# Practical Applications of the Alternating Cycle Decomposition

Antonio Casares <u>Alexandre Duret-Lutz</u> Klara J. Meyer Florian Renkin Salomon Sickert

Joint DIMEA and FORMELA seminar — May 23



### Reactive Synthesis in a Nutshell

A reactive controller produces output as a reaction to its input



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input signals 
$$\left\{\begin{array}{c|c} \bar{a} & \bar{a} & \bar{a} & \dots \\ \hline \bar{b} & \bar{b} & \bar{b} & \dots \end{array}\right\}$$
 output signals

### The reactive synthesis problems

Given a specification relating input signals and output signals over time:

realizability: decide if a controller exist,

synthesis: construct it.

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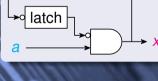
synthesis: construct it.

### Our setup (from SYNTCOMP)

- ▶ the specification is an LTL formula, over ω-words such as " $\bar{a}\bar{b}\bar{x}\bar{y}$ ;  $a\bar{b}xy$ ;  $abx\bar{y}$ ; ..."
- the controller should be an *And-Inverter Graph*

# Reactive Synthesis Example 1. LTL Spec. a ↔ F(x)

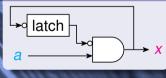




1. LTL Spec.

 $\mathbf{a} \leftrightarrow \mathbf{F}(\mathbf{x})$ 

### 2. DPA



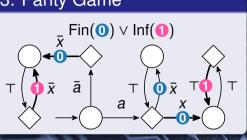
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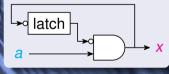
$$\mathbf{a} \leftrightarrow \mathbf{F}(\mathbf{x})$$

2. DPA

 $Fin(\mathbf{0}) \vee Inf(\mathbf{0})$ āx 0

3. Parity Game



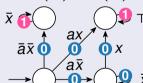


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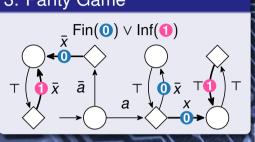
$$\mathbf{a} \leftrightarrow \mathbf{F}(\mathbf{x})$$

2. DPA

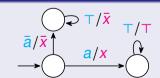
$$Fin(\mathbf{0}) \vee Inf(\mathbf{0})$$

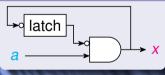


3. Parity Game



4. Winning Strat. as Mealy M.





1. LTL Spec.

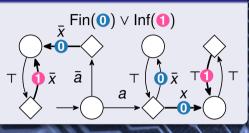
 $\mathbf{a} \leftrightarrow \mathbf{F}(\mathbf{x})$ 

2. DPA

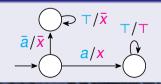
 $Fin(\mathbf{0}) \lor Inf(\mathbf{1})$ 

 $\bar{x}$   $\bar{a}$   $\bar{x}$ 

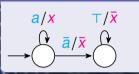
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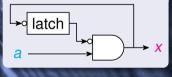


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5. Simpl. Mealy M.







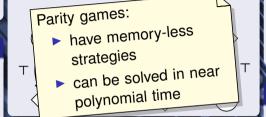
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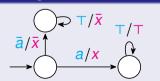
2. DPA

Determinism is required, so we cannot use Büchi acceptance

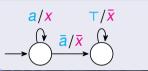
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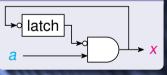


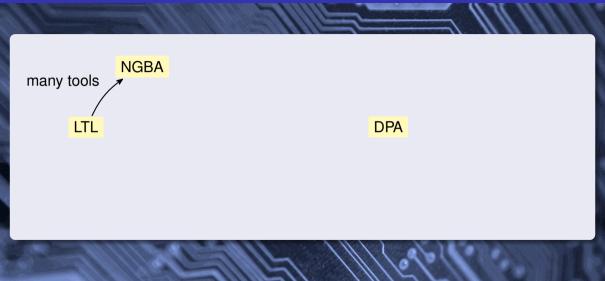
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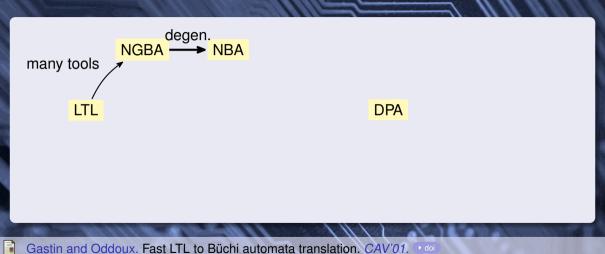


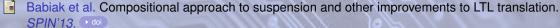
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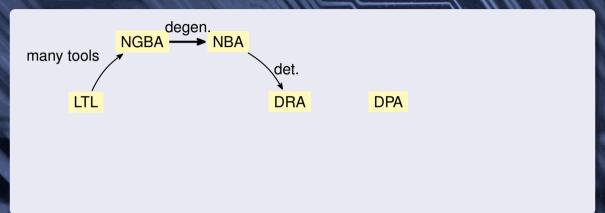




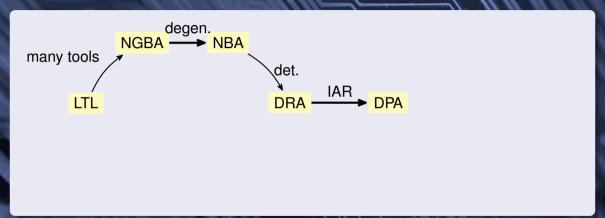




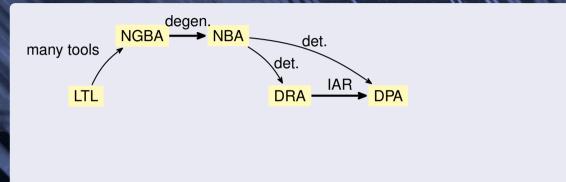






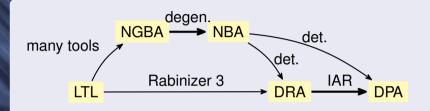








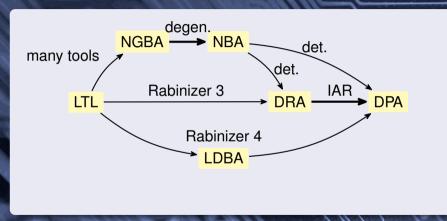
Piterman. From nondeterministic Büchi and Streett automata to deterministic parity automata. LICS.





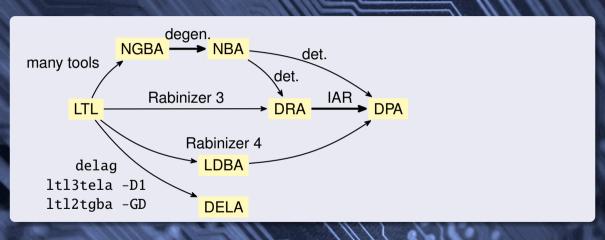
Komárková and Křetínský. Rabinizer 3: Safraless translation of LTL to small deterministic automata.

ATVA'14. odoi

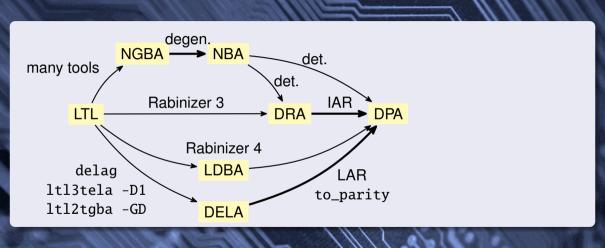




Esparza et al. From LTL and limit-deterministic Büchi automata to deterministic parity automata.

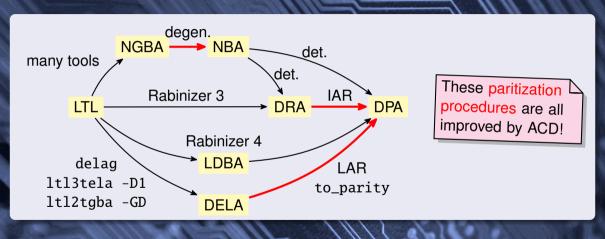








Löding. Optimal bounds for transformations of  $\omega$ -automata. FSTTCS'99.



## Practical Applications of the Alternating Cycle Decomposition

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TACAS'22



### Alternating Cycle Decomposition

### [ICALP'21]

### **Optimal Transformations of Games and Automata Using Muller Conditions**

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CNRS, IRIF, Université de Paris, France

Nathanaël Fijalkow ⊠®

CNRS, LaBRI, Université de Bordeaux, France

The Alan Turing Institute of Data Science, London, UK

We consider the following question: given an automaton or a game with a Muller condition, how can we efficiently construct an equivalent one with a parity condition? There are several examples of such transformations in the literature, including in the determinisation of Büchi automata.

We define a new transformation called the alternating cycle decomposition, inspired and extending Zielonka's construction. Our transformation operates on transition systems, encompassing both automata and games, and preserves semantic properties through the existence of a locally bijective morphism. We show a strong optimality result: the obtained parity transition system is minimal both in number of states and number of priorities with respect to locally bijective morphisms.

We give two applications: the first is related to the determinisation of Büchi automata, and the second is to give crisp characterisations on the possibility of relabelling automata with different

- Defines the ACD structure of Muller automata.
- How to use ACD to paritize an automaton. (With an optimality result.)
- How to use ACD to check automaton types.
- Purely theoretical (no implementation).

2012 ACM Subject Classification Theory of computation  $\rightarrow$  Automata over infinite objects

### Alternating Cycle Decomposition

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But is this ACD construction practical?

2012 ACM Subject Classification  $\,$  Theory of computation  $\rightarrow$  Automata over infinite objects to ad always Automata cost infinite words. Omera resular languages, Determinisation of

### This Paper

- Adaptation of the definition of ACD and acd\_transform() to TELA (Transition-based Emerson-Lei Automata, as supported by the HOA format.)
- Implementation in two tools:



Owl 21.0 owl.model.in.tum.de



Spot 2.10
spot.lrde.epita.fr

Motivation is LTL synthesis with: LTL→DTELA→ DPA→game→controller

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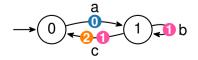
- Comparison to other existing paritization procedures
- State-based version of acd\_transform()
- Comparison to degeneralization procedures (transition-based generalized Büchi → state-based Büchi)

### Emerson-Lei Automata (Using the HOA Syntax)

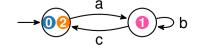
Acceptance condition = any positive Boolean combination of Inf(n) or Fin(n) terms.

```
Büchi
                                       Inf(\mathbf{0})
                                       Inf(0) \wedge Inf(1) \wedge Inf(2) \wedge ...
generalized Büchi
co-Büchi
                                       Fin(\mathbf{0})
                                       Fin(0) \vee Fin(1) \vee Fin(2) \vee ...
generalized co-Büchi
                                        (\operatorname{Fin}(\mathbf{0}) \wedge \operatorname{Inf}(\mathbf{0})) \vee (\operatorname{Fin}(\mathbf{2}) \wedge \operatorname{Inf}(\mathbf{3})) \vee \dots
Rabin
                                        (Inf(\mathbf{0}) \vee Fin(\mathbf{0})) \wedge (Inf(\mathbf{0}) \vee Fin(\mathbf{0})) \wedge ...
Streett
                                       Inf(0) \lor (Fin(1) \land (Inf(2) \lor (Fin(3) \land ...)))
parity min even
                                       Fin(0) \wedge (Inf(1) \vee (Fin(2) \wedge (Inf(3) \vee ...)))
parity min odd
```

transition-based acceptance (TELA):

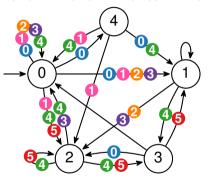


state-based acceptance:

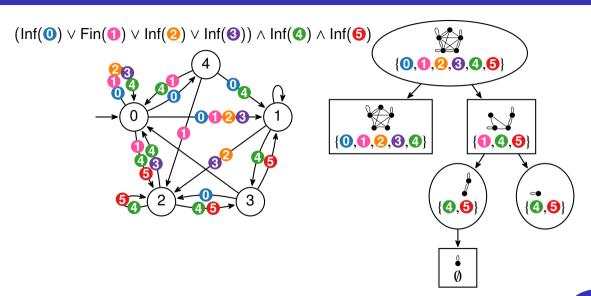


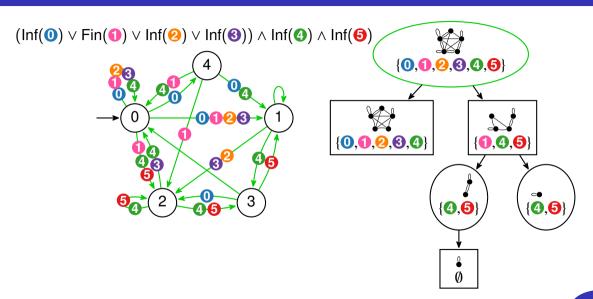
### **TELA**

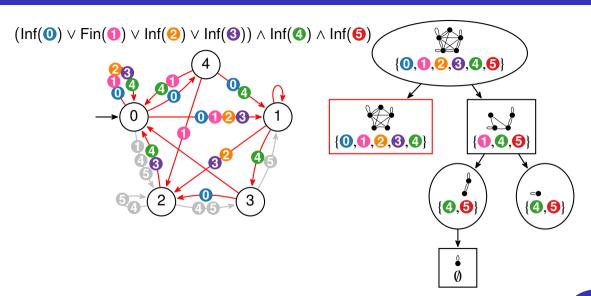
 $(Inf(\mathbf{0}) \vee Fin(\mathbf{1}) \vee Inf(\mathbf{2}) \vee Inf(\mathbf{3})) \wedge Inf(\mathbf{4}) \wedge Inf(\mathbf{5})$ 

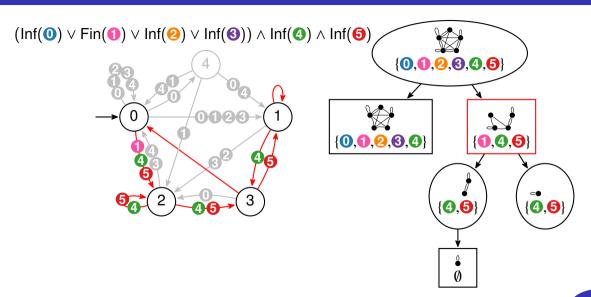


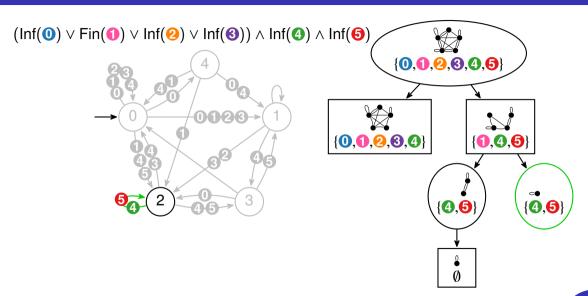
Automaton labels/alphabet hidden for simplicity.

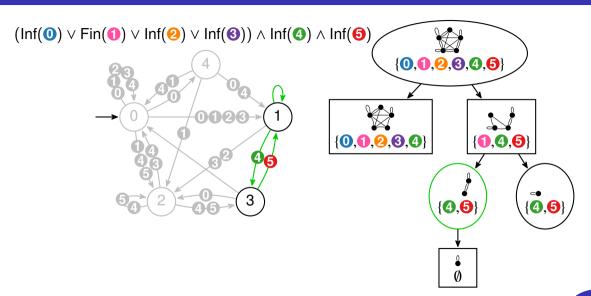


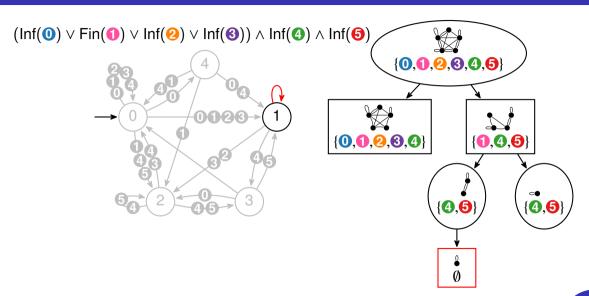




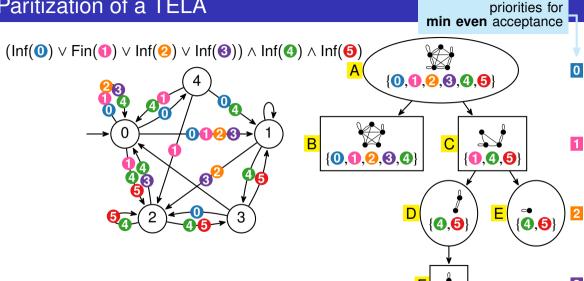


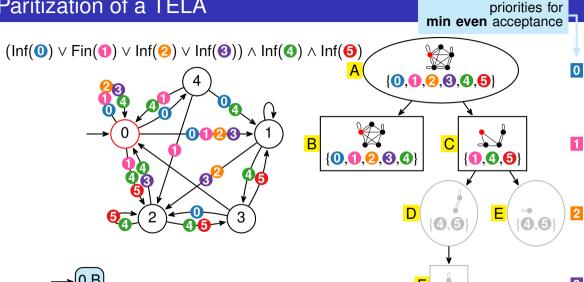


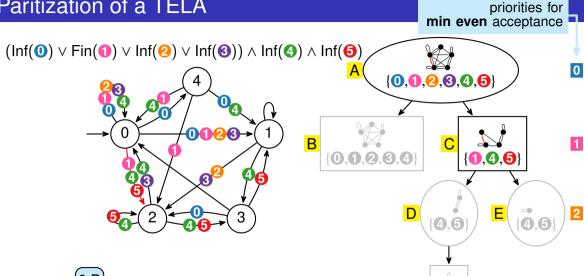


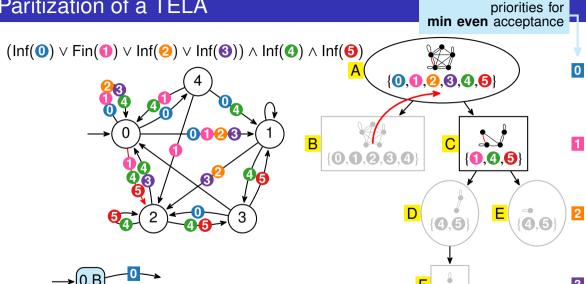


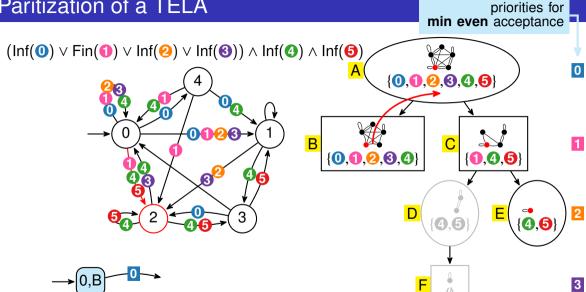
### Paritization of a TELA

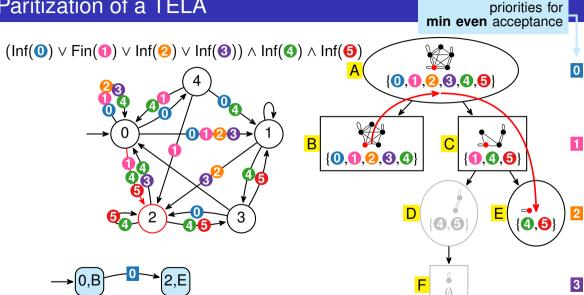


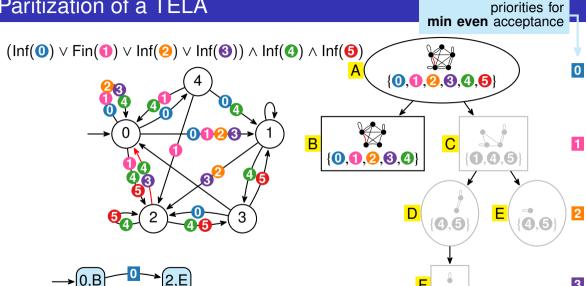


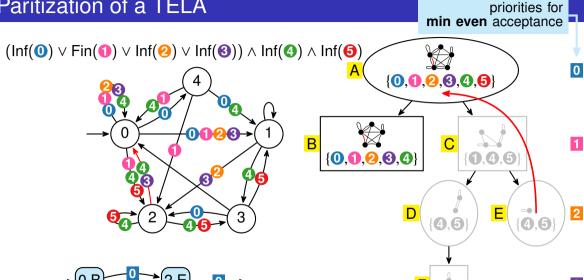


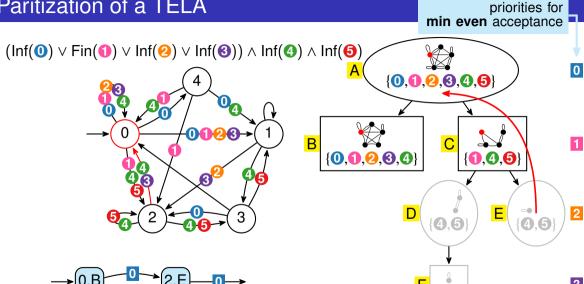


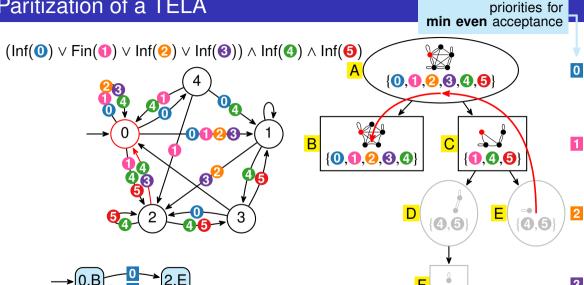


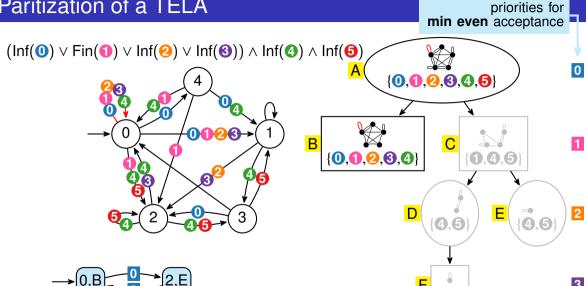


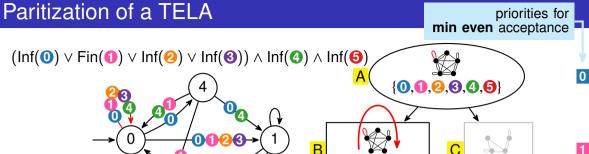


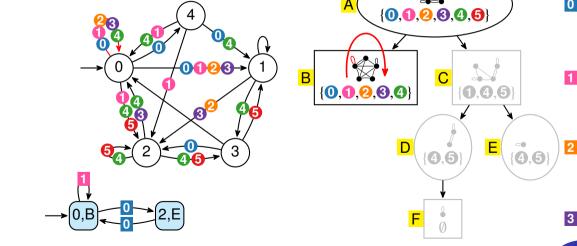


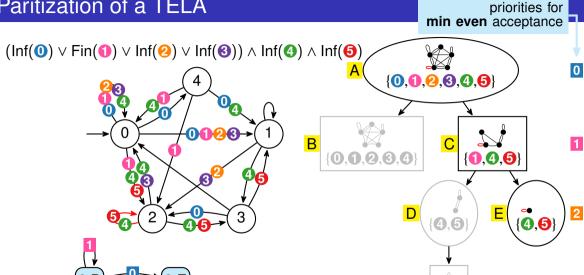


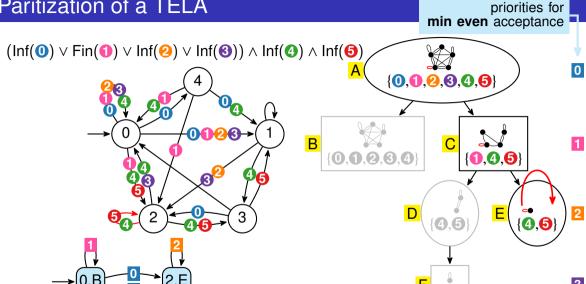


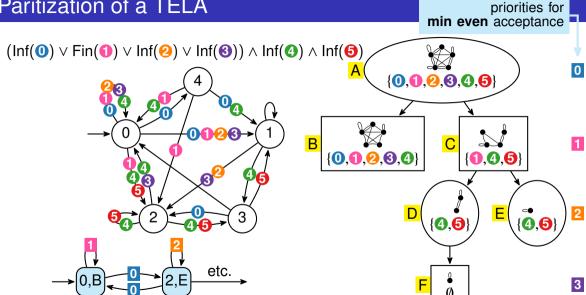












Many paritization procedures create states (s, m) where  $\begin{cases} s: \text{ original state} \\ m: \text{ memory} \end{cases}$ Latest Appearance Record (LAR) works on any TELA,

m is an order of all colors

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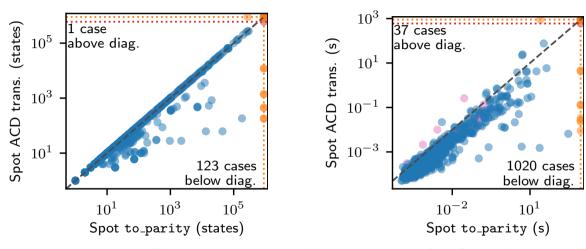


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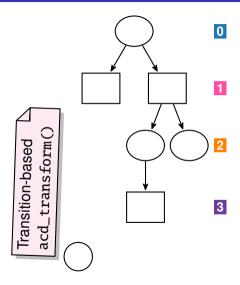


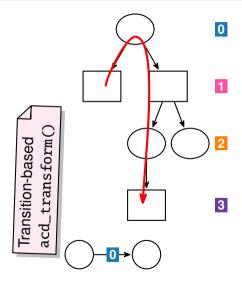
Casares, Colcombet, and Fijalkow. Optimal transformations of games and automata using Muller conditions. *ICALP'21*.

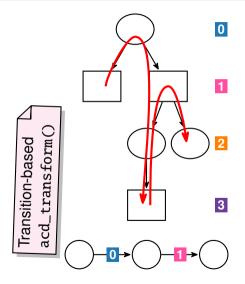
## Comparison between to\_parity() and acd\_transform()

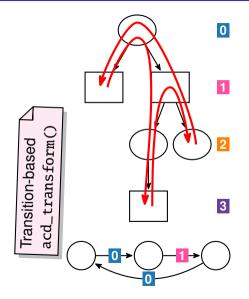


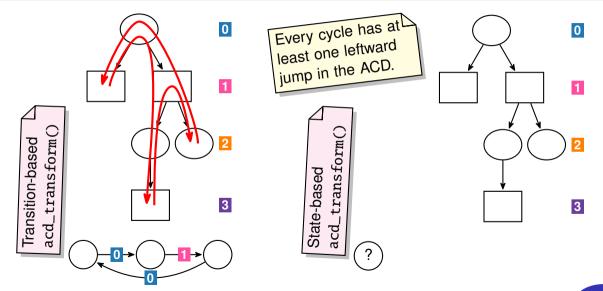
Benchmarck of 1065 TELA generated from LTL formulas from SyntComp. These TELA have between 2 and 55 colors (median 5) and up to 245761 states (median 20).

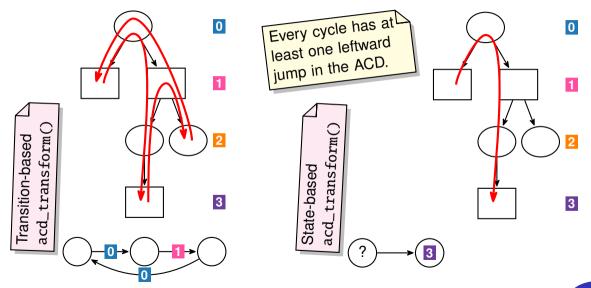


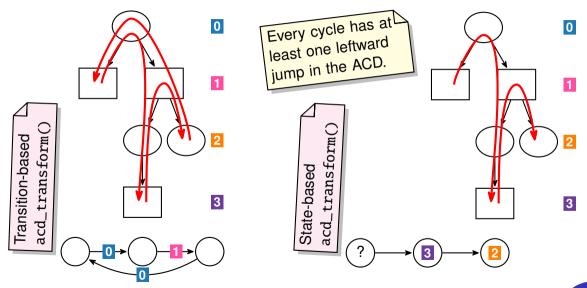


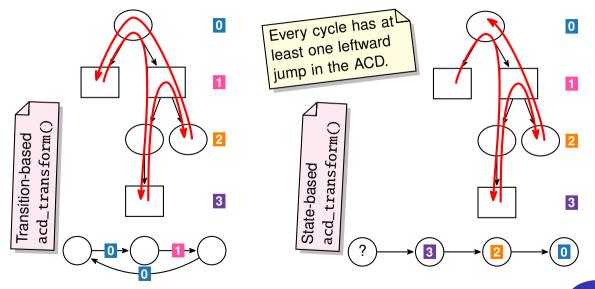


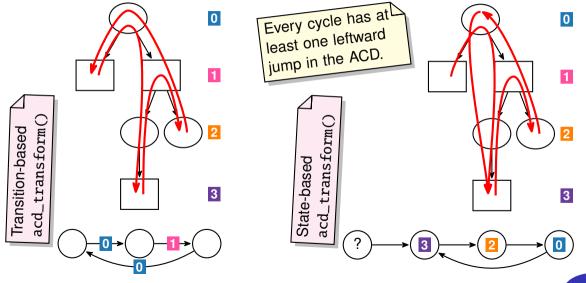


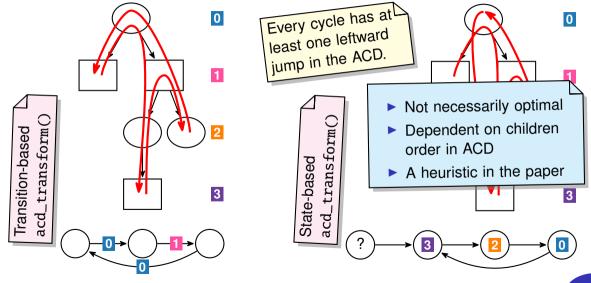




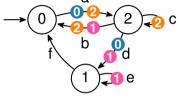


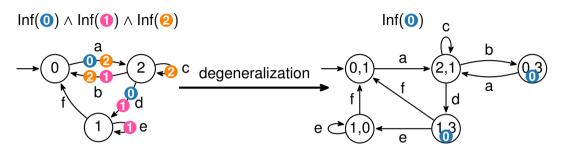


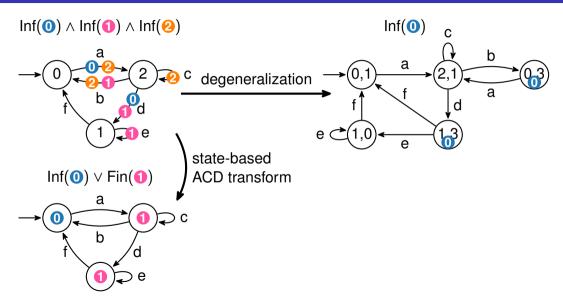


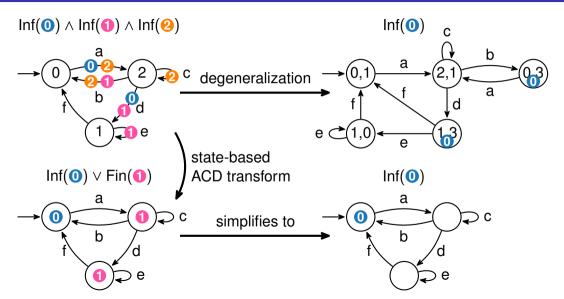


 $Inf(\mathbf{0}) \wedge Inf(\mathbf{1}) \wedge Inf(\mathbf{2})$  a





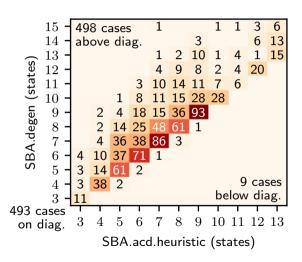




## Application to Degeneralization: Benchmark

Application of the state-based version of acd\_transform() on 1000 random transition-based generalized Büchi automata, with 3–4 states and 2–3 colors.

Comparison with Spot 2.10's best degeneralization routine.

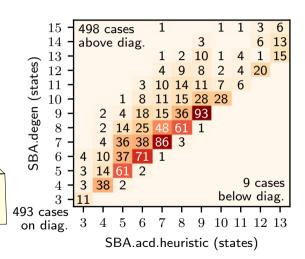


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Compared to min. sizes (found via SAT), ACD gives **+0.17** states on avg., and Spot's degen. gives **+1.21** states on avg.



## Conclusion

ACD is versatile, and can replace several existing algorithms:

- paritization of automata with arbitrary acceptance (always better than LAR, IAR, and their combinations)
- degeneralization (better than traditional algorithms)
- minimization of the number of priorities in a parity automaton
- several typeness checks (looking at the shape of the ACD)



Two implementations can be a basis for further experiments:



Owl 21.0

owl.model.in.tum.de



Spot 2.10 demo

spot.lrde.epita.fr