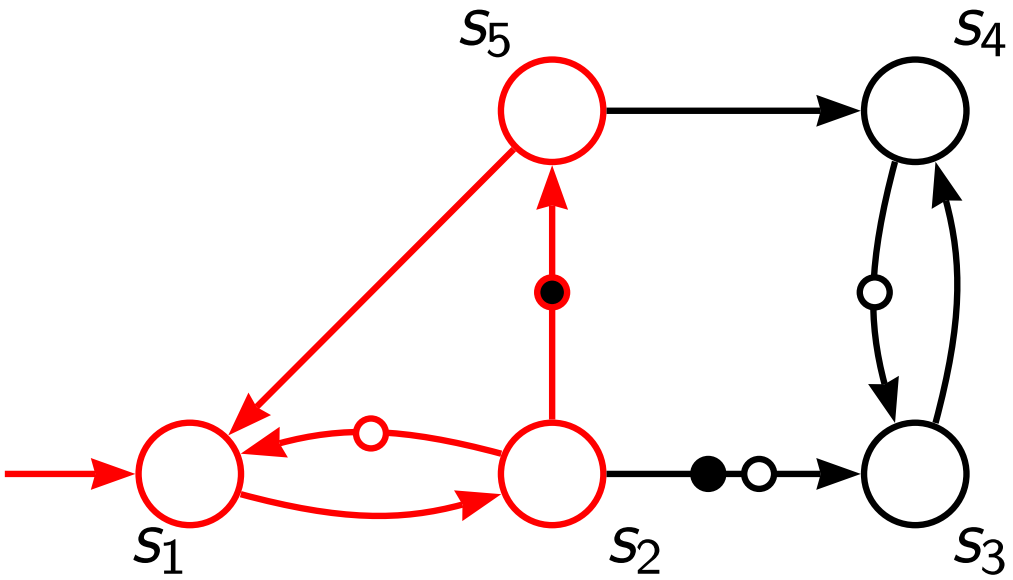

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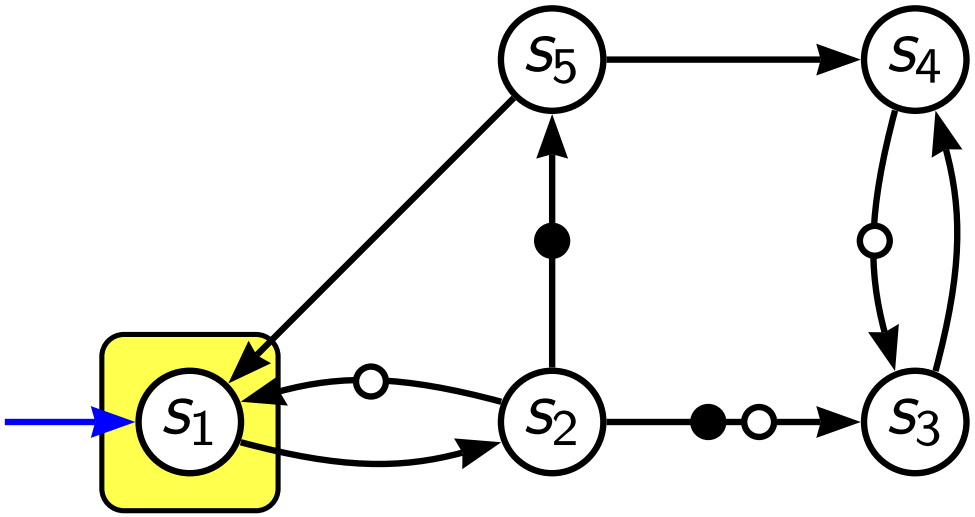
Emptiness Checks

	degeneralized (**one** acceptance condition)	generalized (m acceptance conditions)
Nested DFS	DFS for acc. transitions + Nested DFS to find cycle • 2 bits per state • immediate counterexamples • require degeneralization (size $\times m$) • slow	DFS for acc. transitions + **several** Nested DFS • $\log_2(m+1)$ bits per state • visit states several times • slow
SCC	(pointless)	Compute SCCs on the fly, abort on accepting SCC • fastest • visit states once • indifferent to m • SCC information useful • one integer per state

Couvreur's SCC-Based Emptiness Check



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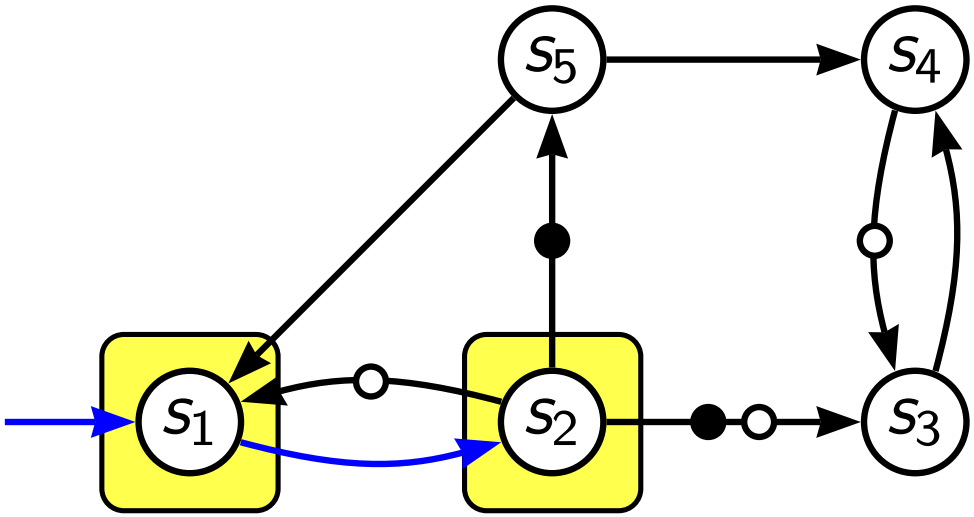

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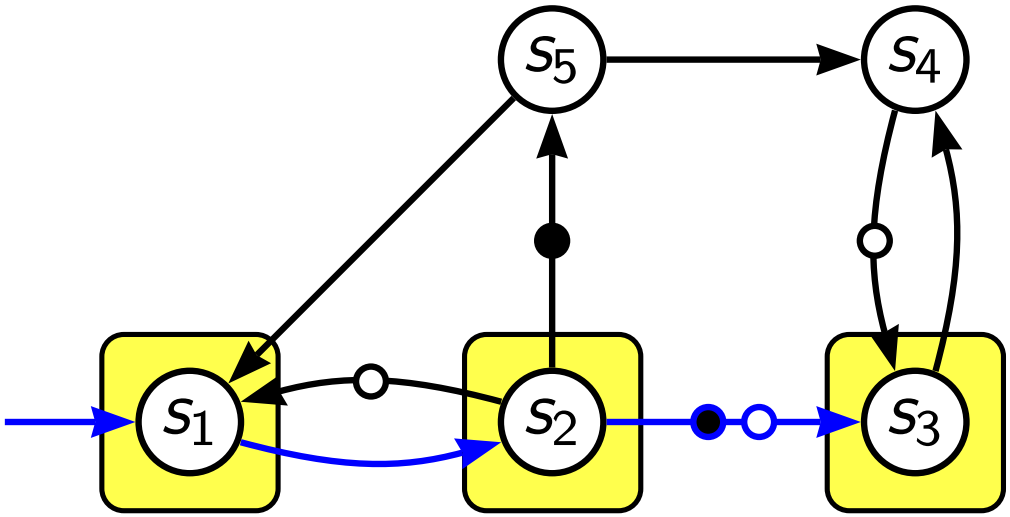

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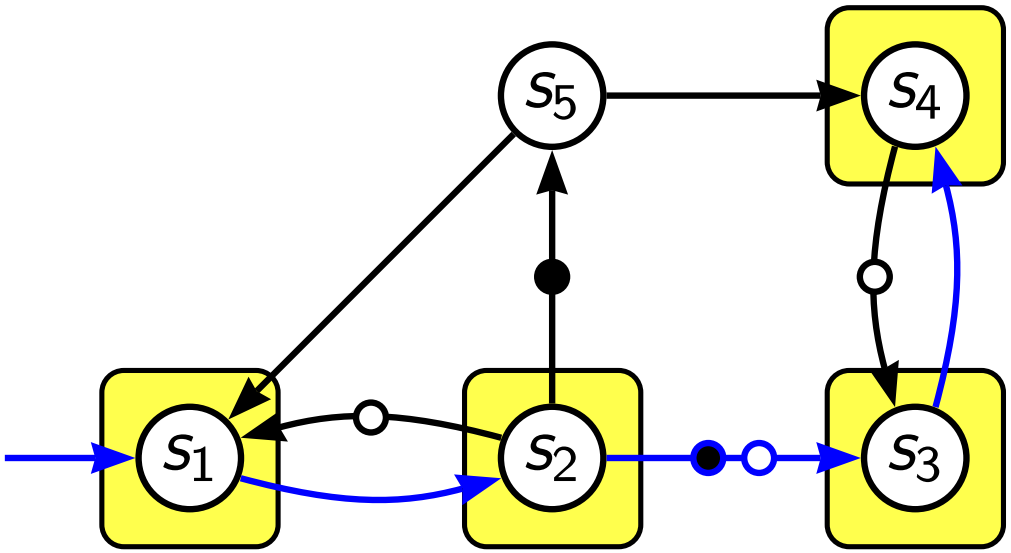

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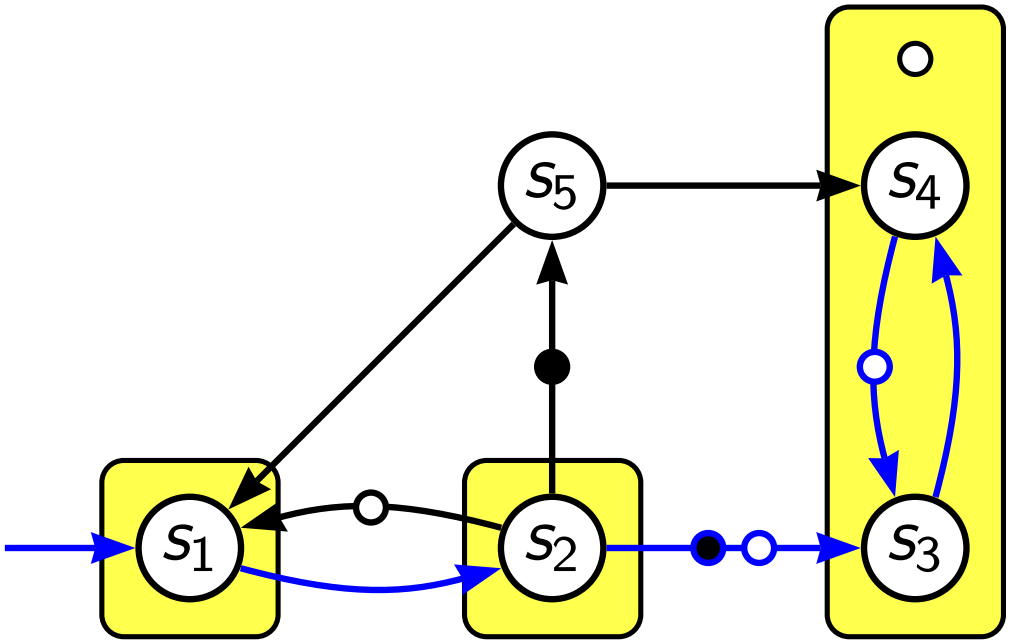

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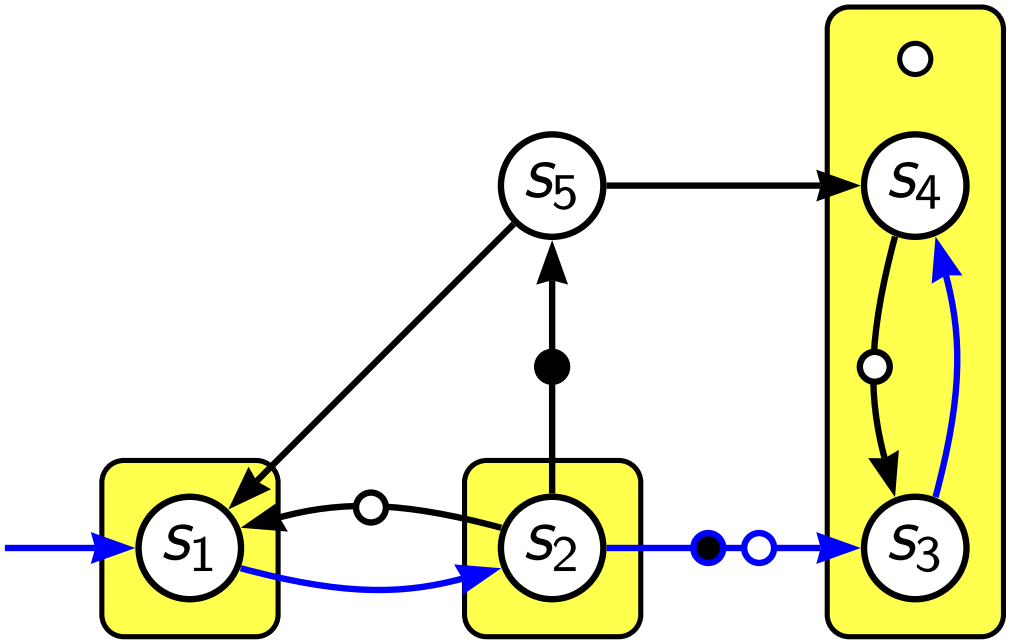

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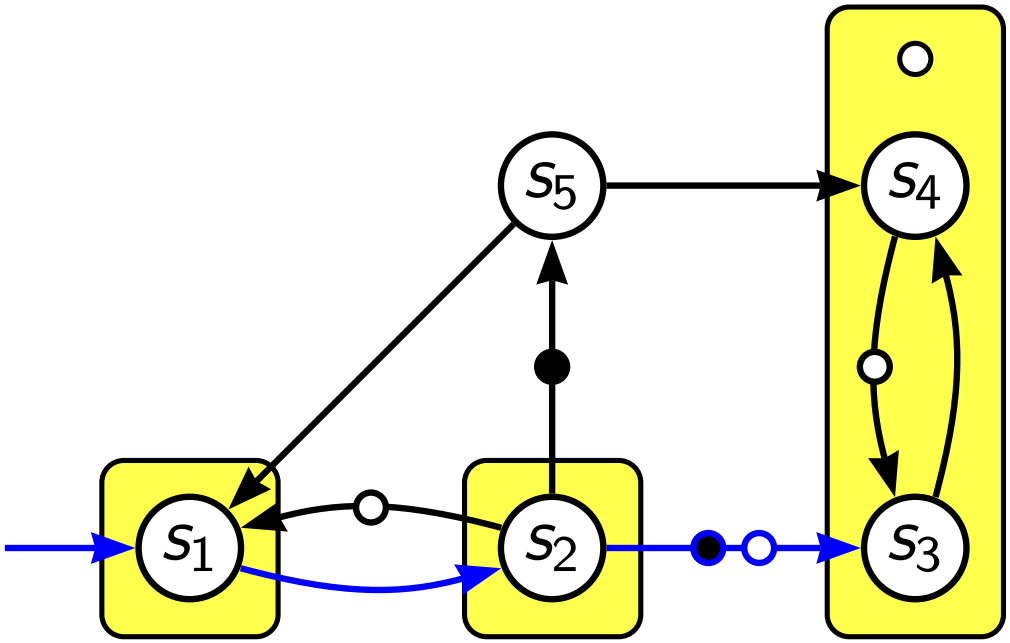

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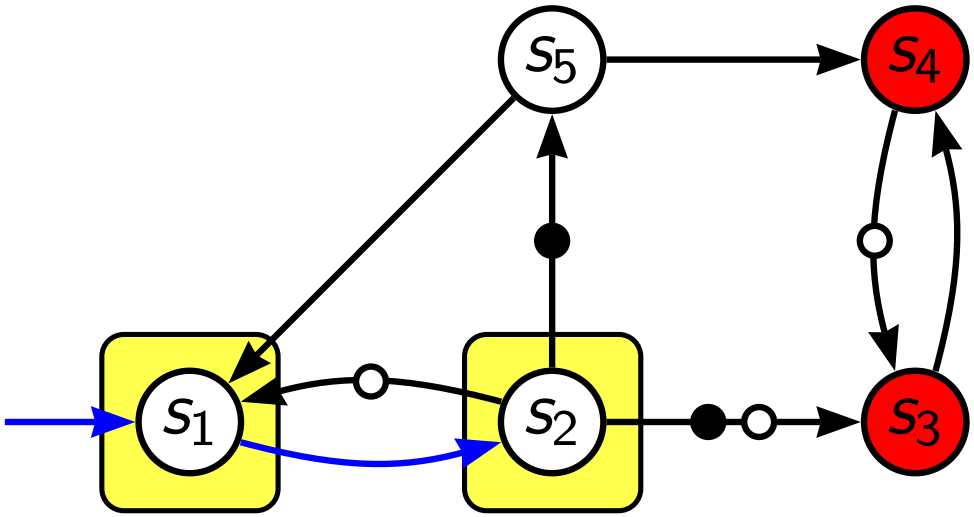

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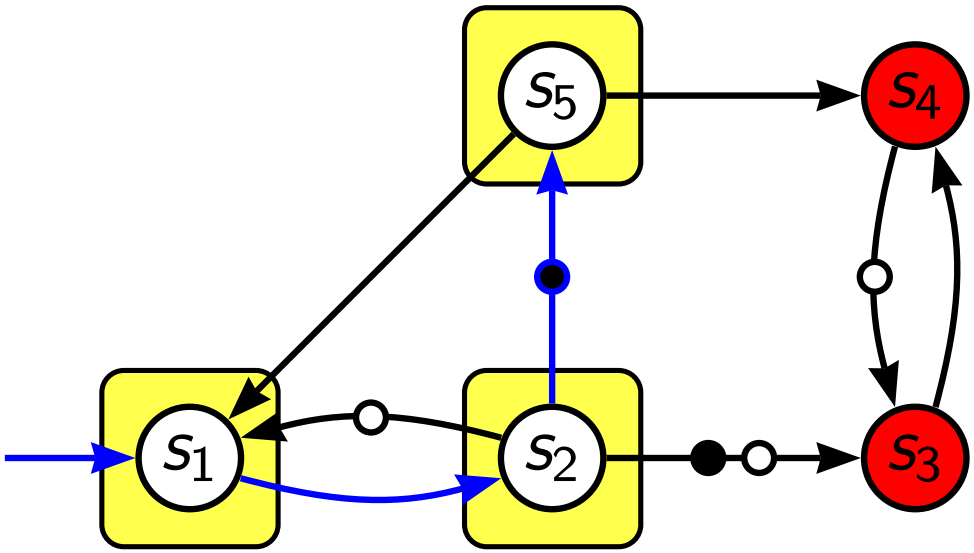

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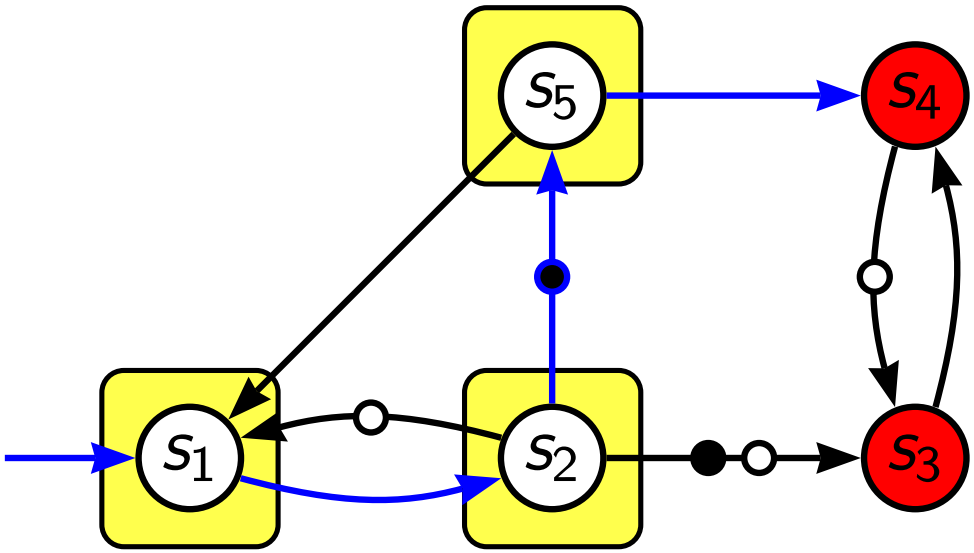

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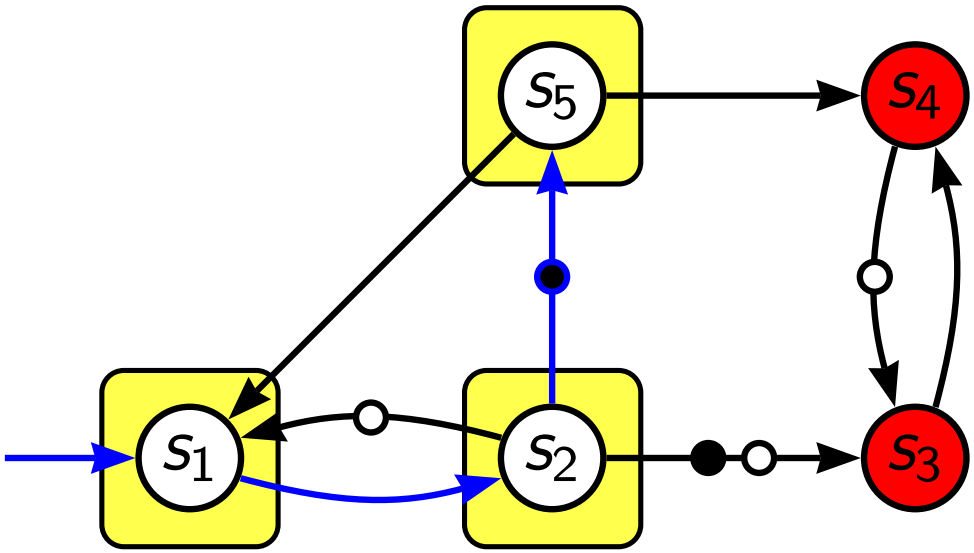

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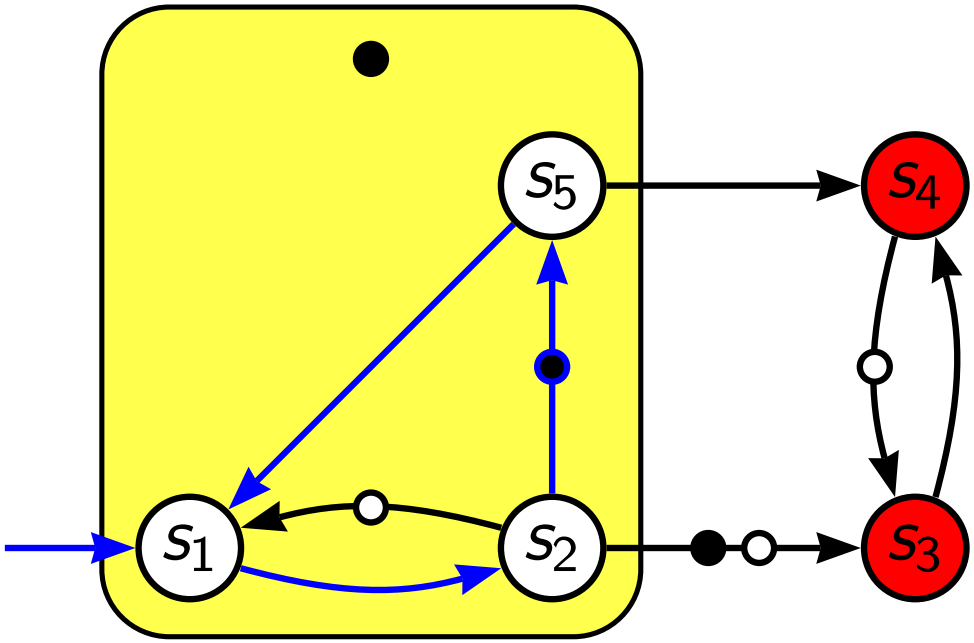

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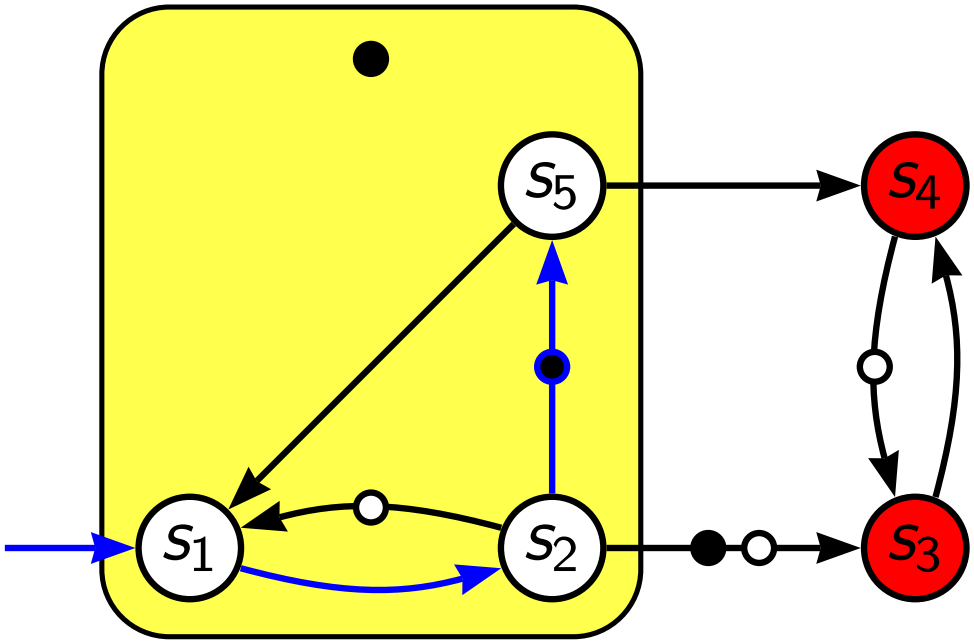

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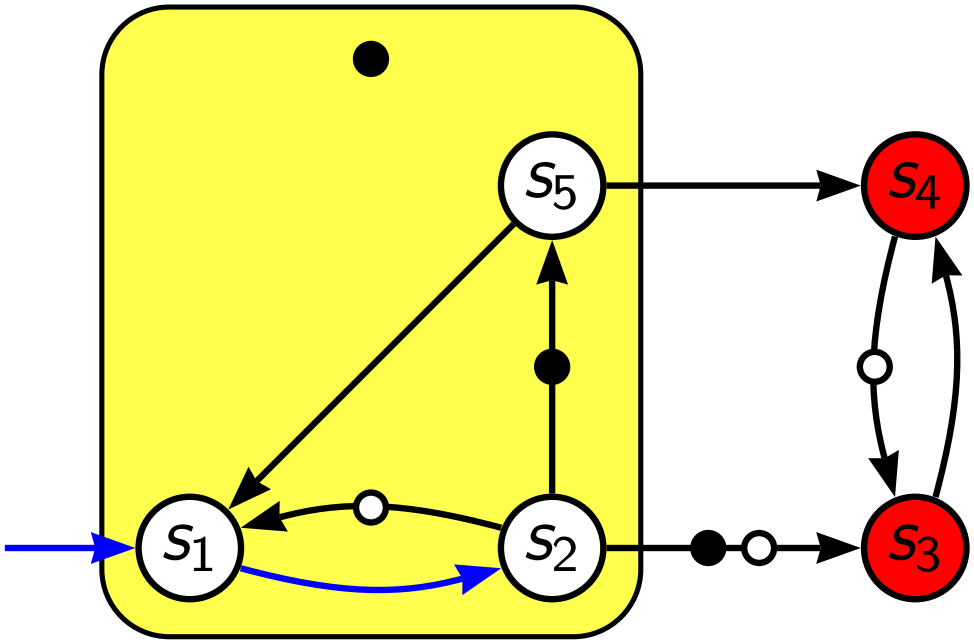

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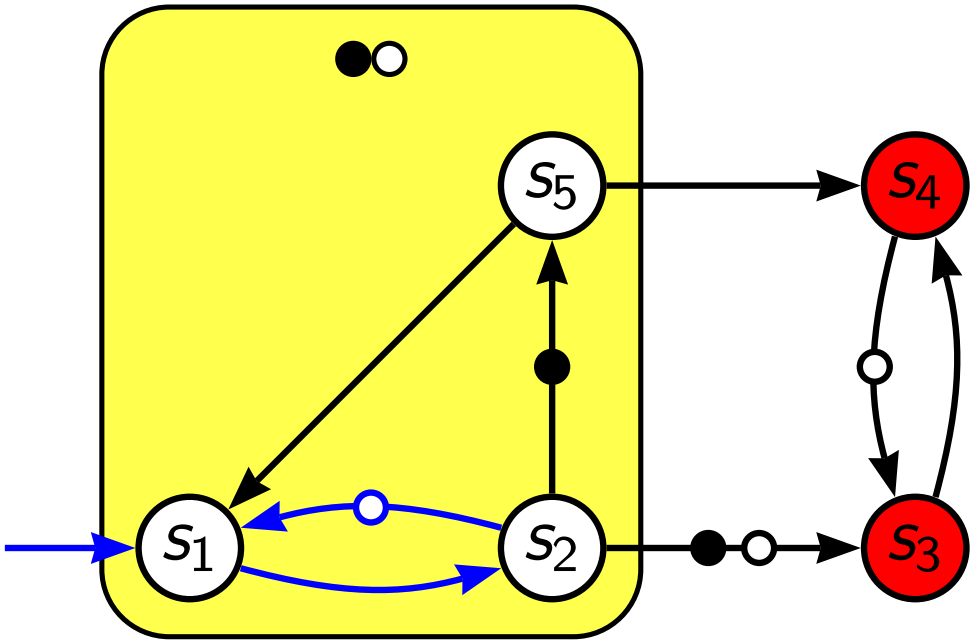

Found!

Emptiness Check for GBA, with Inclusion Checks

Context

State space reduction using symmetries:

- Global Symmetries
  (e.g., client/server system with many identical clients)

- Partial Symmetries [Haddad, Ilié, Ajami: FORTE'00]
  (e.g., two clients have different access priorities to the server)

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  - Define equivalence classes of states.
  - Reduction straightforward.
  - But globally symmetric state spaces are uncommon.
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- Partial Symmetries [Haddad, Ilié, Ajami: FORTE'00]
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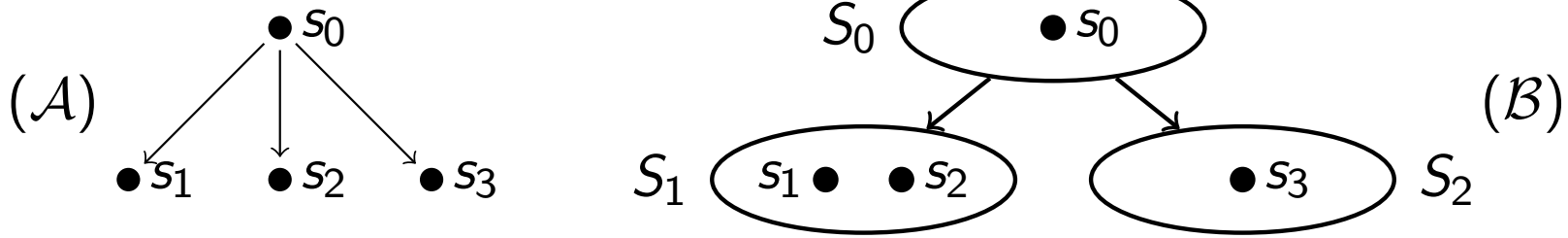
  - Exploits symmetries locally.
  - Partitions the successors of a state:



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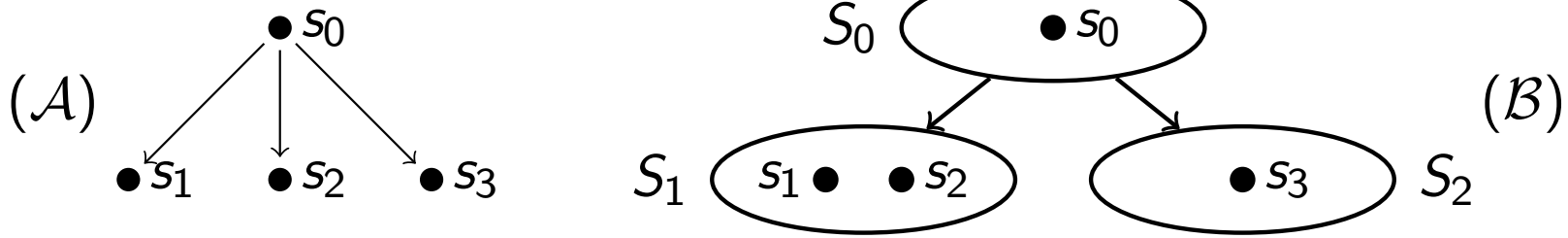

  - Bounds of the "reduction": $|\mathcal{B}| \leq 2^{|\mathcal{A}|}$...

Benchmark

			basic			p. sym.		
model	n		st.	tr.	T	st.	tr.	T
WCS3	28	nonempty prod.	28	80	0.05	25	58	0.06
WCS4	28							
WCS5	28							
PO22	18							
PO23	18							
PO32	22							
WCS3	22	empty products	99	279	0.06	94	255	0.13
WCS4	22							
WCS5	22							
PO22	32							
PO23	32							
PO32	28							

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model	n		st.	tr.	T	st.	tr.	T
WCS3	28	nonempty prod.	28	80	0.05	25	58	0.06
WCS4	28		78	250	0.06	77	202	0.14
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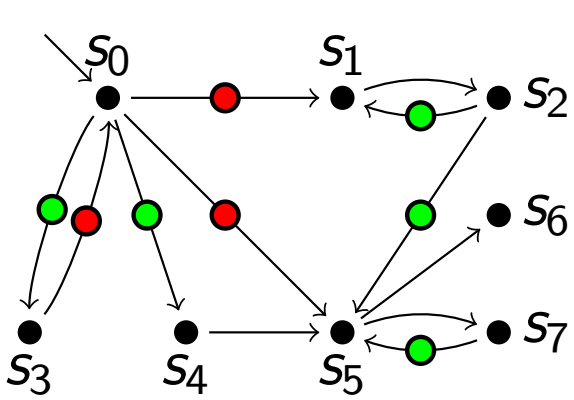
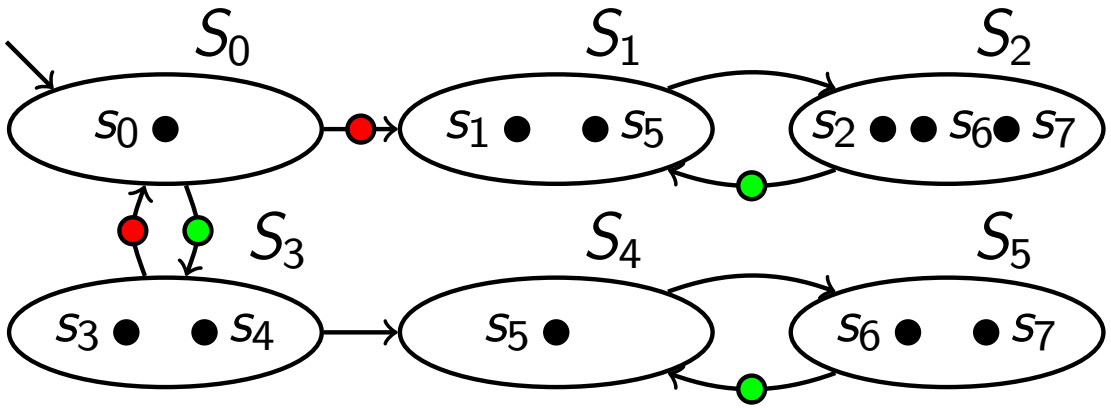
PO22	18		252	431	0.13	690	1307	0.95
PO23	18		292	511	0.17	770	1441	1.48
PO32	22		1173	2235	0.65	2184	4730	7.40
WCS3	22	empty products	99	279	0.06	94	255	0.13
WCS4	22		434	1485	0.13	602	2063	1.60
WCS5	22		1889	7430	0.60	4224	17744	144
PO22	32		2484	5482	0.91	866	1817	2.16
PO23	32		3253	7200	1.56	952	2030	3.68
PO32	28		4617	10651	2.48	1334	2848	4.97

Generalization, from the Emptiness Check POV



$(\mathcal{A})$



$(\mathcal{B})$

Any method that can construct such a $(\mathcal{B})$ instead of $(\mathcal{A})$ so that

- $\forall s \multimap s', \forall S \ni s, \exists S' \ni s'$ such that $S \multimap S'$
- $\forall S \multimap S', \forall s' \in S', \exists s \in S$ such that $s \multimap s'$

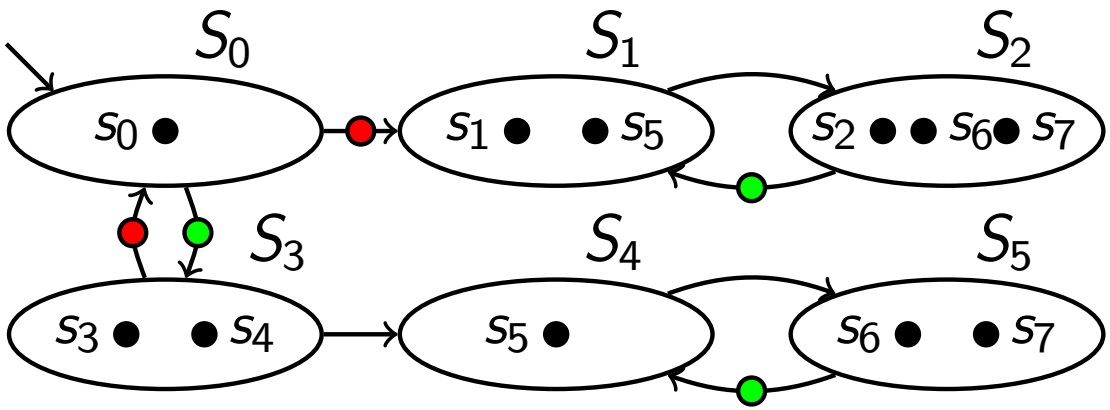
will guarantee emptiness check equivalence

$$\mathrm{Acc}(\mathcal{A}) = \emptyset \iff \mathrm{Acc}(\mathcal{B}) = \emptyset$$

Inclusion and Dead States

$$S \supseteq S' \quad \Longrightarrow \quad \Big(\mathrm{Acc}(\mathcal{B}[S]) = \emptyset \implies \mathrm{Acc}(\mathcal{B}[S']) = \emptyset\Big)$$

$(\mathcal{B})$



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WCS4	28	nonempty prod.	78	250	0.06	77	202	0.14	66	176	0.17
WCS5	28	nonempty prod.	290	979	0.13	416	1309	2.76	294	1118	6.44
PO22	18	nonempty prod.	252	431	0.13	690	1307	0.95	738	1505	1.10
PO23	18	nonempty prod.	292	511	0.17	770	1441	1.48	750	1550	1.69
PO32	22	nonempty prod.	1173	2235	0.65	2184	4730	7.40	1400	3031	3.75
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WCS5	22	empty products	1889	7430	0.60	4224	17744	144	3905	16719	107
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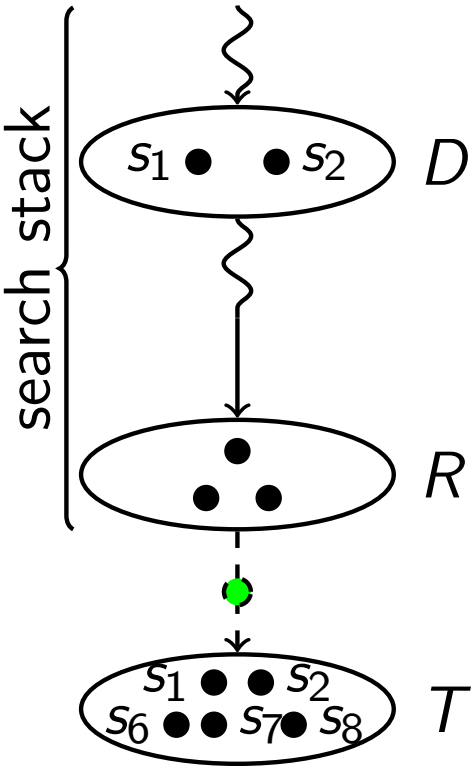
Inclusion and Search Stack States



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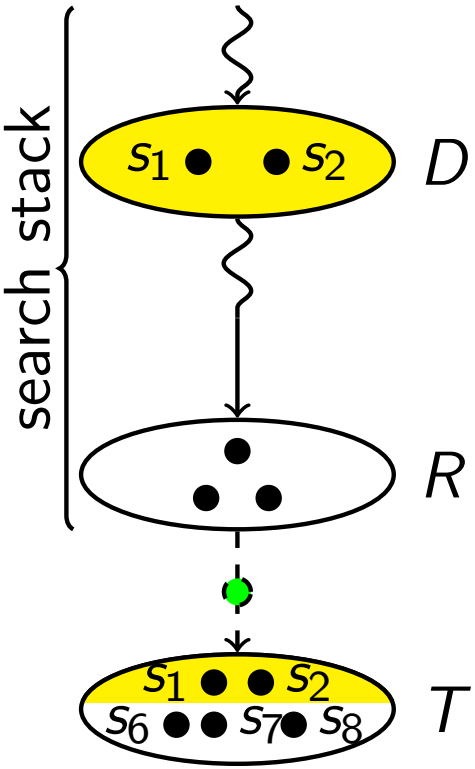

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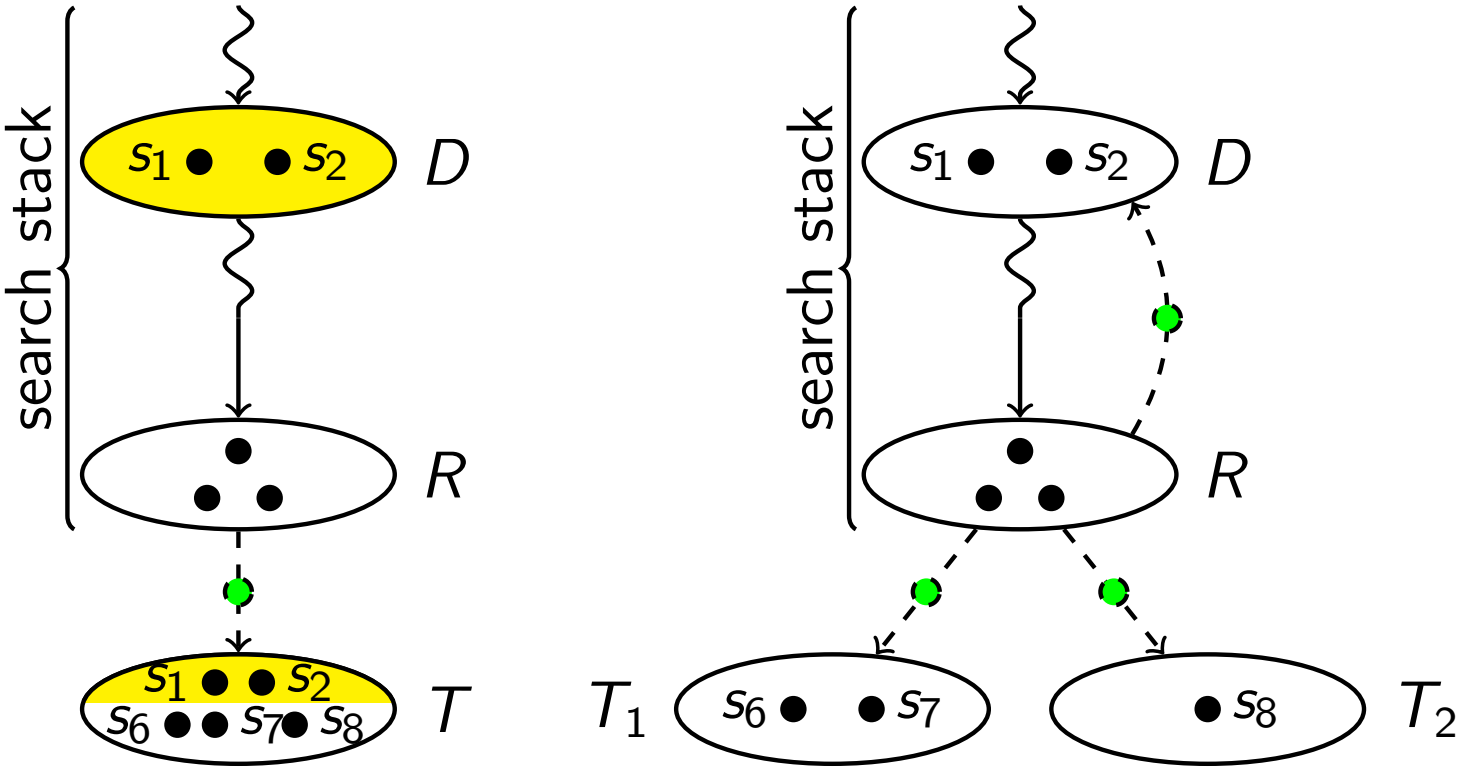

- Directs the on-the-fly construction of the state space.
- Explicits a loop in the stack: good to abort early
- Tries to reuse existing states: a heuristic attempt to fight $2^{|\mathcal{A}|}$

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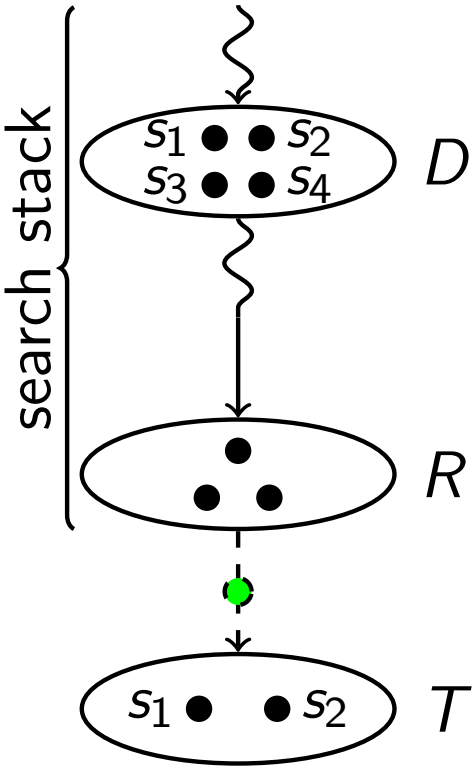
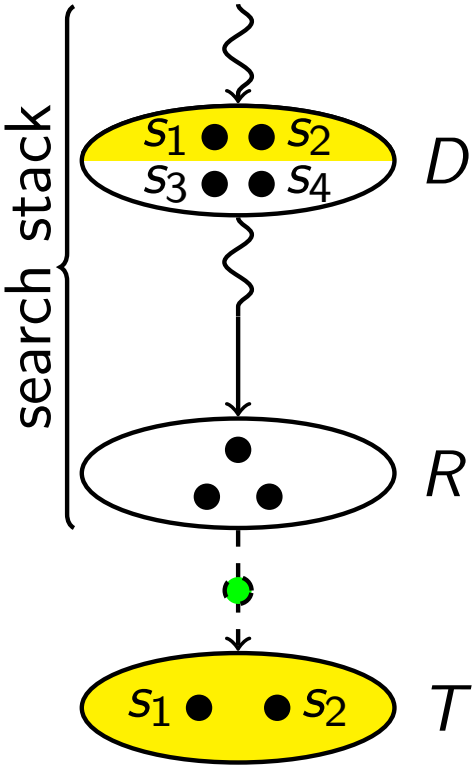


Reverse Inclusion and Search Stack States



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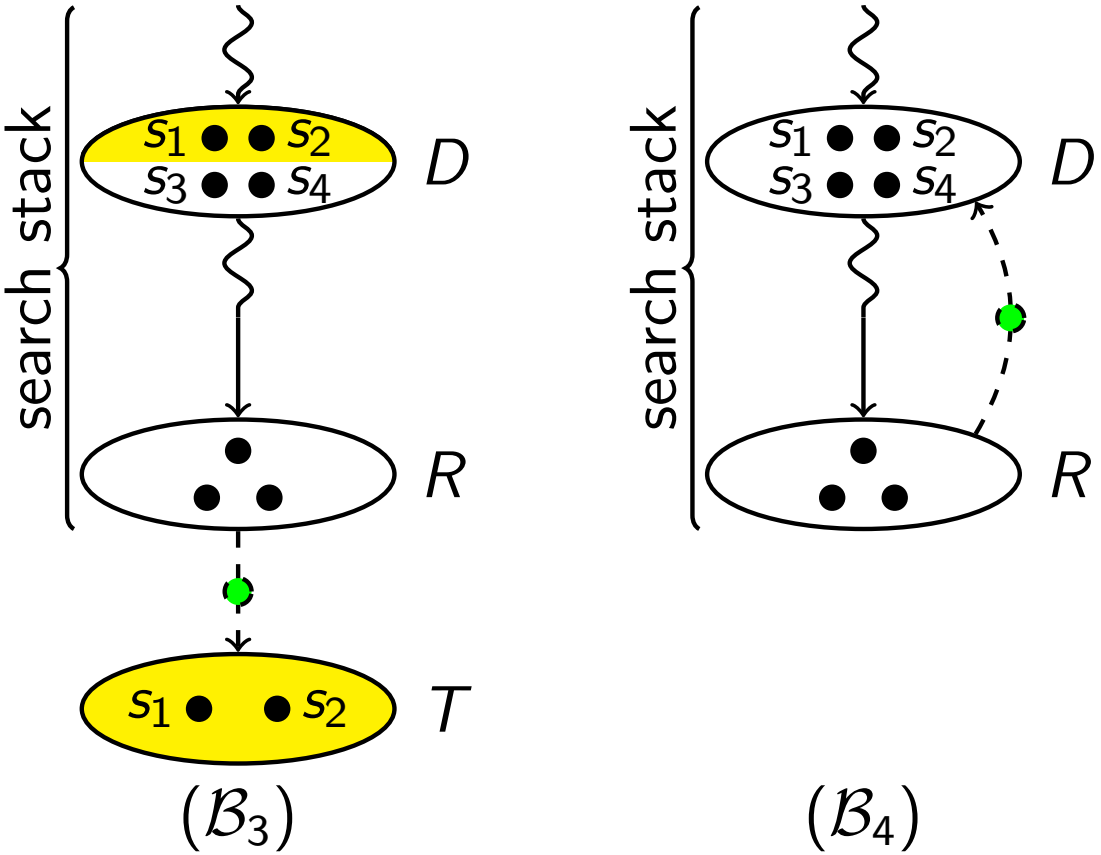

- In this case we only have $\mathrm{Acc}(\mathcal{B}_4) = \emptyset \implies \mathrm{Acc}(\mathcal{B}_3) = \emptyset$
- The emptiness check returns "*empty*" or "*I don't know*"

Benchmark

			basic			p.sym.+DSI+Incl			p.sym.+DST+RIncl		
model	n		st.	tr.	T	st.	tr.	T	st.	tr.	T
WCS3	28	nonempty prod.	28	80	0.05	24	45	0.05	21	39	0.05
WCS4	28		78	250	0.06	43	96	0.10	29	57	0.06
WCS5	28		290	979	0.13	106	287	0.95	39	82	0.07
PO22	18		252	431	0.13	738	1505	1.10	738	1505	1.10
PO23	18		292	511	0.17	750	1550	1.69	750	1550	1.69
PO32	22		1173	2235	0.65	1400	3031	3.75	1392	2982	3.71
WCS3	22	empty products	99	279	0.06	73	194	0.13	30	70	0.07
WCS4	22		434	1485	0.13	297	940	1.07	64	177	0.15
WCS5	22		1889	7430	0.60	1370	4815	23.7	136	428	0.46
PO22	32		2484	5482	0.91	865	1814	2.14	864	1813	2.14
PO23	32		3253	7200	1.56	868	1831	3.09	868	1830	3.08
PO32	28		4617	10651	2.48	1294	2784	4.78	1294	2783	4.79

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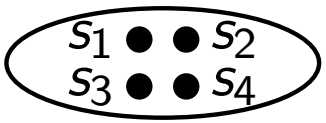
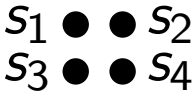
Summary and Perspectives

Requirements:

- A representation of 

smaller than that of 

- A construction that guarantees
  - $\forall s \longrightarrow s', \forall S \ni s, \exists S' \ni s'$ such that $S \longrightarrow S'$
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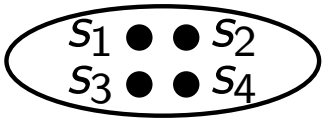
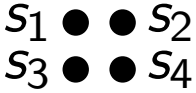
- A representation of 

smaller than that of 

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- Approximative, with the reverse inclusion

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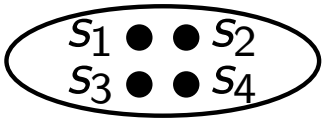
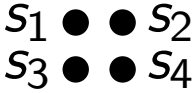
- A representation of 

smaller than that of 

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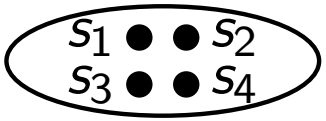
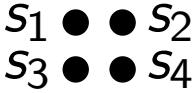
- A representation of 

smaller than that of 

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