On-the-Fly Emptiness Checks for Generalized Büchi Automata

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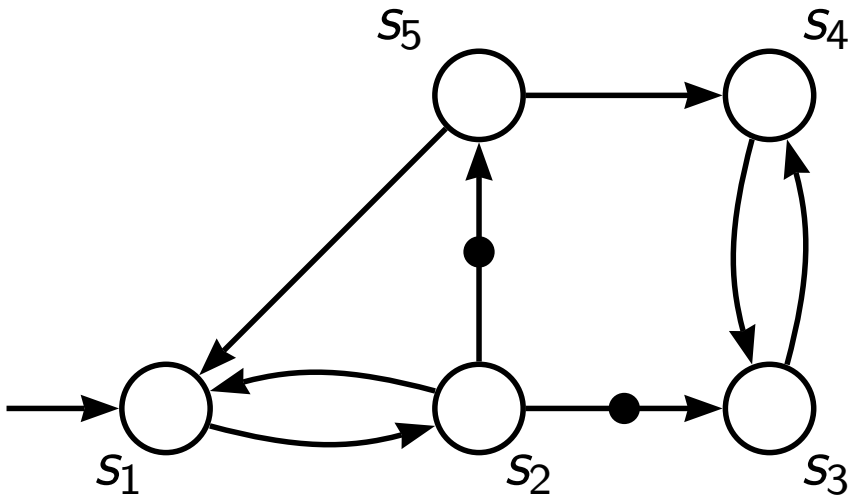
Büchi Automata

A (transition-based) Büchi automaton has:

- A set of states, with a designated initial state,
- A set of transitions between states,
- A set of accepting transitions.

An infinite run of this automaton is accepting if it visits an accepting transition infinitely often.



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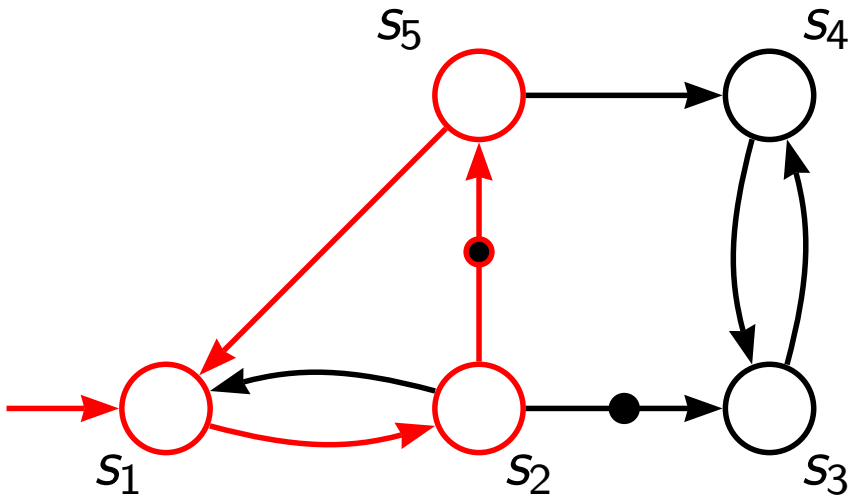

Emptiness Check

Emptiness Check = Does an automaton have no accepting run?

$\Longrightarrow$ Search for an accepting cycle reachable from the initial state.

Emptiness Checks History

nested DFS	Courcoubetis et al.	'90
	Godefroid & Holzmann	'93
	Holzmann et al.	'96
	Gastin et al.	'04
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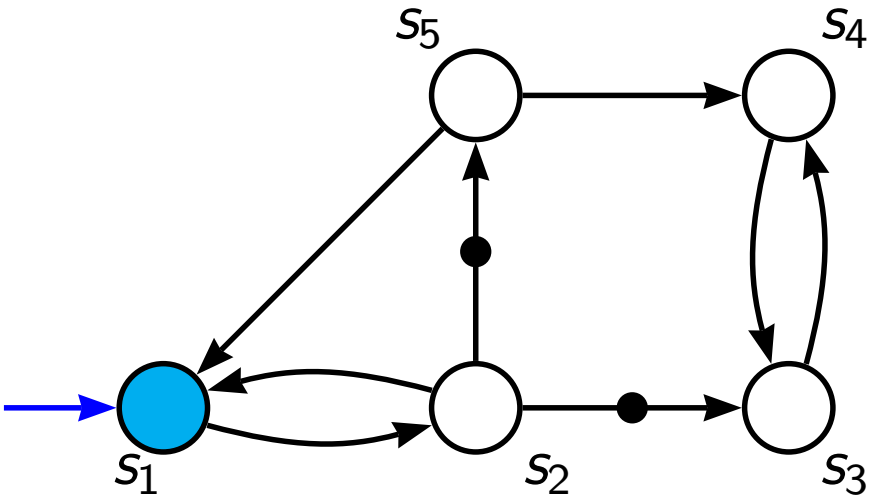
Nested DFS



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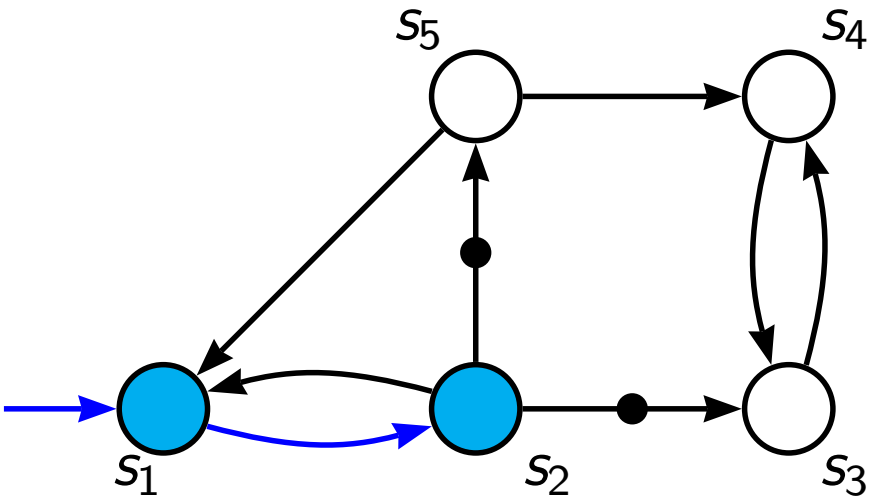

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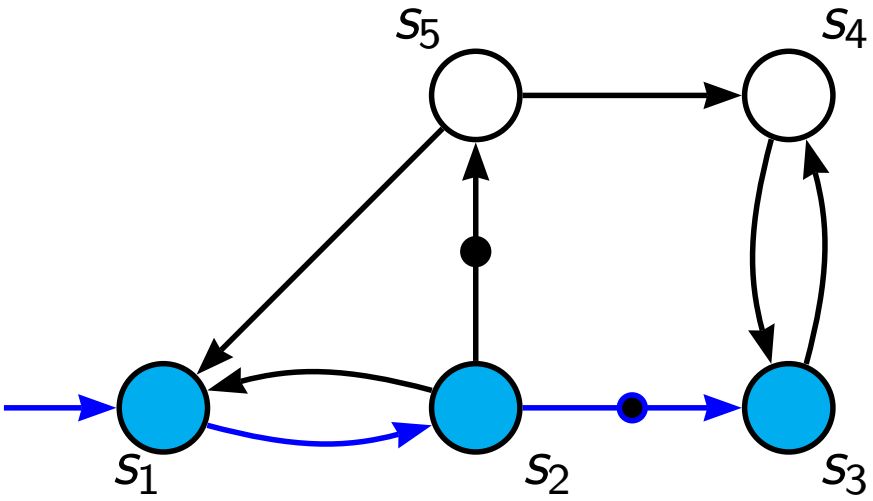

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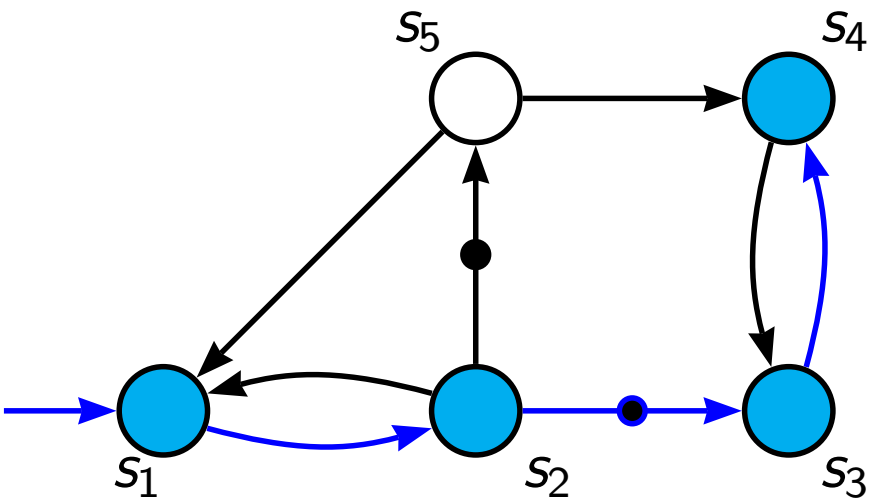

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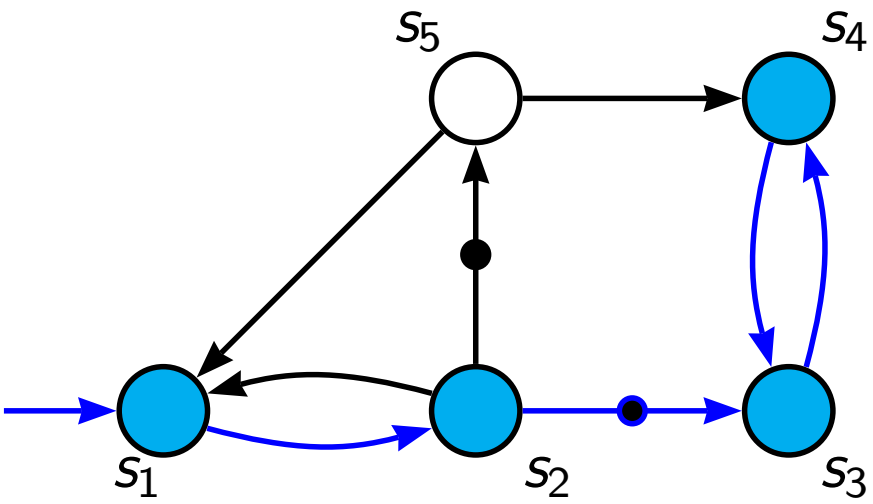

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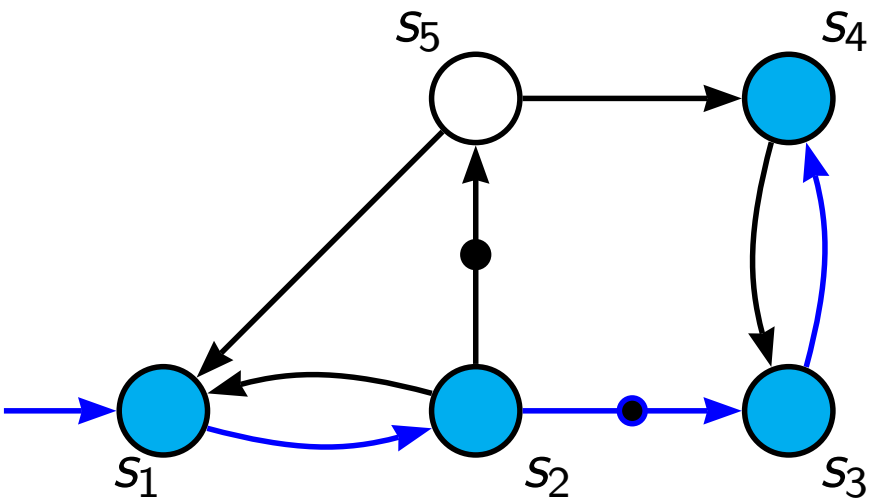

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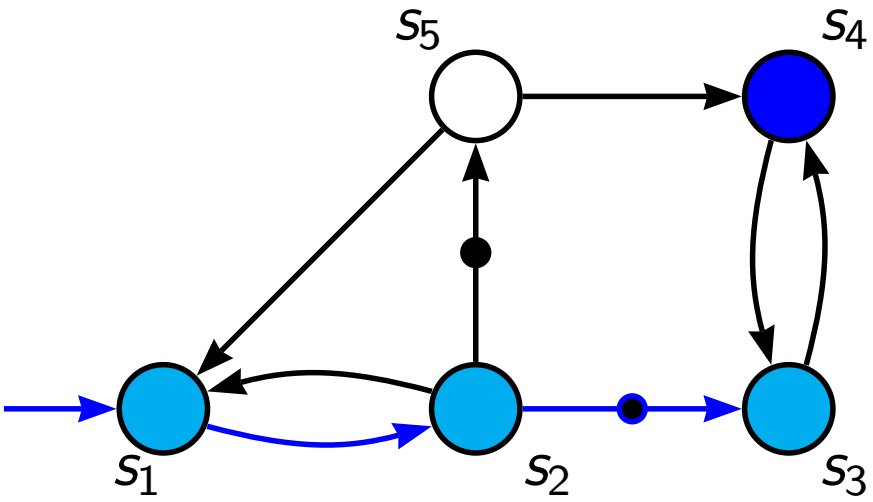

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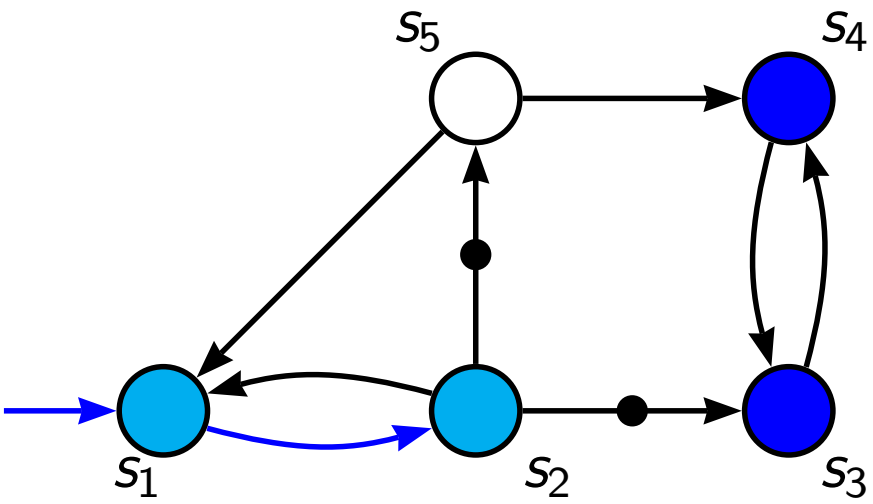

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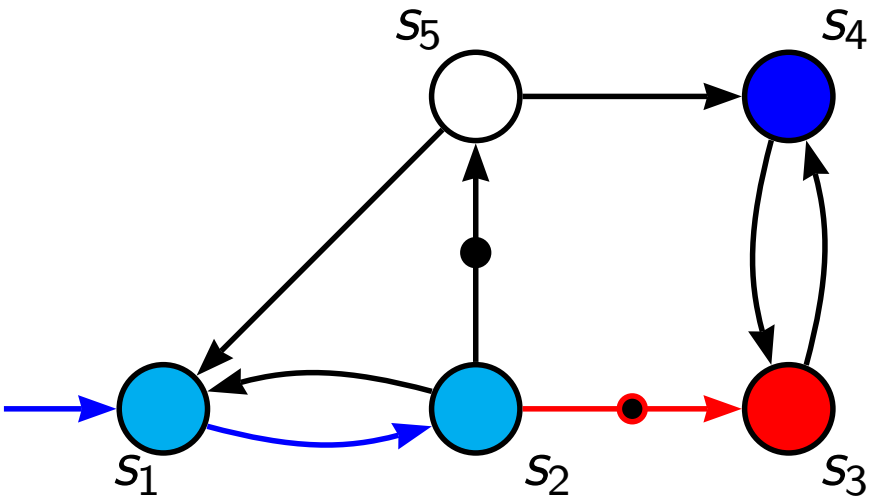

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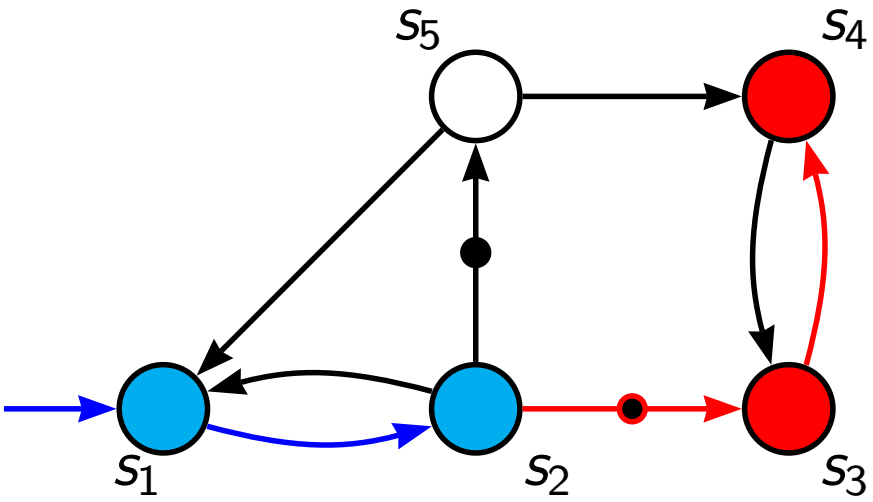

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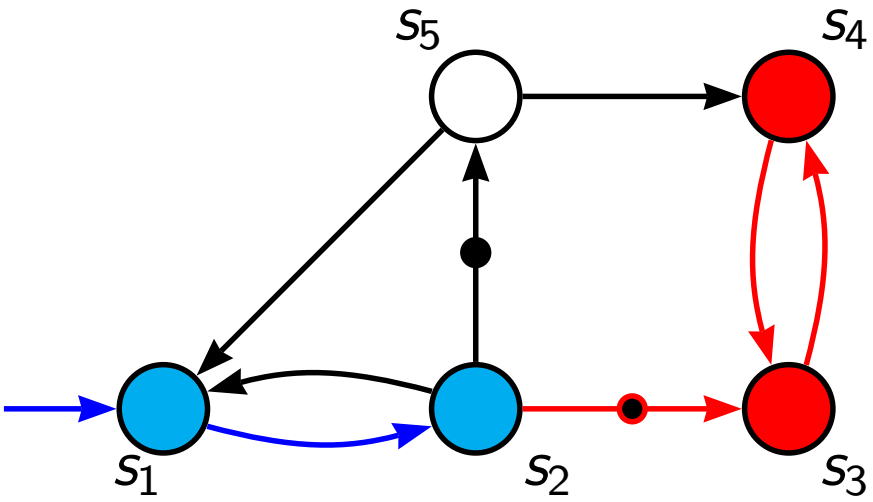

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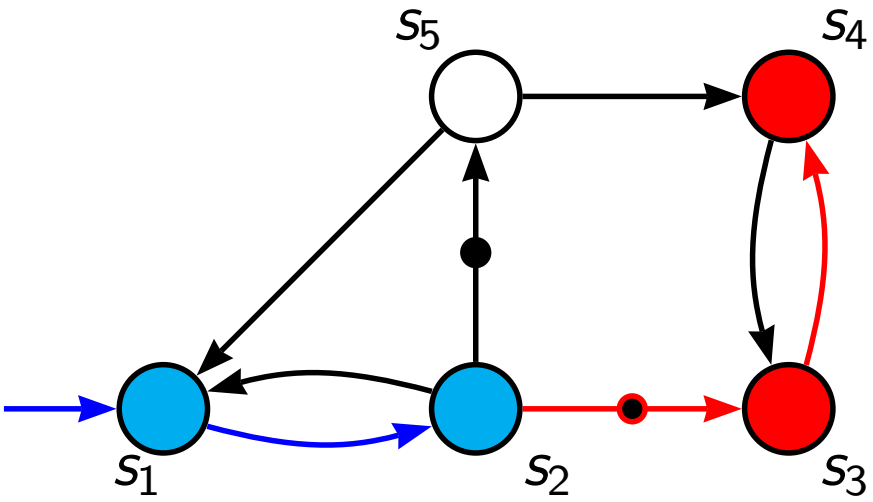

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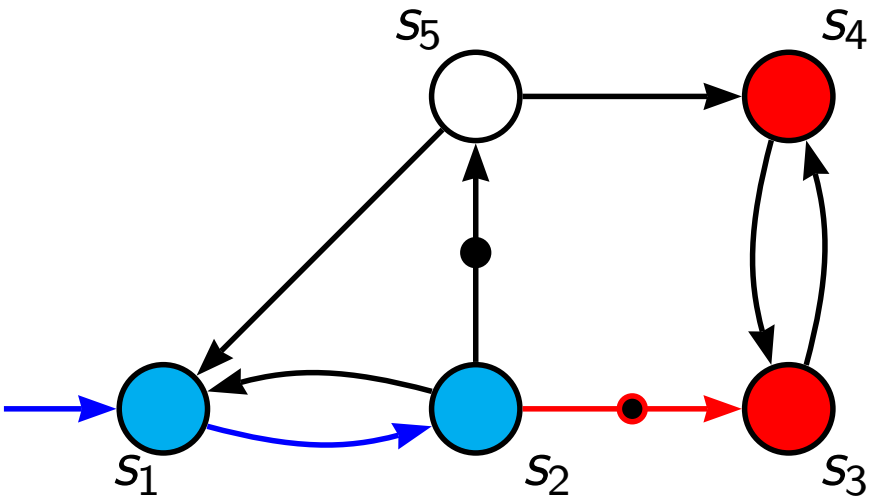

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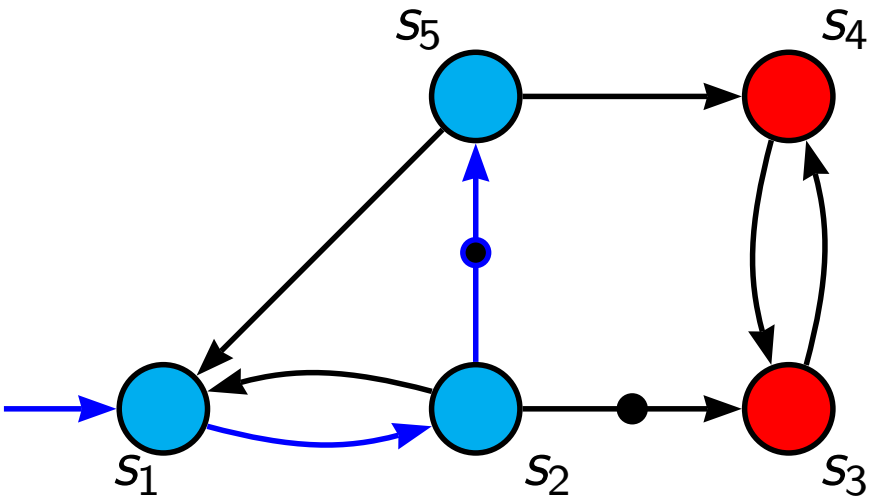

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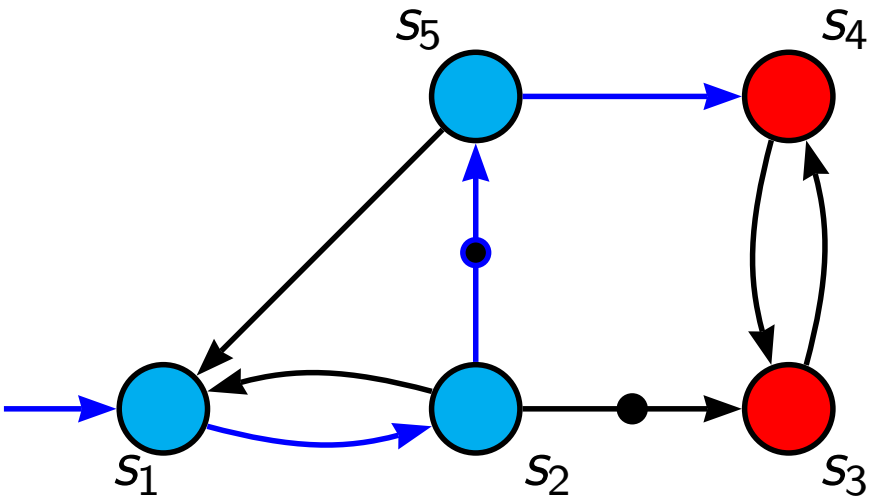

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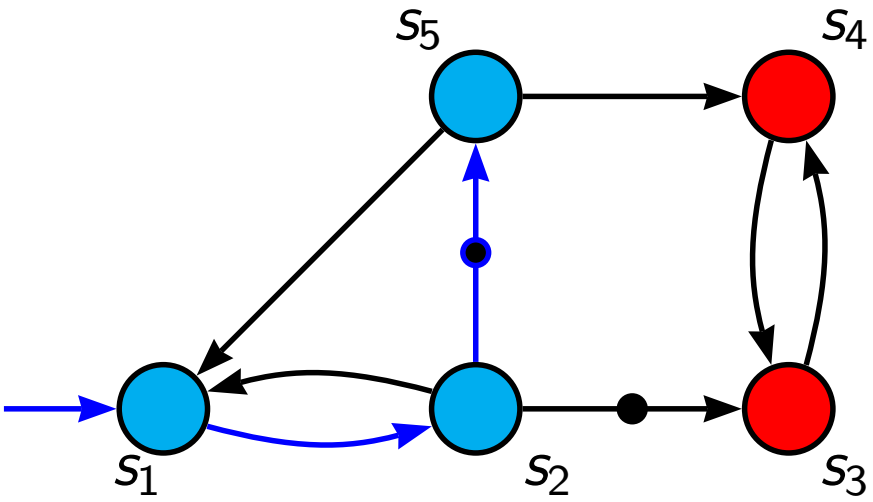

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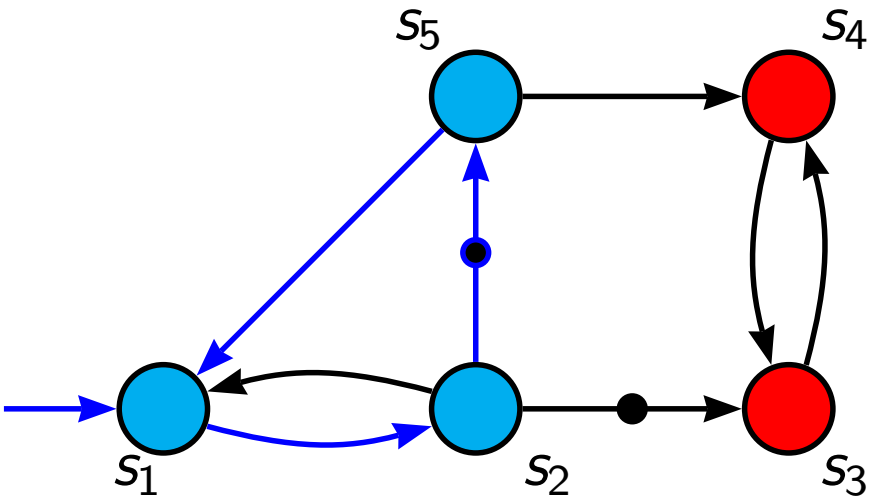

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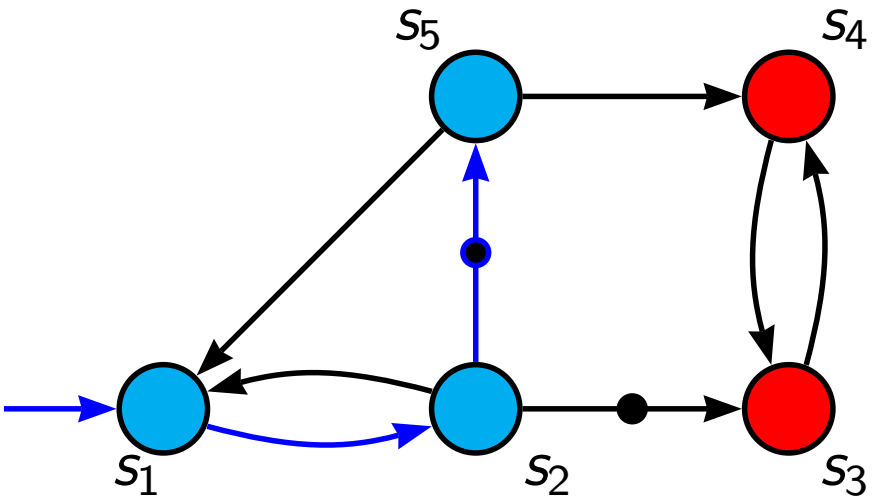

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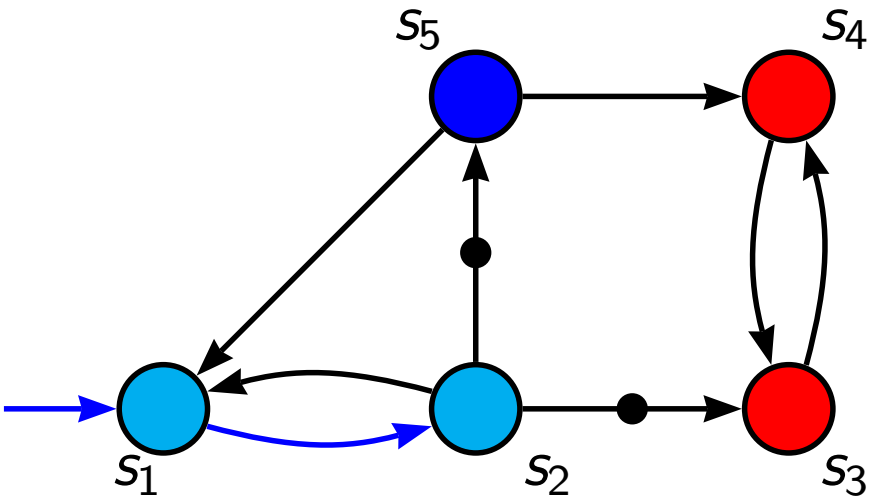

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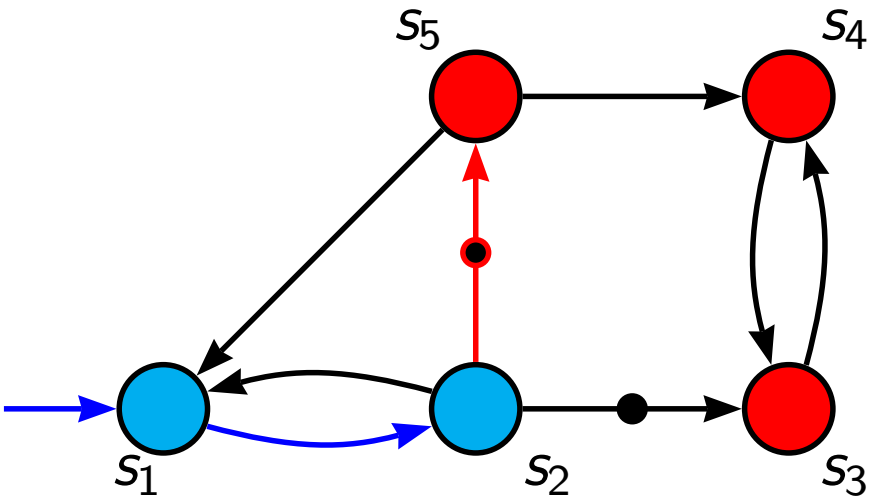

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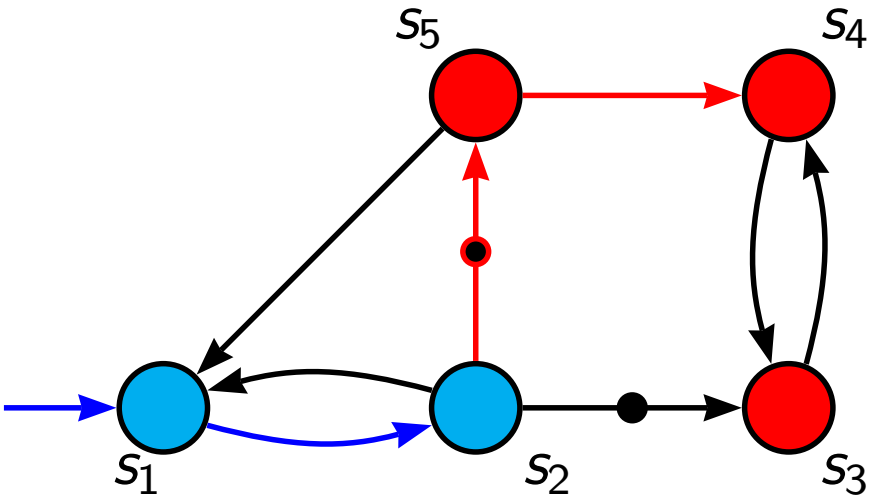

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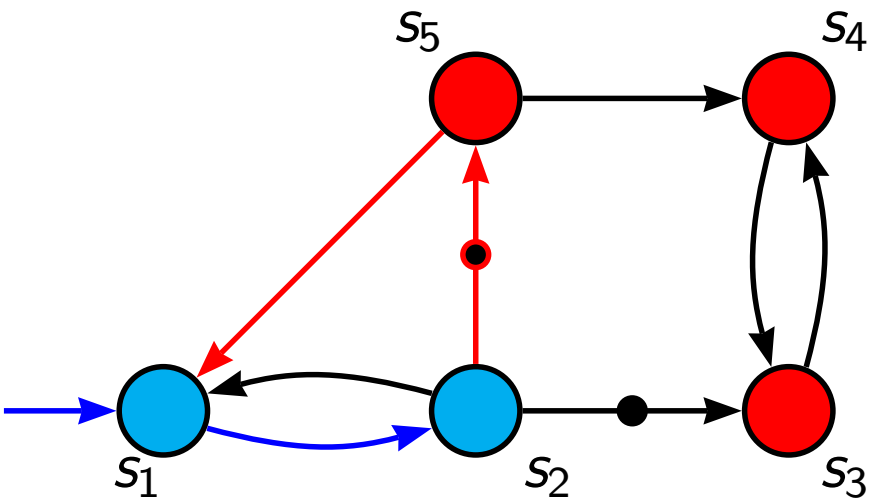

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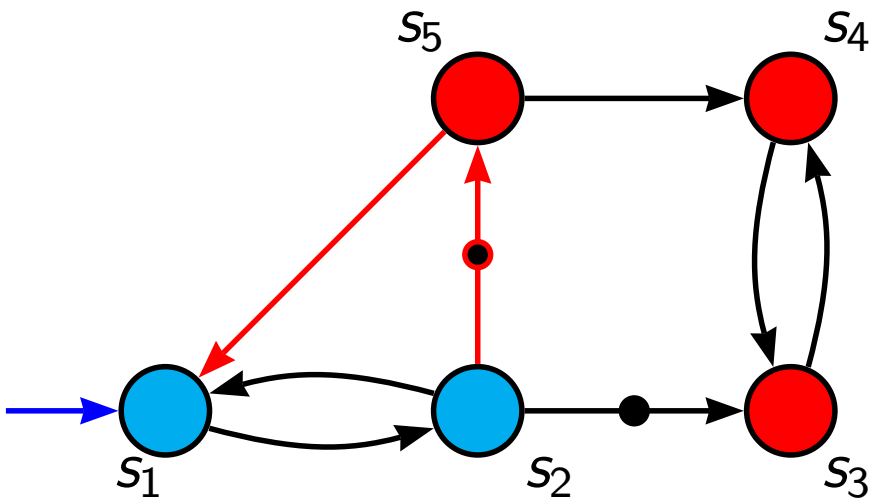

Found!

	entries in hash table	hash table size in bits	search stack depth	states traversed
upper bounds:	n	$n(s+2)$	n	$2n$

n = number of states; s = bits per state.

Generalized Büchi Automata

A Generalized (transition-based) Büchi automaton has:

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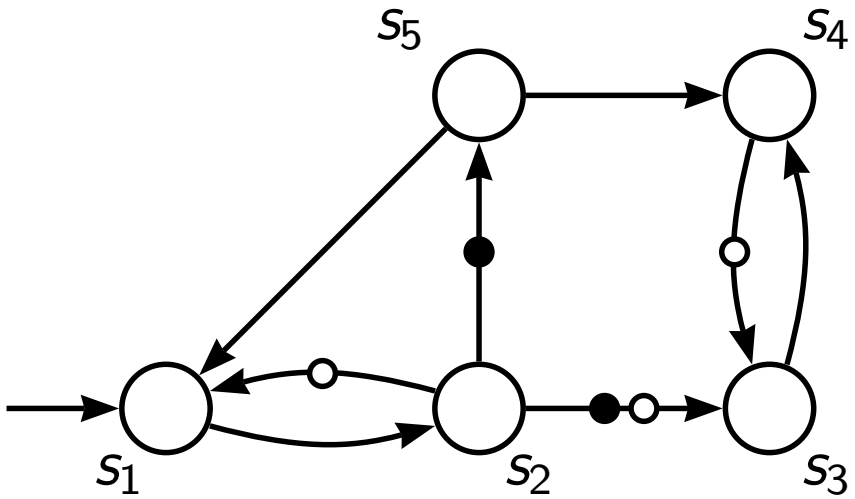
An infinite run of this automaton is accepting if it visits a transition from each accepting set infinitely often.



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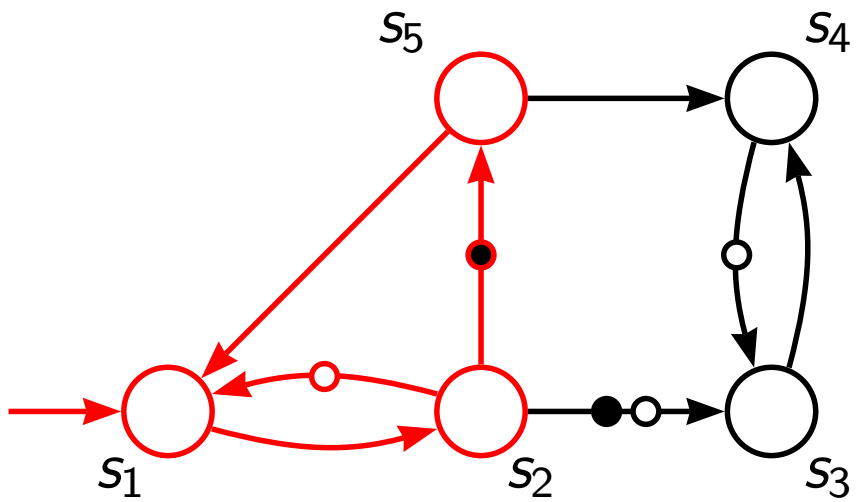

Degeneralization

A generalized automaton with

- n states
- m acceptance conditions

can be degeneralized into an automaton with

- nm states at worst
- 1 acceptance condition

Nested DFS on Generalized Büchi Automata

entries in hash table	hash table size in bits	search stack depth	states traversed
n	$n \quad (s \quad + 2)$	n	$2n$

n states,

s bits per state.

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n states, m acceptance conditions,
s_d bits per degeneralized state.

Nested DFS on Generalized Büchi Automata

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n	$n(s_g + 2m)$	nm	$2nm$

n states, m acceptance conditions,
s_d bits per degeneralized state, s_g bits per generalized state ($s_g \leq s_d$).

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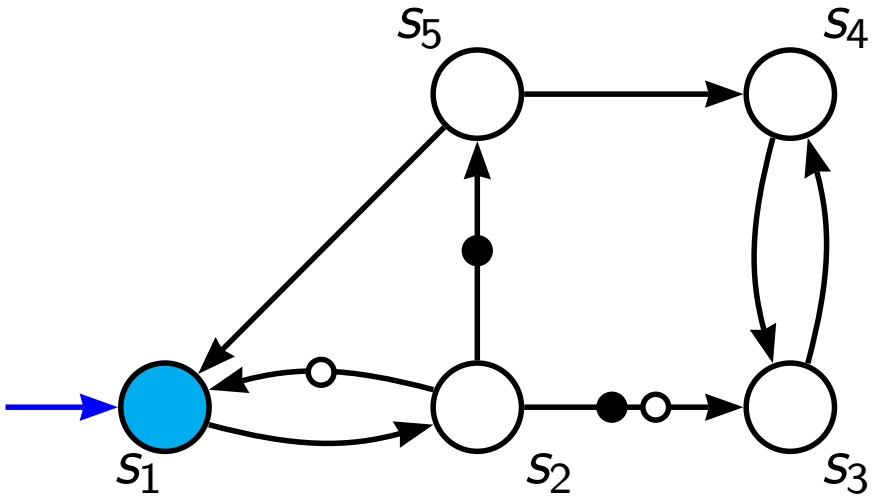

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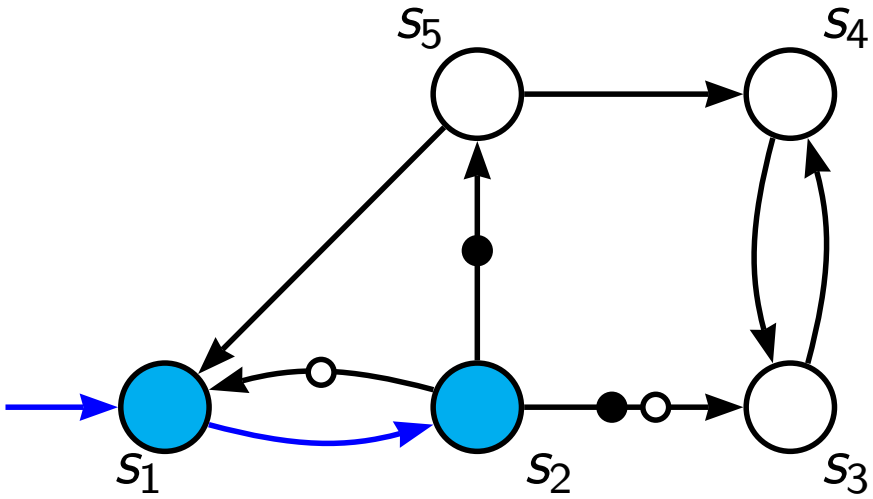

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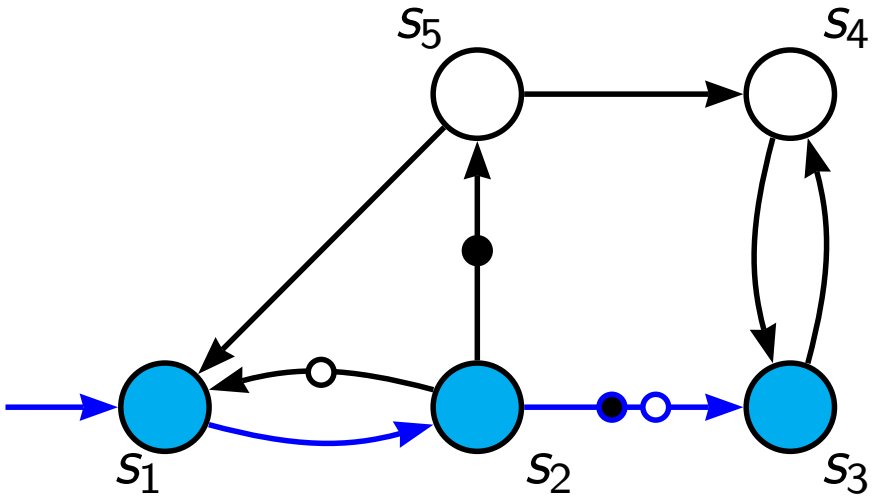

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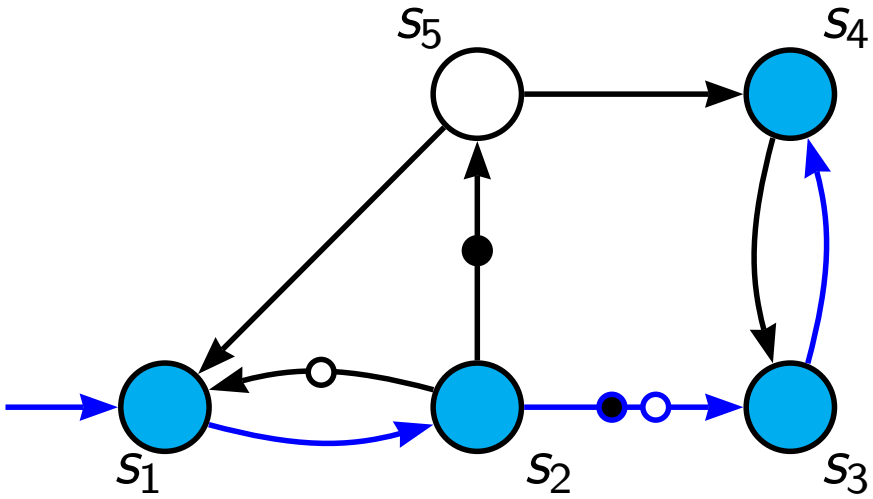

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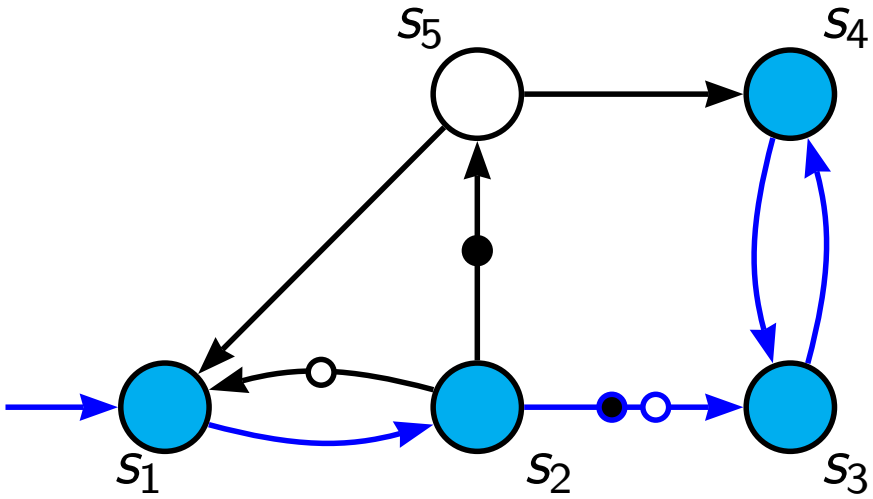

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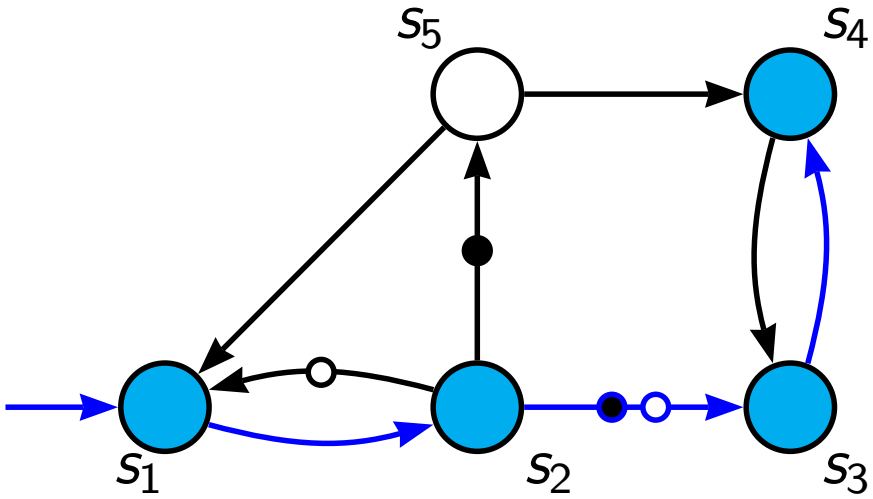

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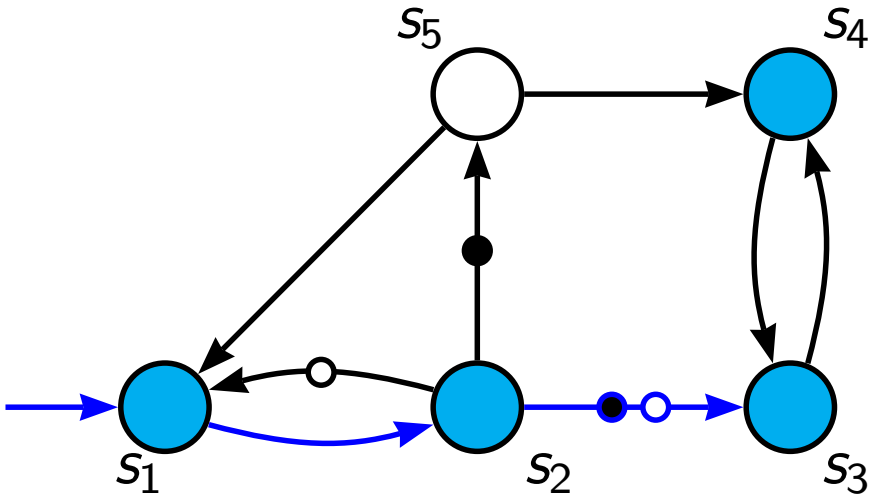

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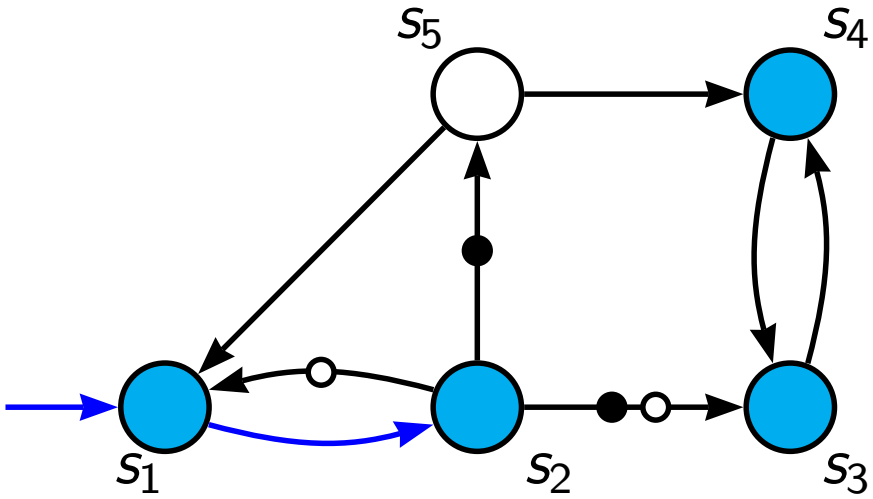

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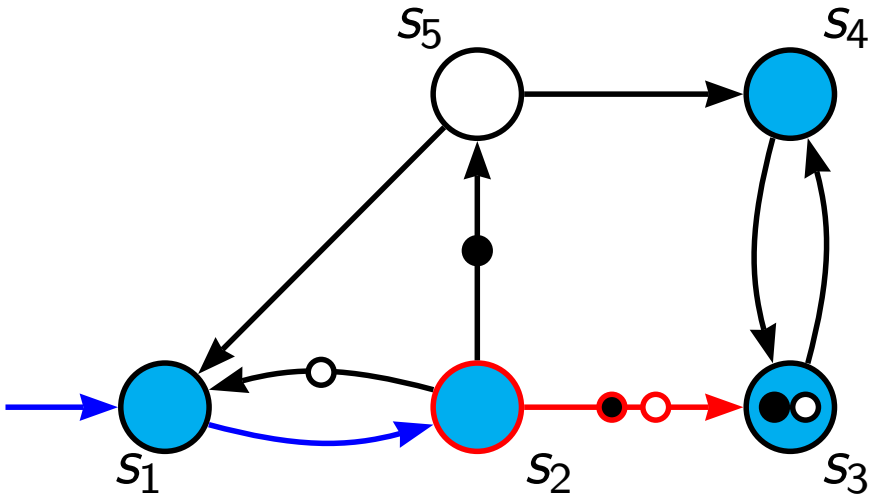

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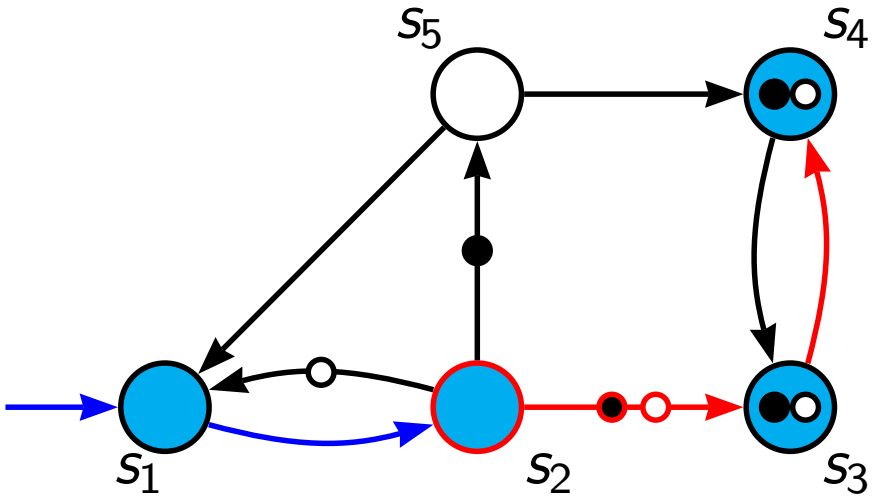

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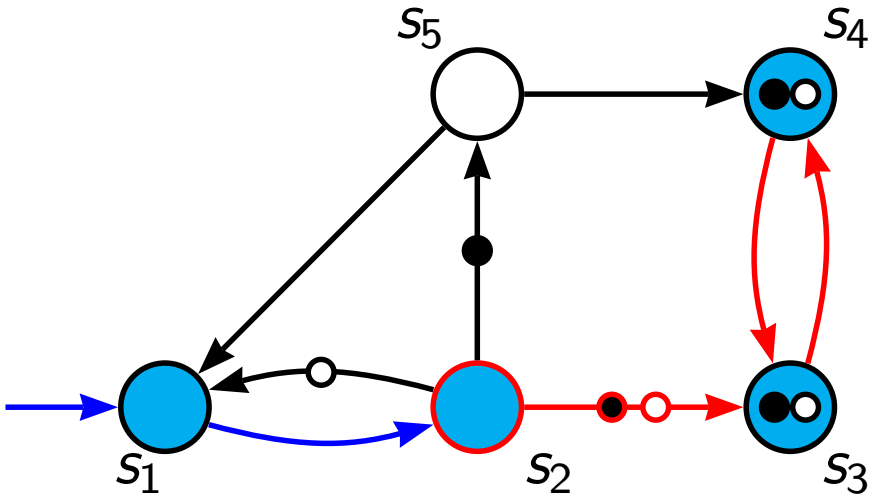

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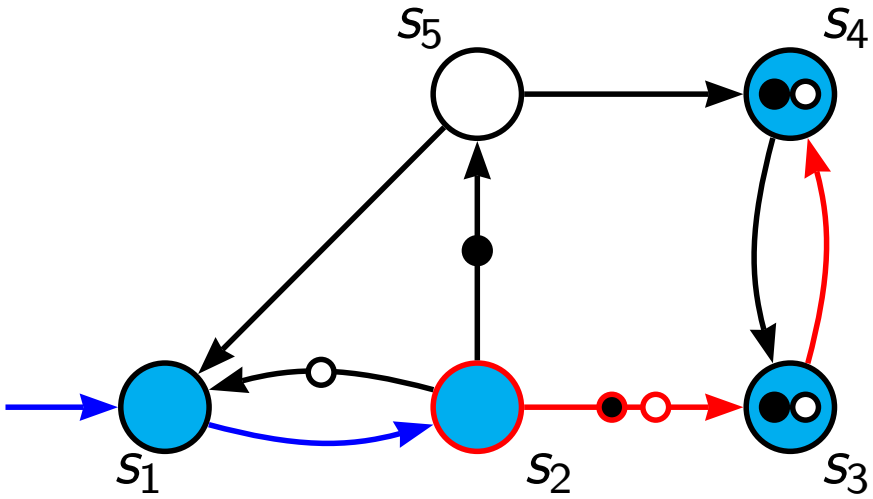

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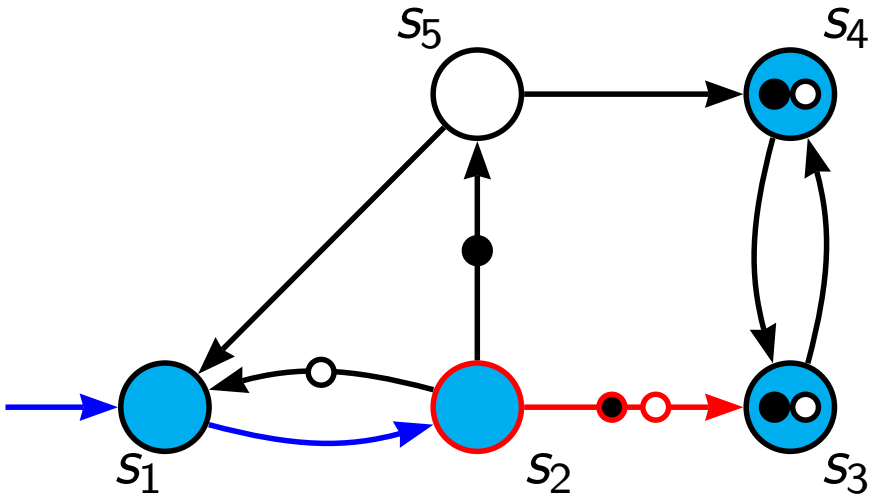

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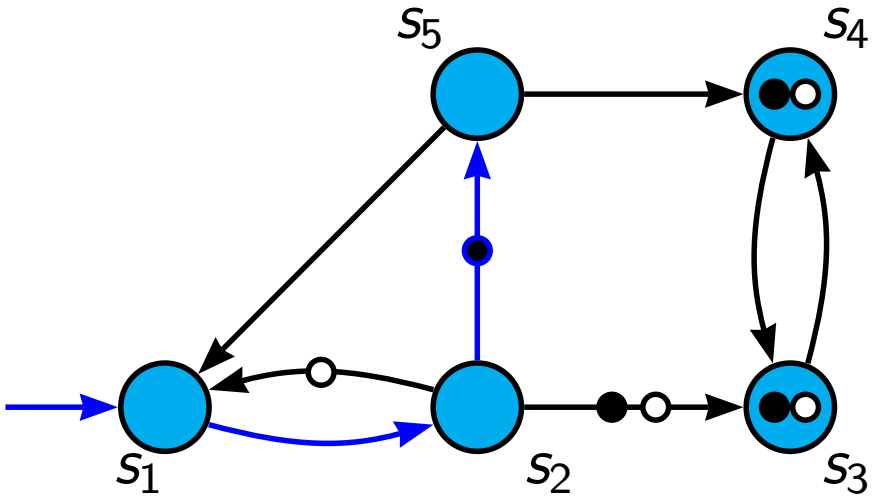

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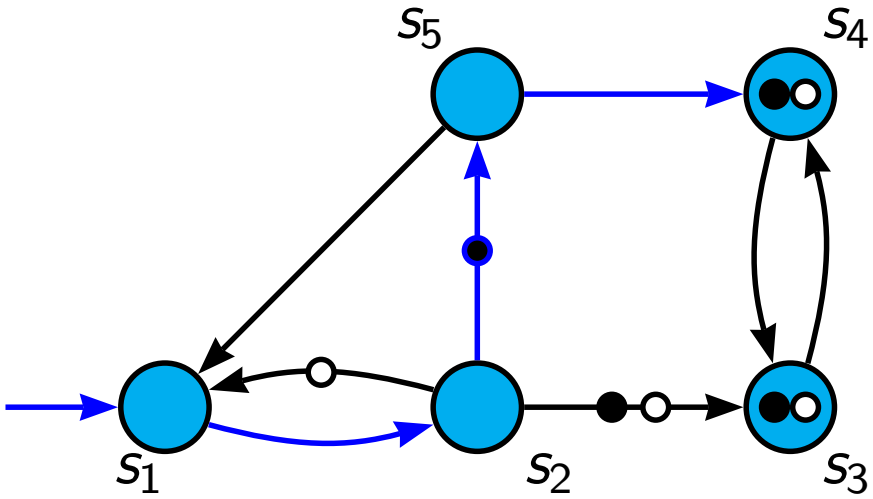

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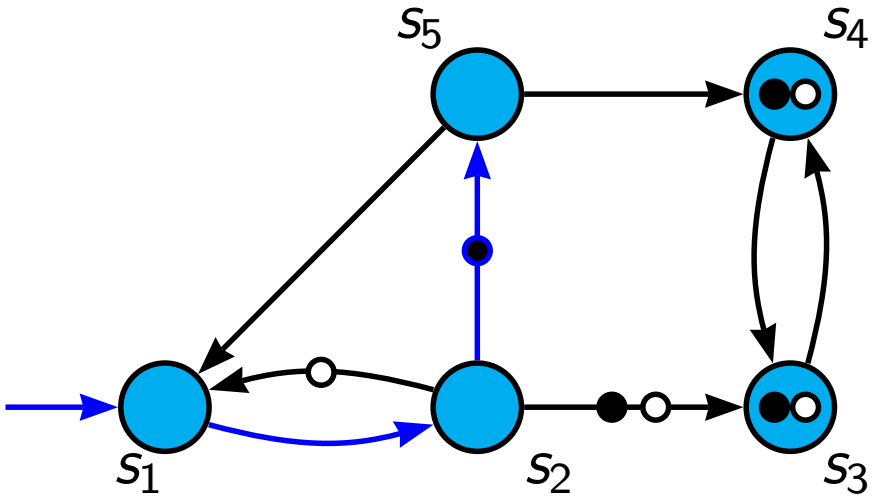

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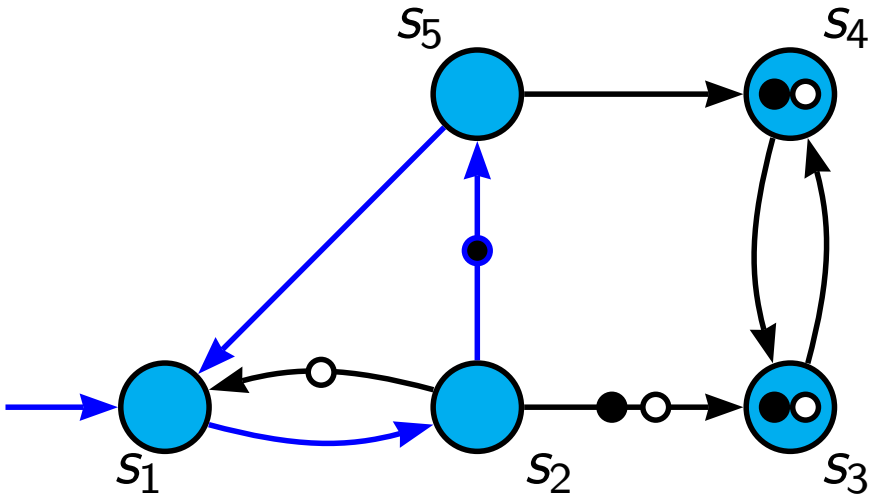

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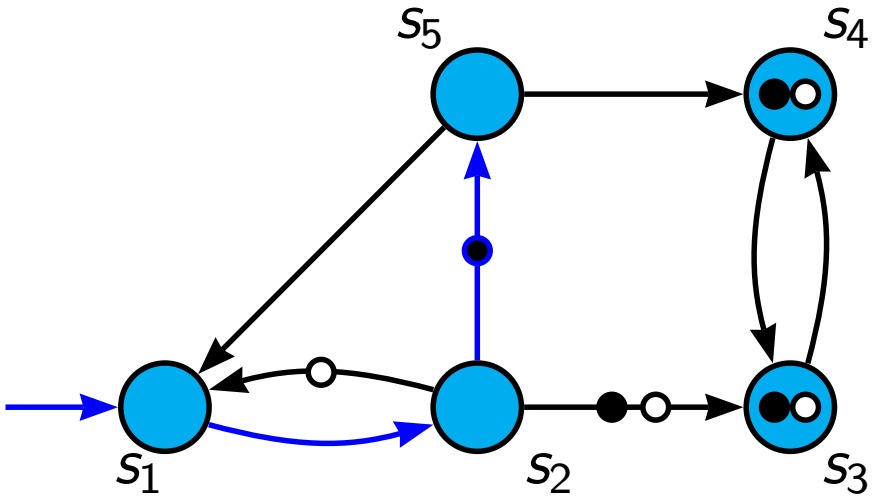

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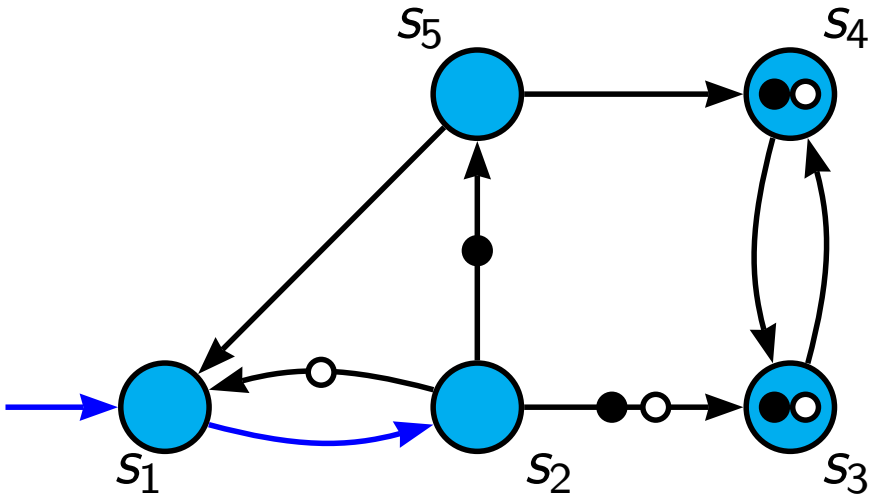

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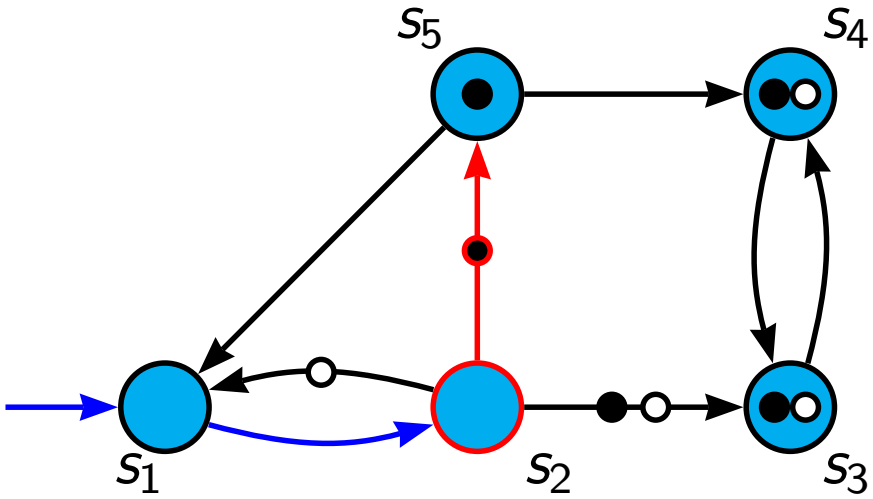

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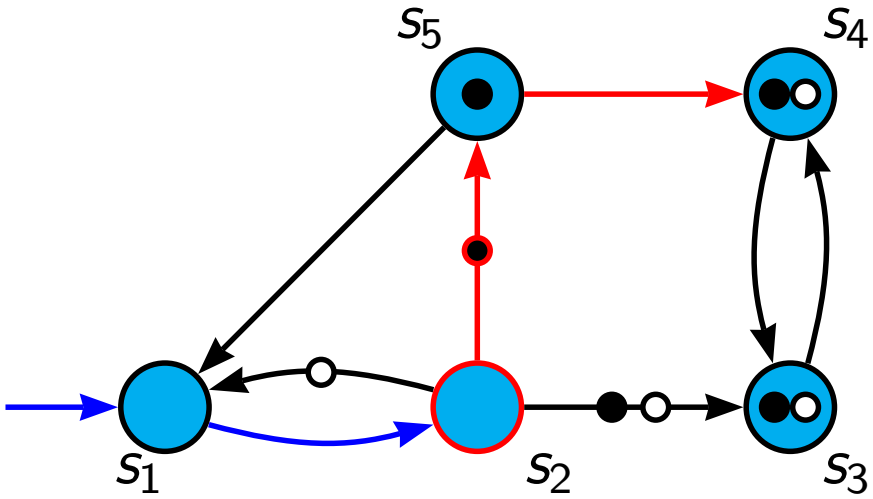

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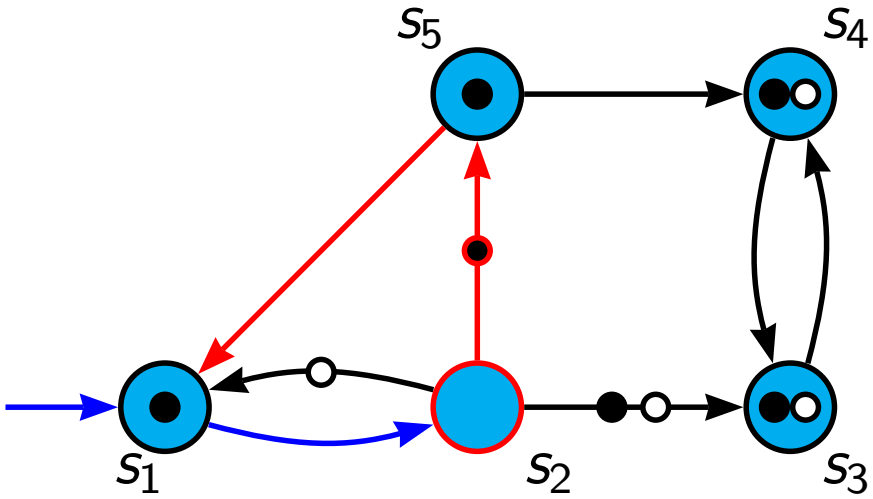

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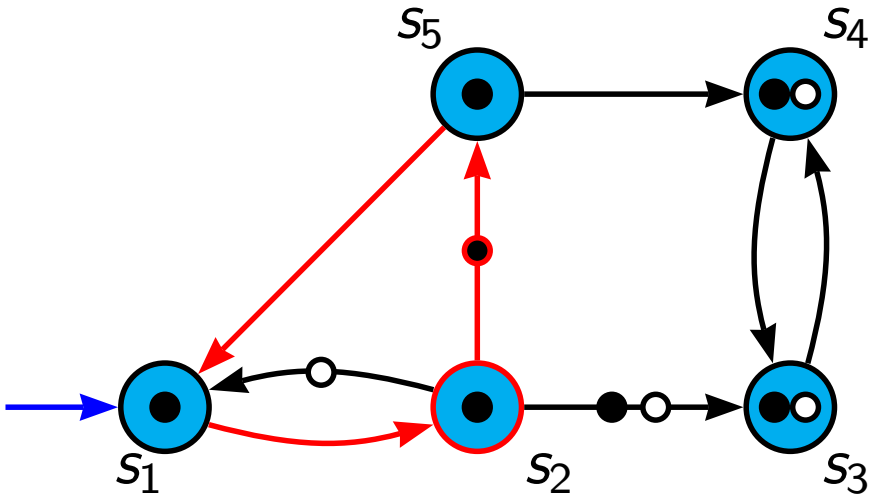

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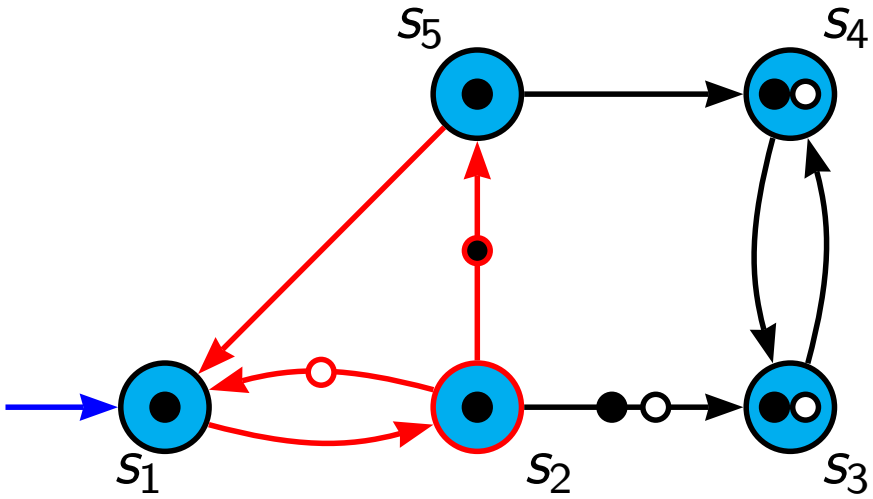

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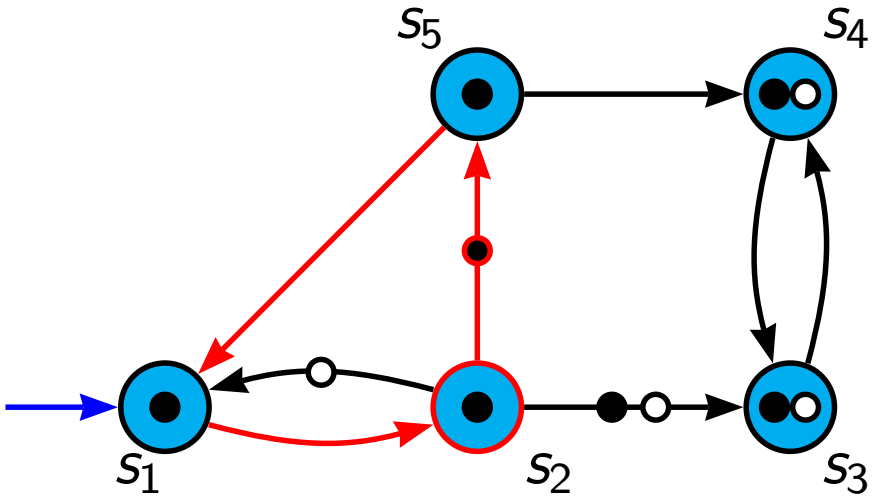

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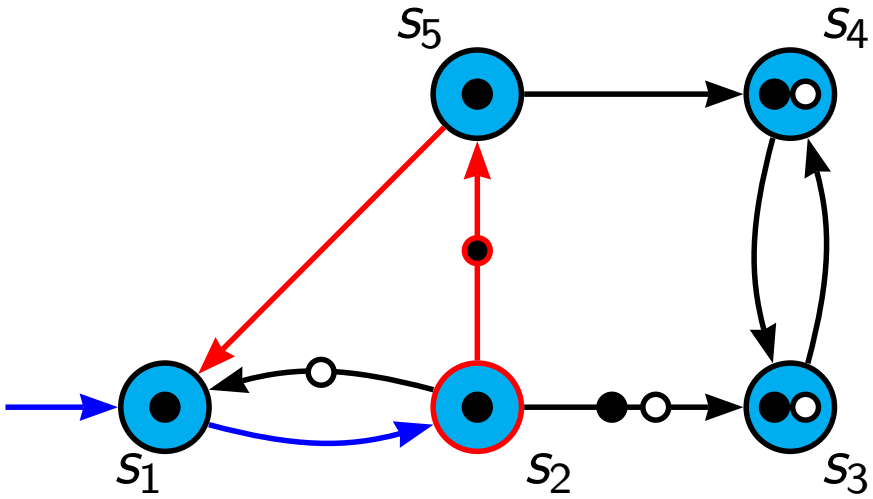

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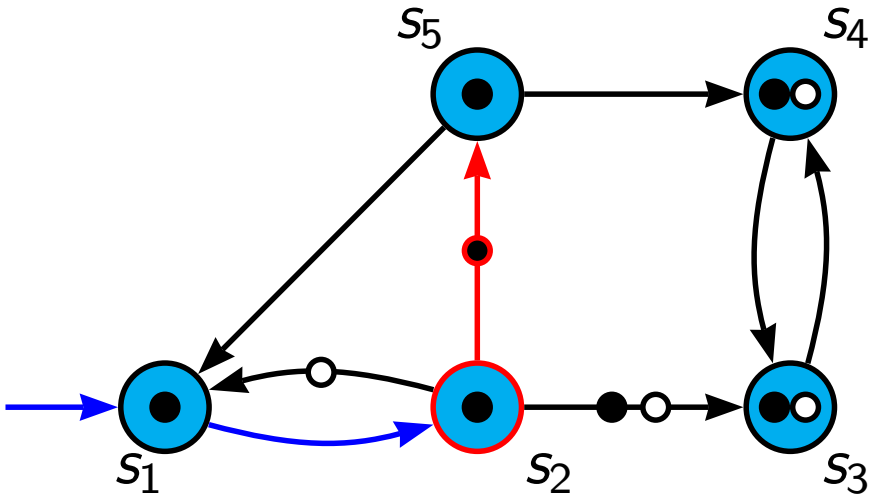

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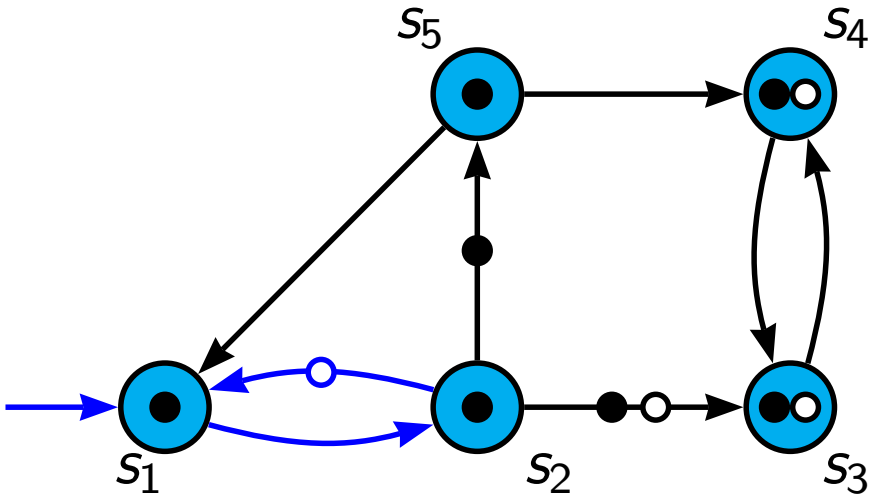

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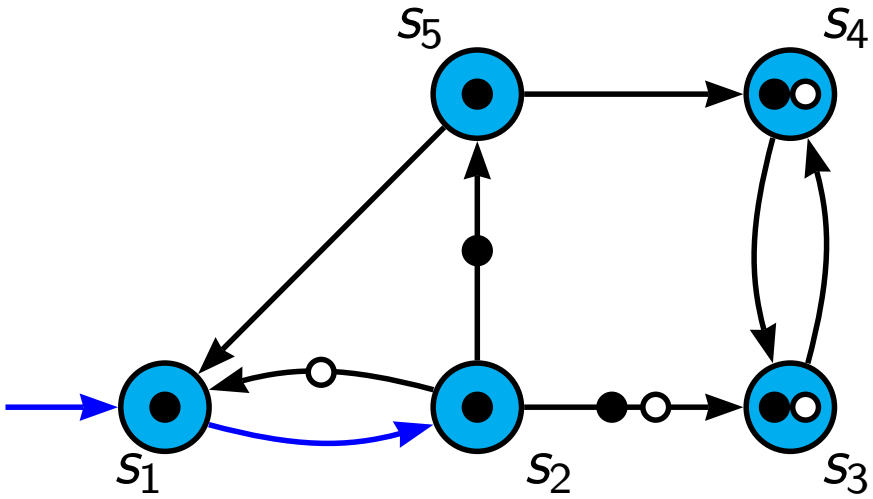

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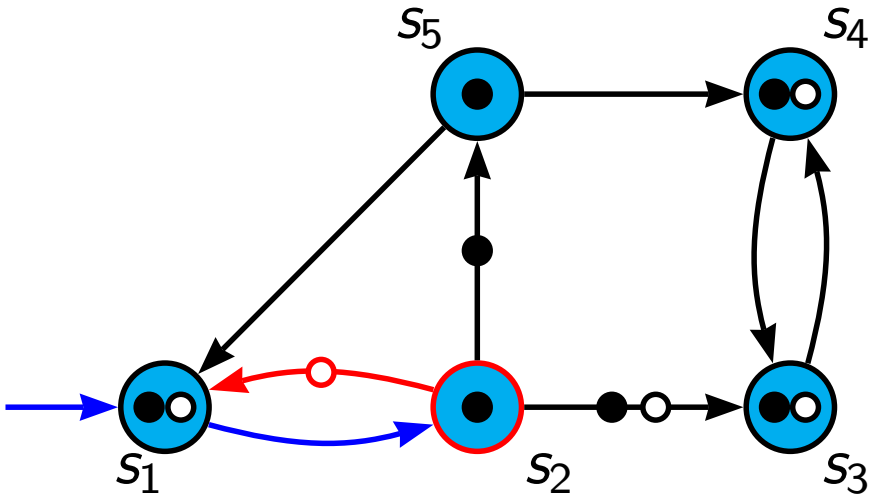

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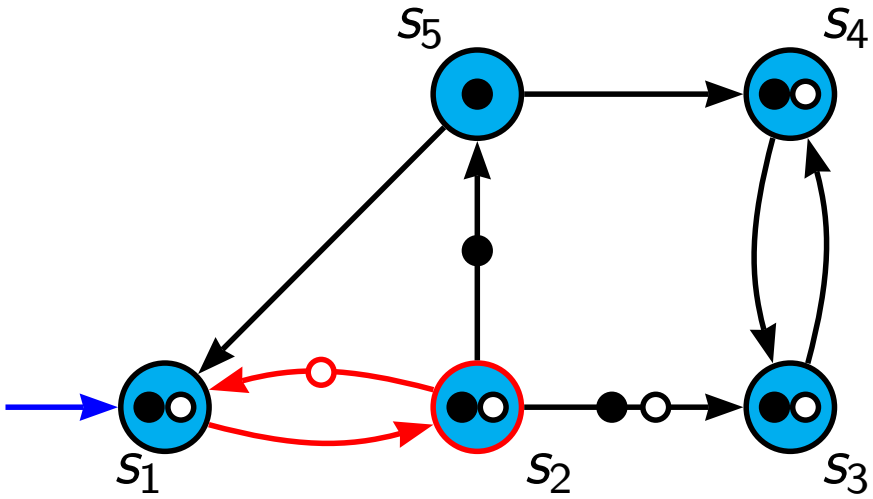

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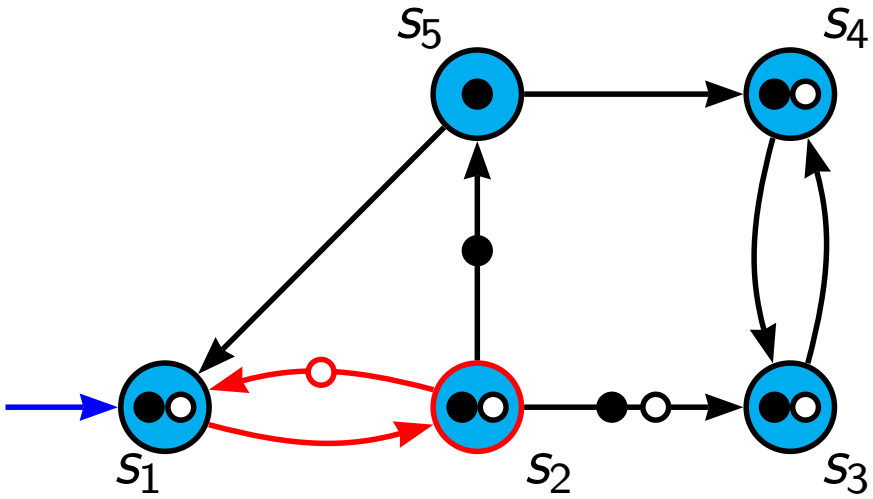

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	entries in hash table	hash table size in bits	search stack depth	states traversed
degen+NDFS	n	$n(s_g + 2m)$	nm	$2nm$
gen. NDFS	n	$n(s_g + m)$	$2n$	$n(m+1)$

Emptiness Checks History

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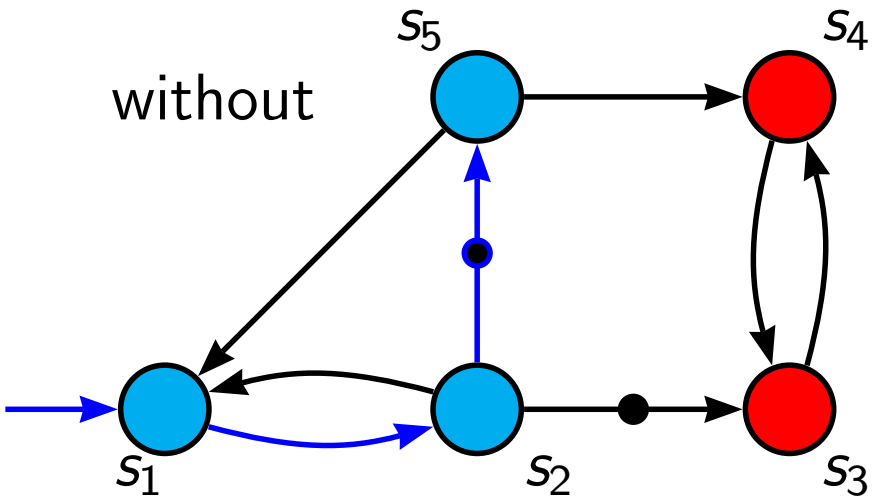

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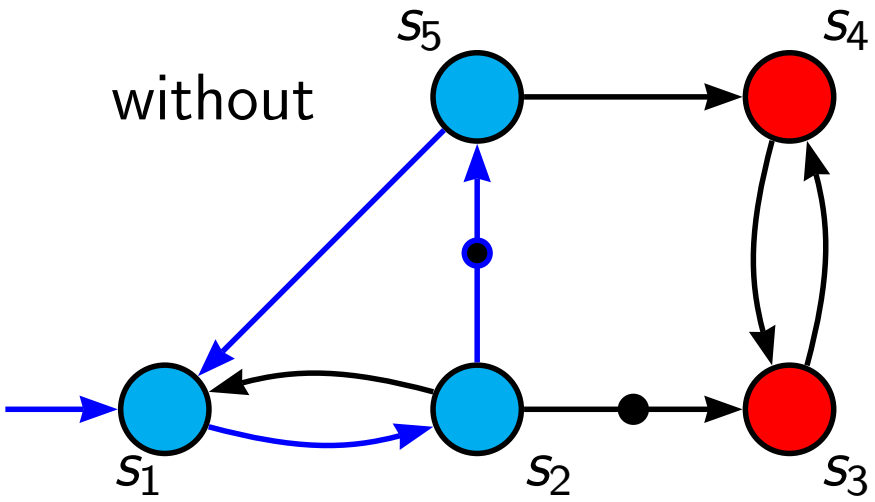

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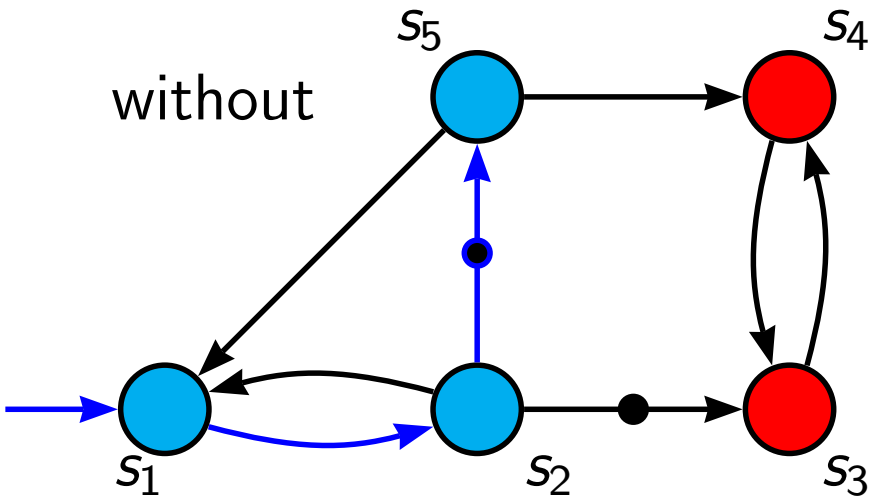

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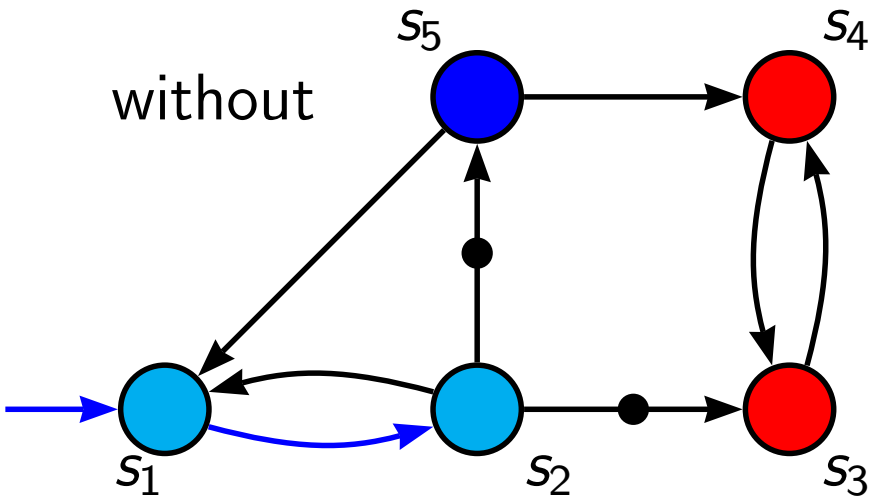

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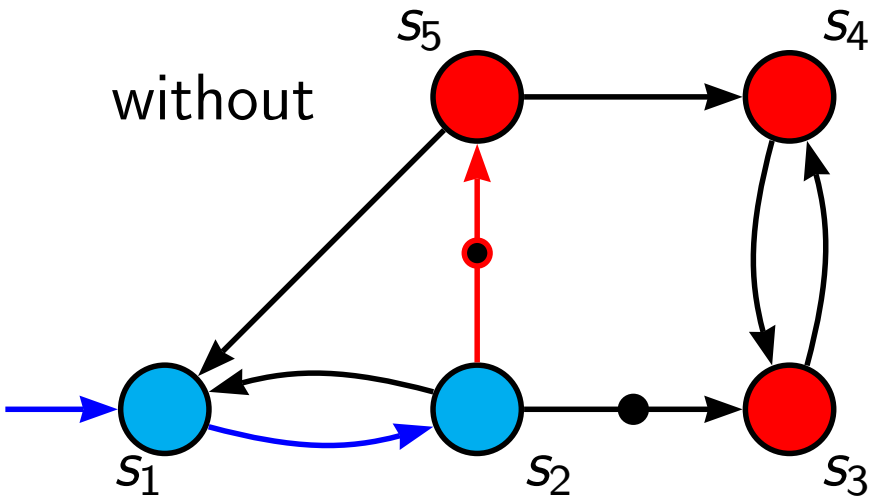

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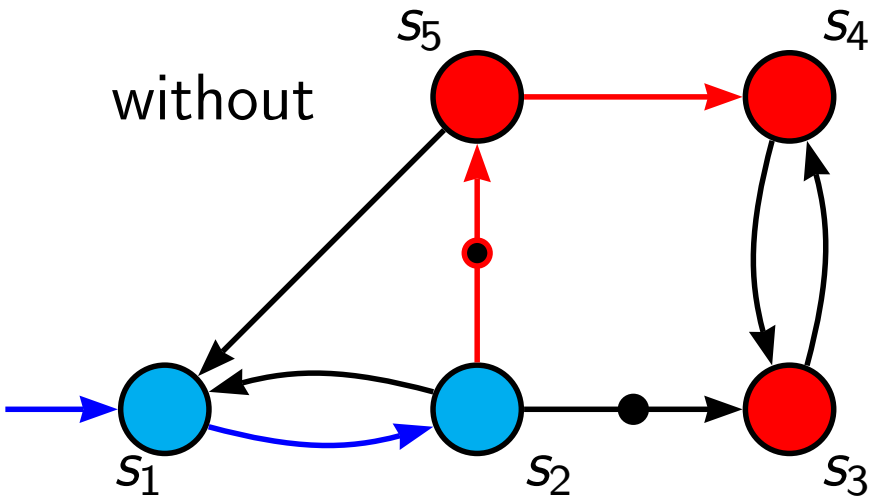

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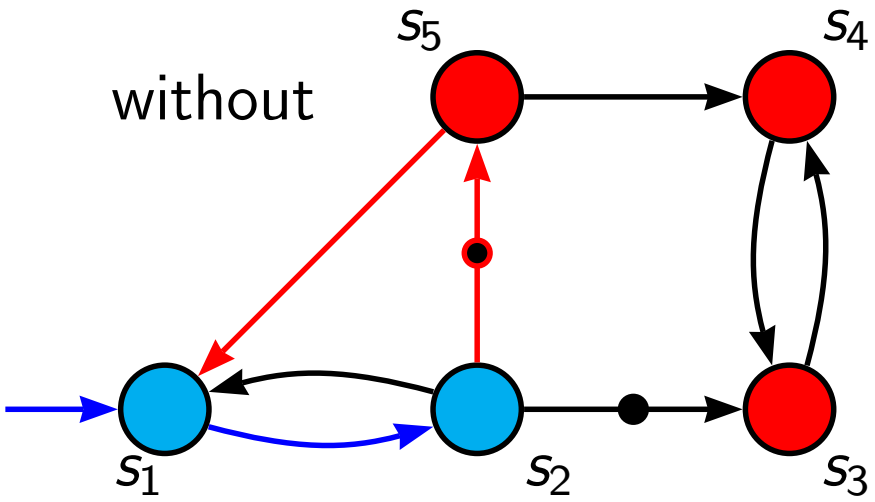

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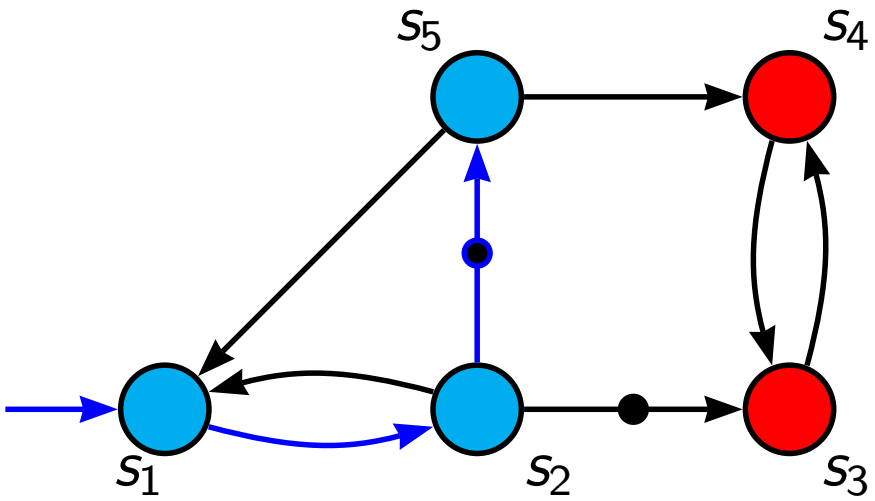

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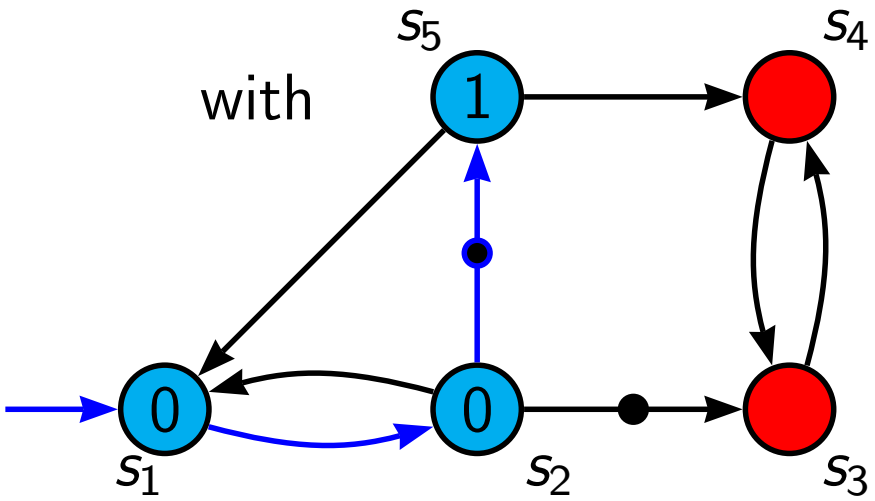

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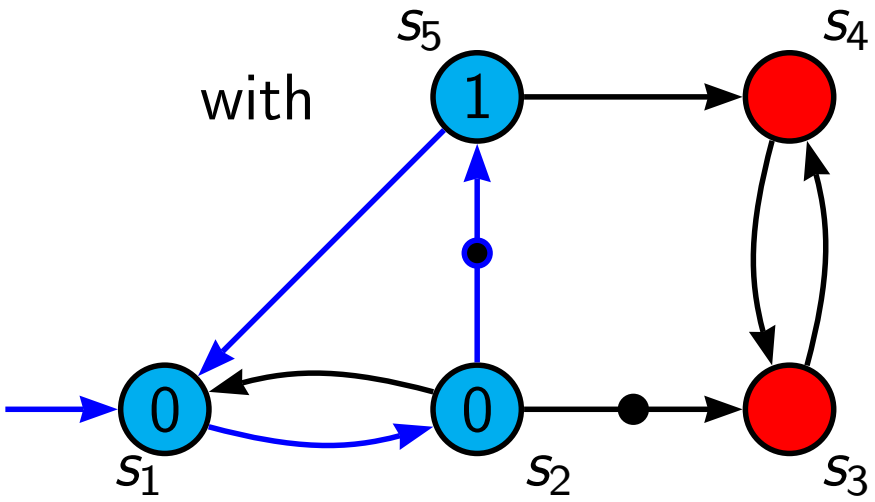

Found!

Generalizable with m counters per state in the blue search.

Emptiness Checks History

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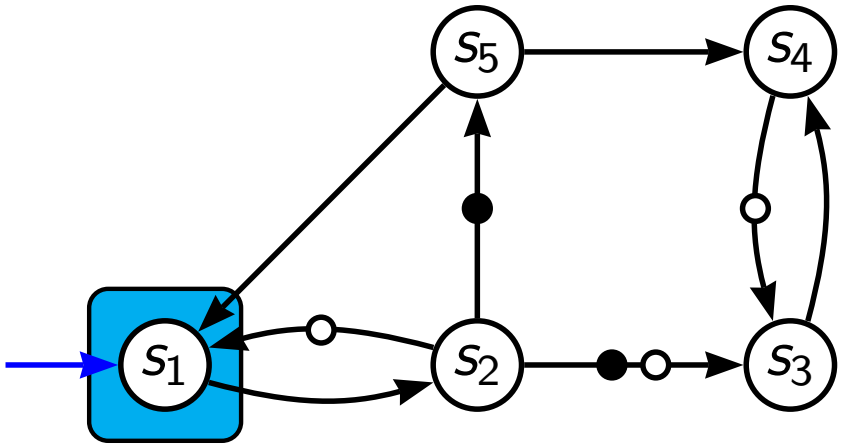
SCC-Based Emptiness Check



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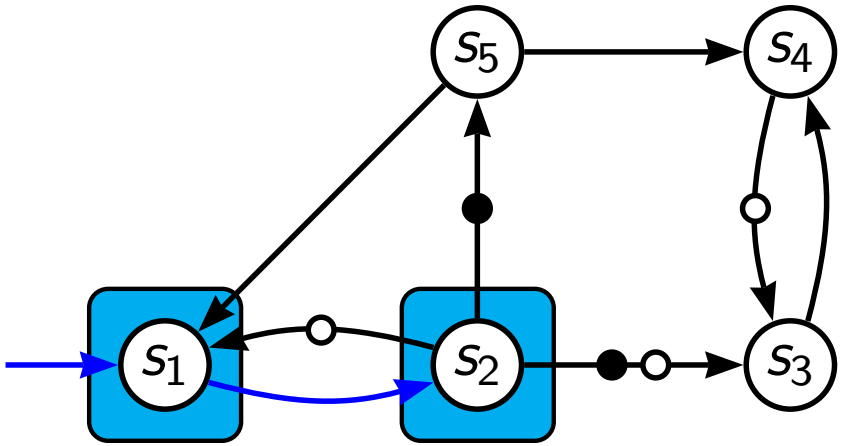

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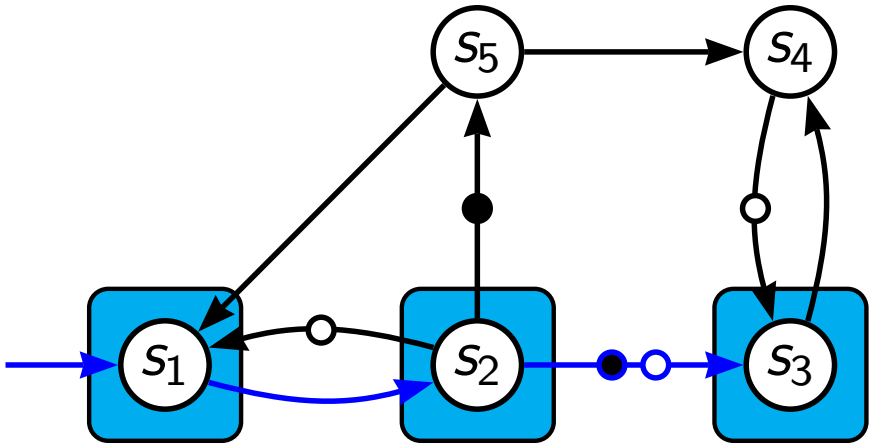

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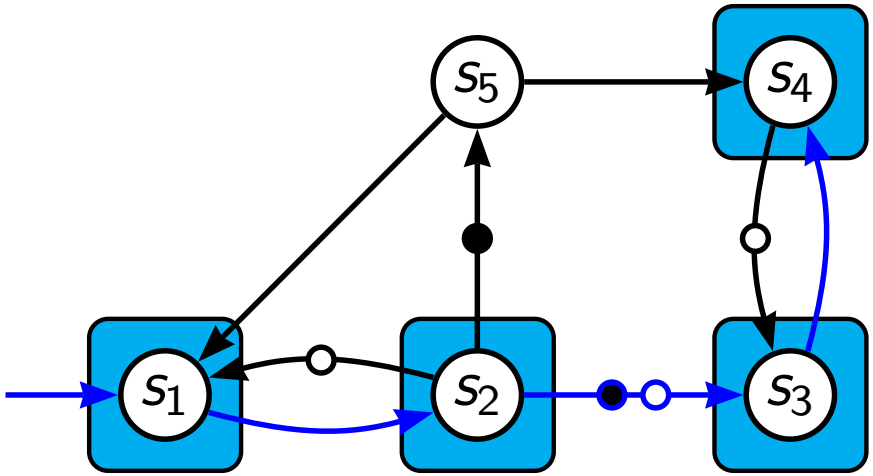

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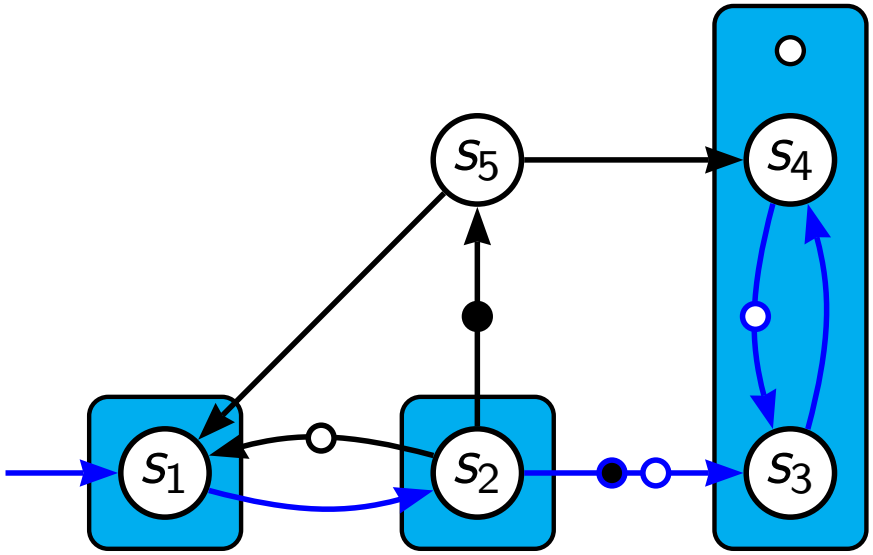

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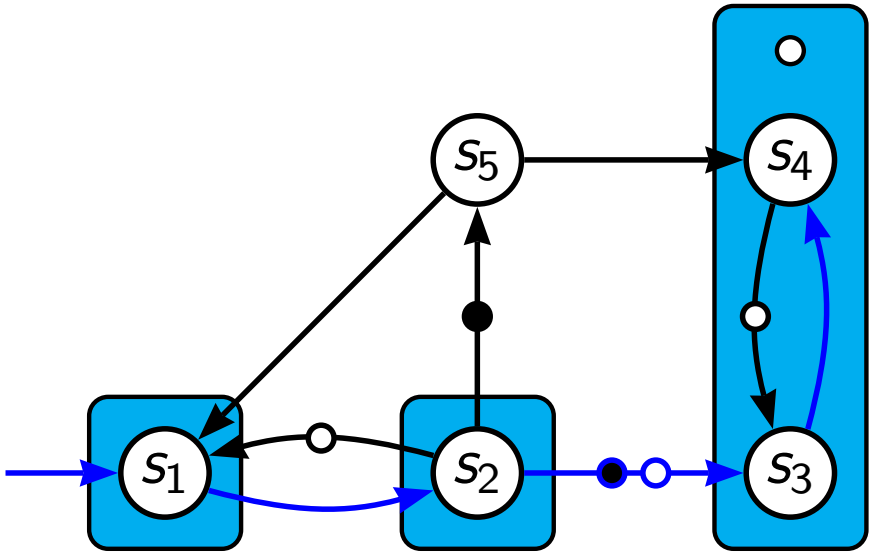

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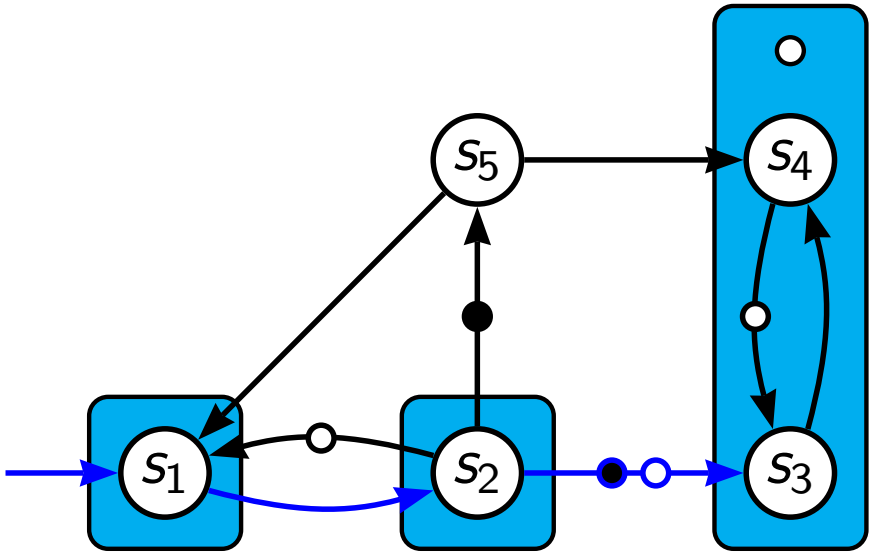

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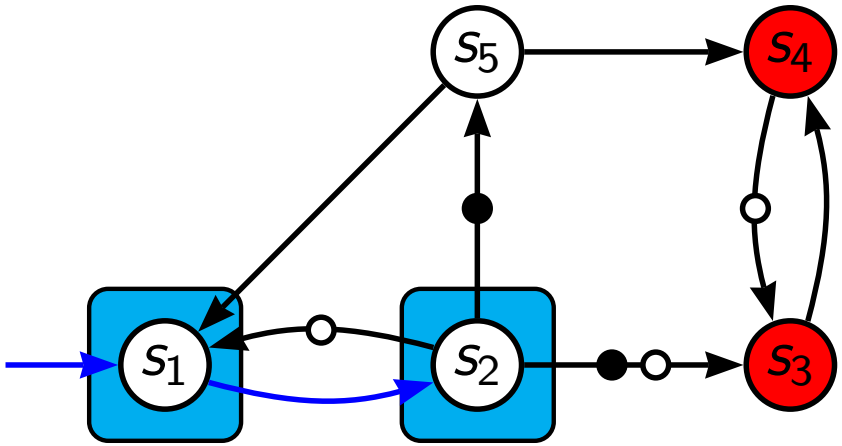

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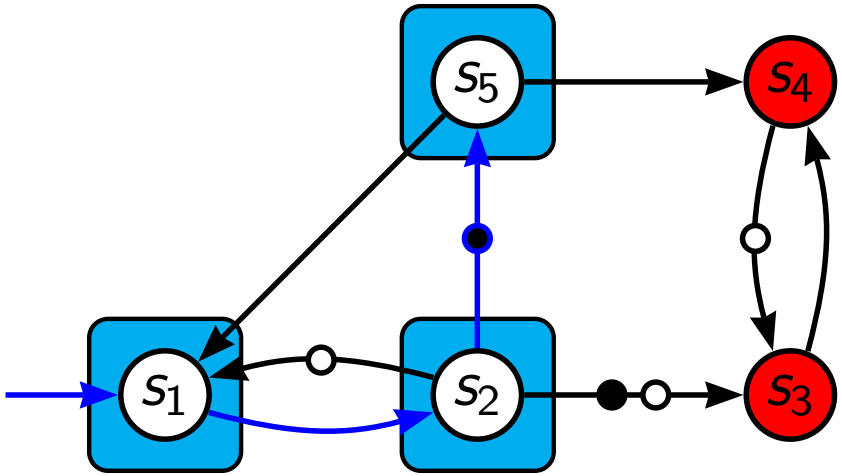

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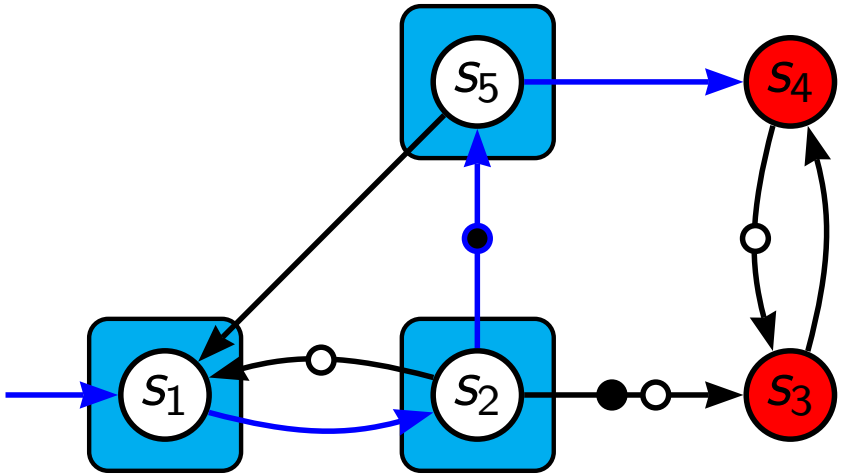

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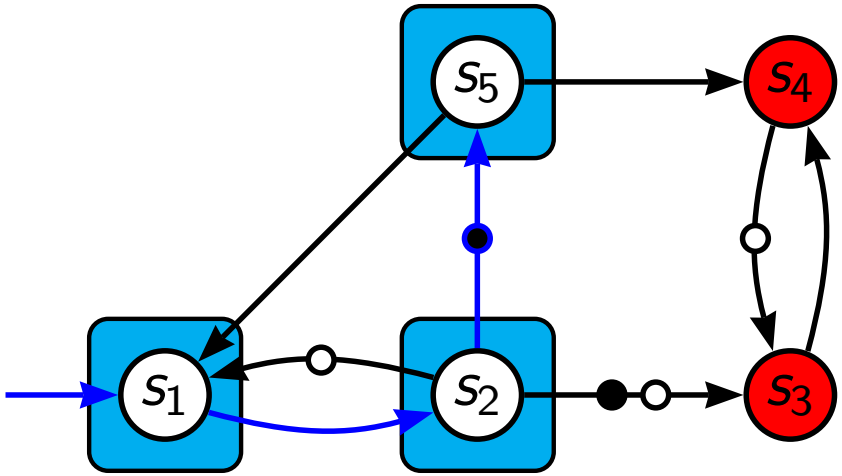

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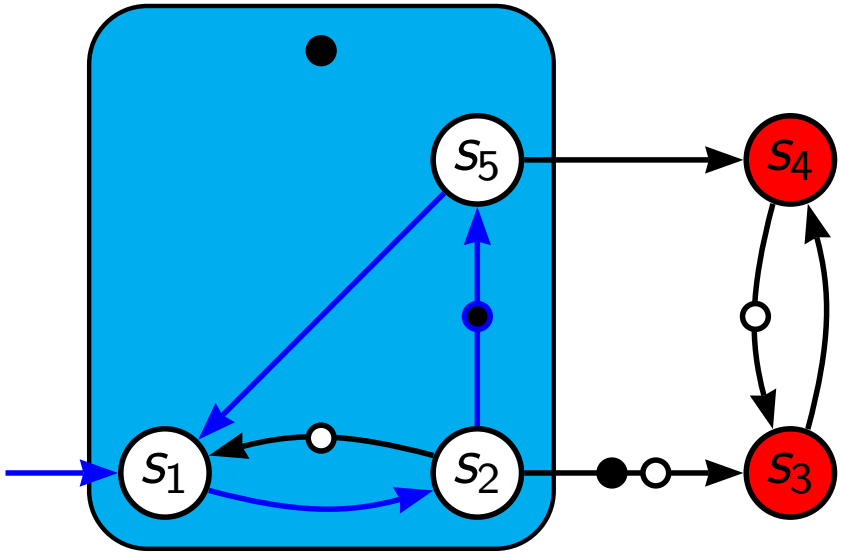

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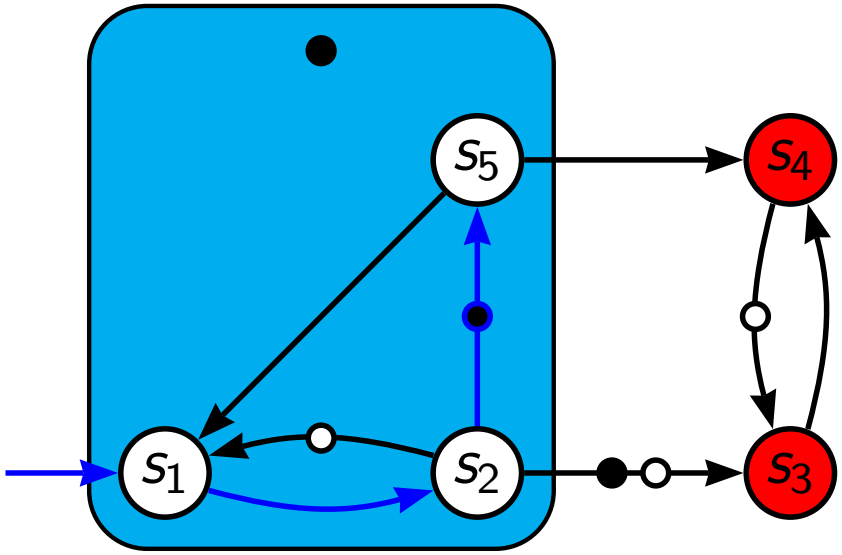

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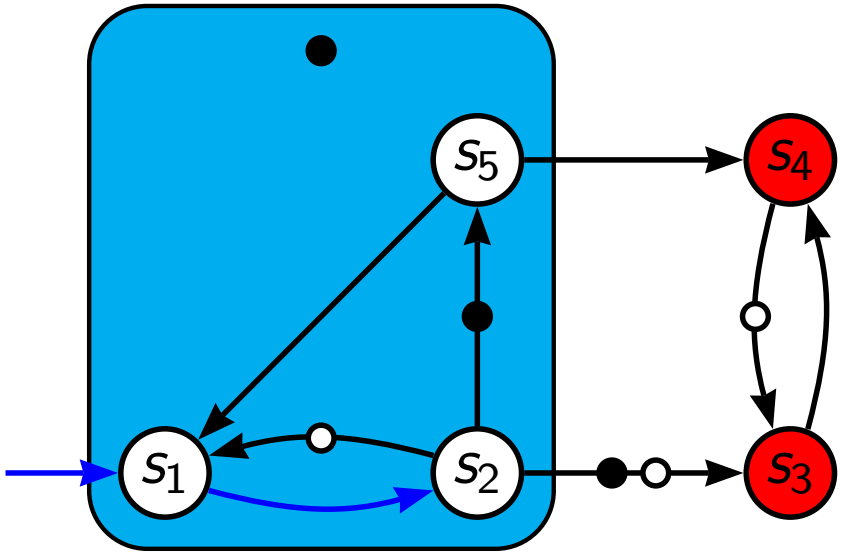

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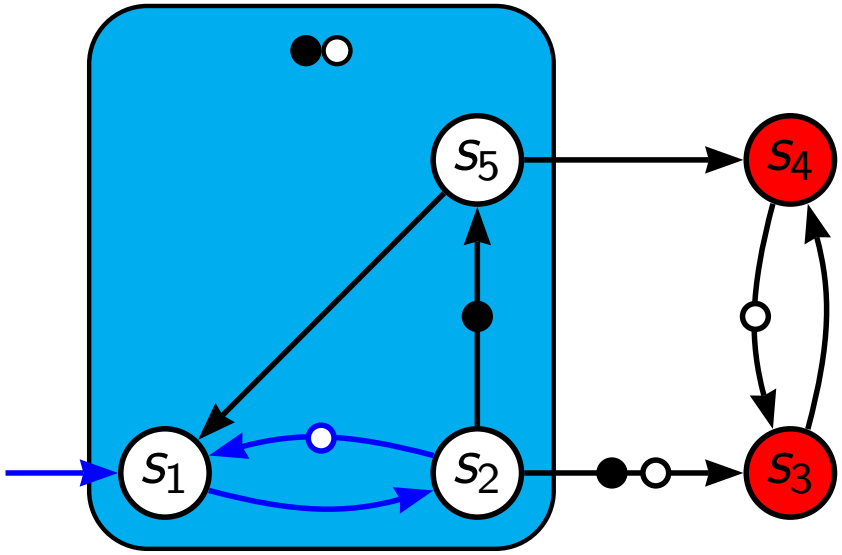

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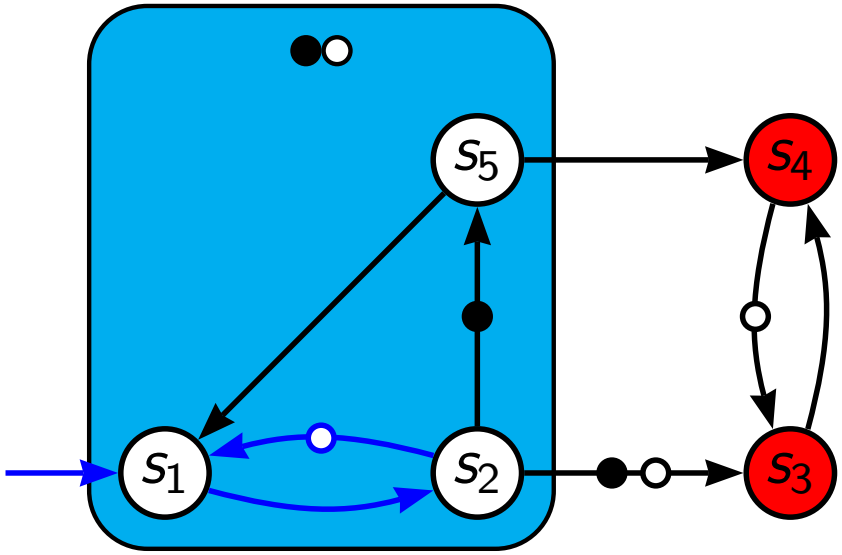

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entries in hash table	hash table size in bits	search stack depth	states traversed
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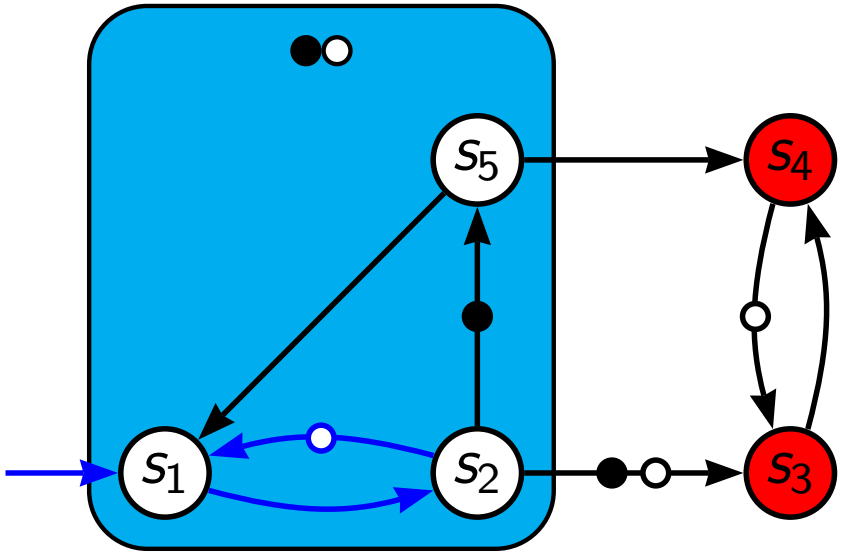

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Can be reduced to n if the search stack explicitly stores the states of each component (require more memory).

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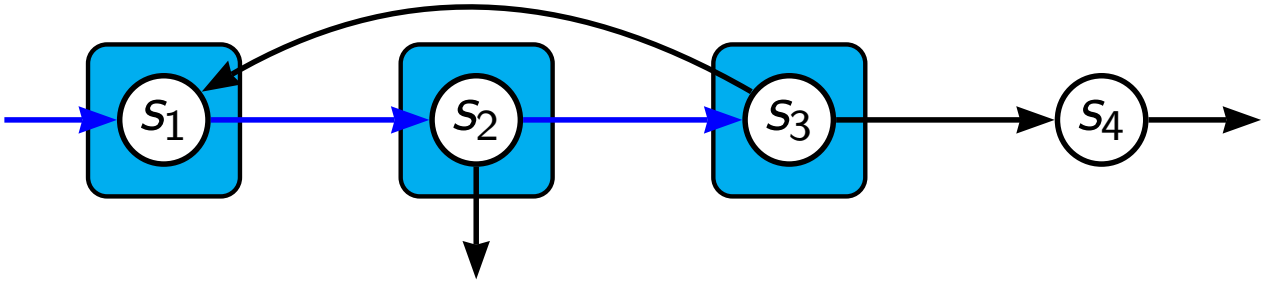
Two Heuristics for SCCs

- H1: visit transitions that go to visited states first.



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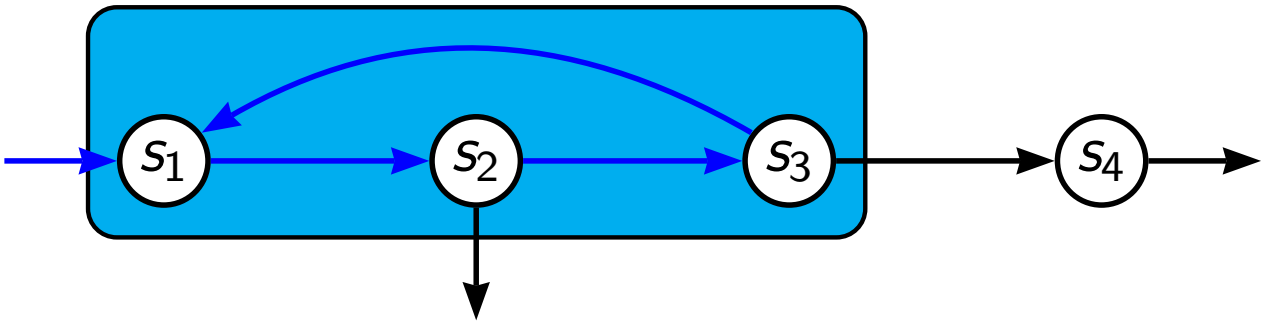

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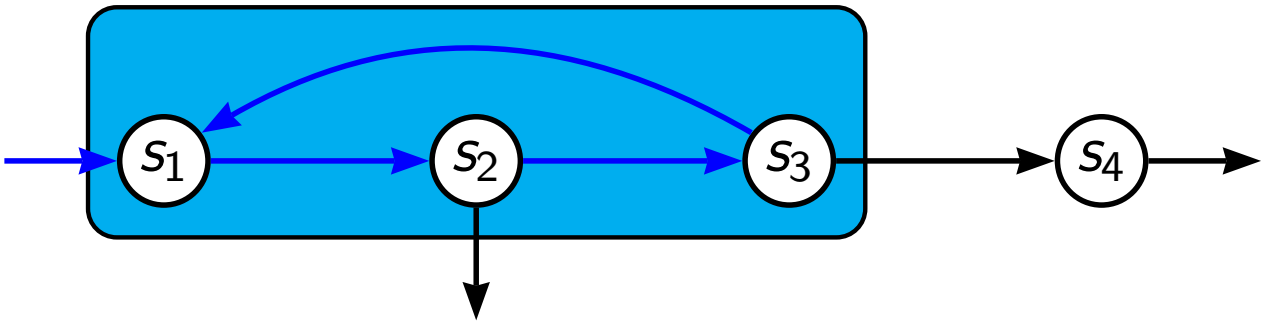

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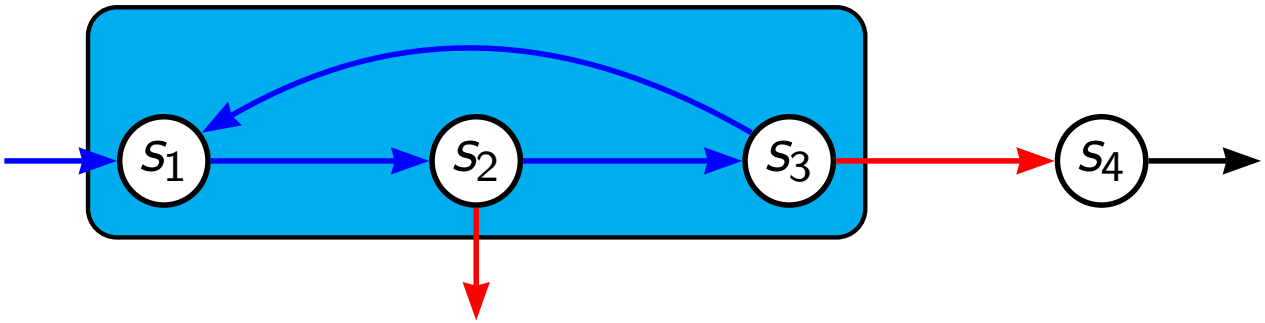

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Benchmarks

- Upper bounds easy to have.
- Objective : evaluate all these algorithms on the average, on non-empty automata.

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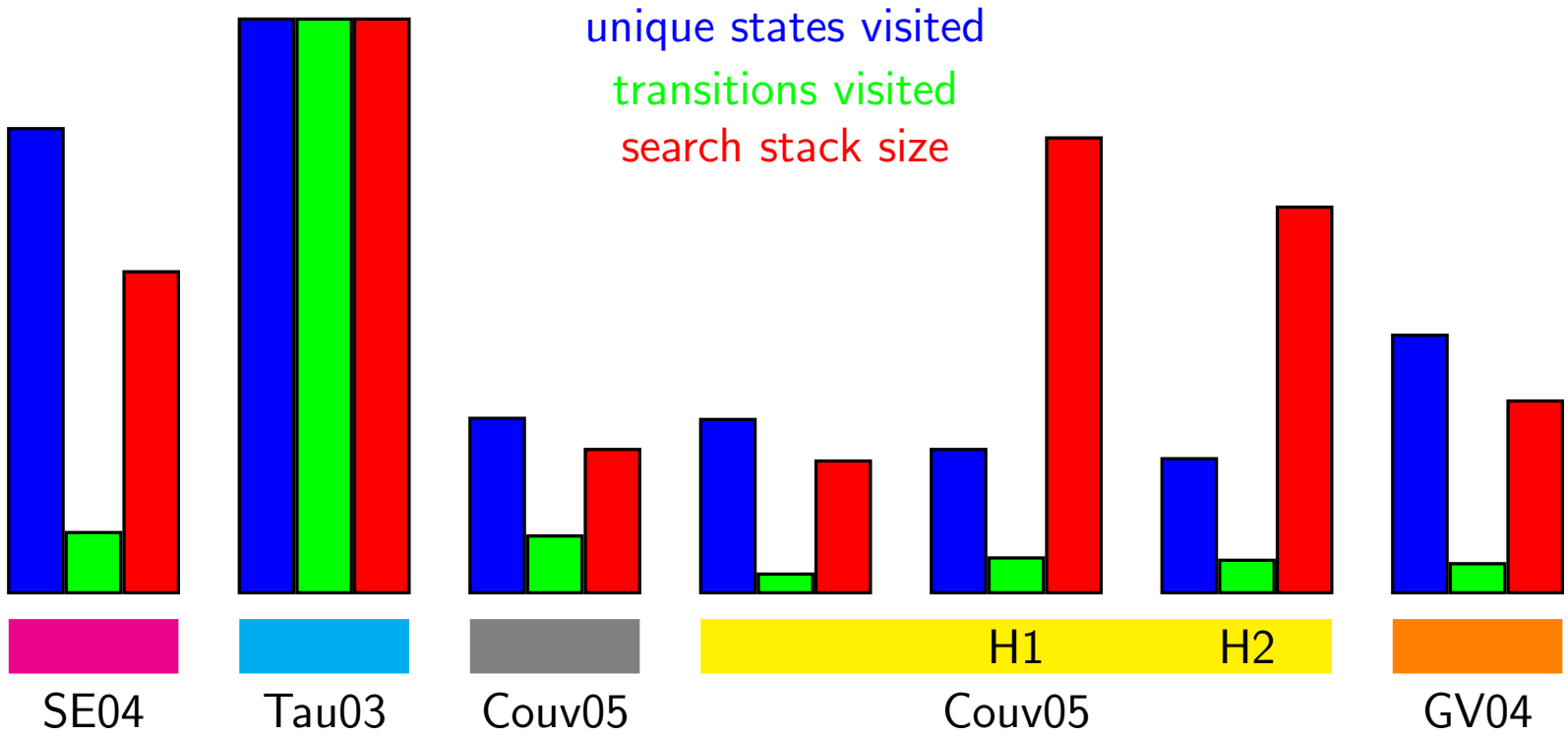

Conclusions

- generalized vs. non-generalized:
  - generalized algorithms require less memory
  - generalized algorithms produce more meaningful counterexamples
  - weak fairness expressible using generalized conditions
  - non-generalized NDFSs produce counterexamples directly
- NDFS vs. SCC algorithms:
  - SCC algorithms check emptiness faster
  - SCC algorithms scale to generalized conditions and fairness conditions easily
  - NDFSs require less memory
- All these algorithms and the benchmark framework are implemented in our model-checking library:

  `http://spot.lip6.fr`