Translation of Semi-Extended Regular Expressions using Derivatives

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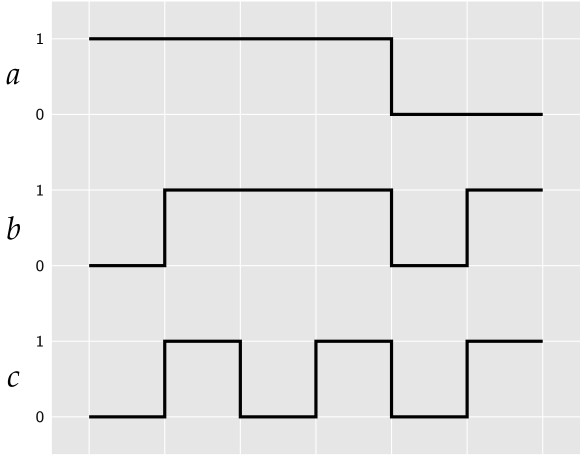
[1]LRE (EPITA), [2]SiPearl

CIAA — Akita — September 5, 2024

Context: Signals



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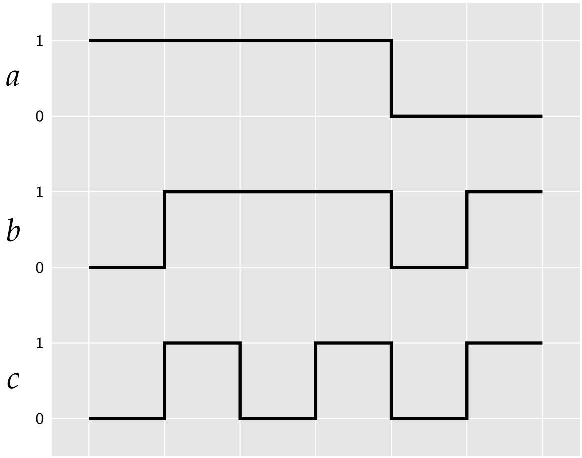
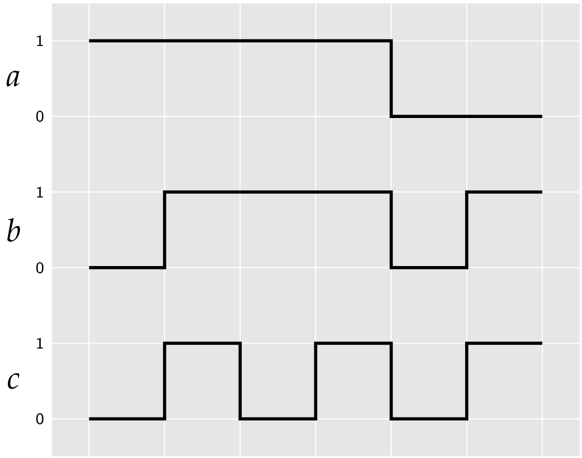


$$a\bar{b}\bar{c} \;;\; abc \;;\; ab\bar{c} \;;\; abc \;;\; \bar{a}\bar{b}\bar{c} \;;\; \bar{a}bc$$

Context: Signals



$a\bar{b}\bar{c}$; abc ; $ab\bar{c}$; abc ; $\bar{a}\bar{b}\bar{c}$; $\bar{a}bc$ } Word

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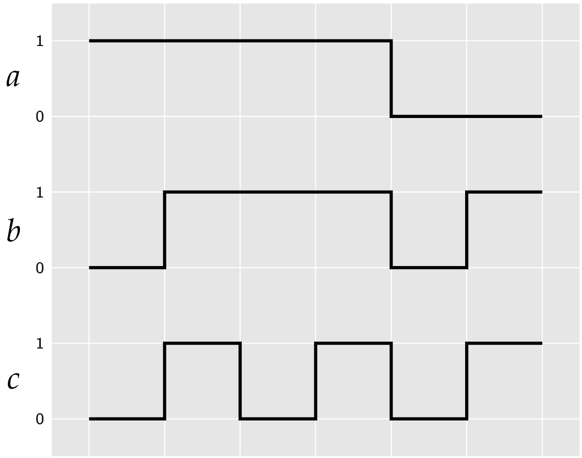

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Alphabet

$$\Sigma = 2^{\{\mathsf{a},\mathsf{b},\mathsf{c}\}}$$

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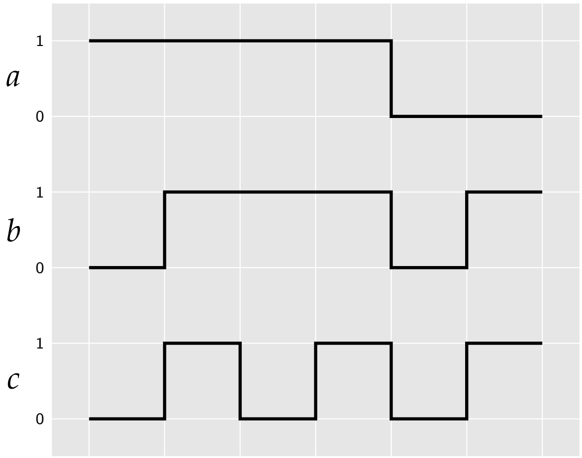

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Alphabet

$$\Sigma = 2^{\{\mathtt{a},\mathtt{b},\mathtt{c}\}} = 2^{AP}$$

Context: PSL[1] and SVA[2] translation to Büchi automata

$$\{(\mathtt{a} \wedge \mathtt{b})* : (\mathtt{c}\ ;\ !\,\mathtt{a})\}\,[\,]\!\!->\mathtt{G}\ \mathtt{c}$$

[1] *Property Specification Language Reference Manual v1.1*, 2004.

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Context: PSL[1] and SVA[2] translation to Büchi automata

{(a ∧ b)* : (c ; ! a)}[]->G c

Alphabet is 2^{AP}

Boolean formulas as abbreviations

$$a \wedge b = \{abc, ab\bar{c}\}$$

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Linear-time Temporal Logic

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Context: PSL[1] and SVA[2] translation to Büchi automata

$$(a \wedge b)* : (c \; ; \; ! \; a)$$

Semi-Extended Regular Expressions

- Regular operators: ";", "*", "|"

Alphabet is 2^{AP}

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Context: PSL[1] and SVA[2] translation to Büchi automata

$$(\texttt{a} \wedge \texttt{b})* : (\texttt{c} \; ; \; ! \, \texttt{a})$$

Semi-Extended Regular Expressions

- Regular operators: ";", "*", "|"
- Additional operators:
  - Intersection "&"
  - First Match "fm"
  - Fusion ":"

Alphabet is 2^{AP}

Boolean formulas as abbreviations

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SERE syntax: fusion operator

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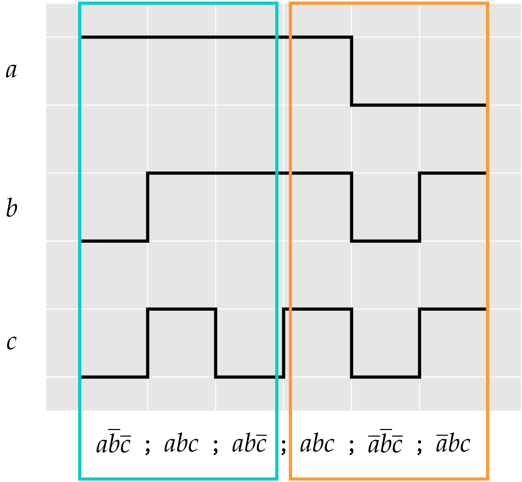
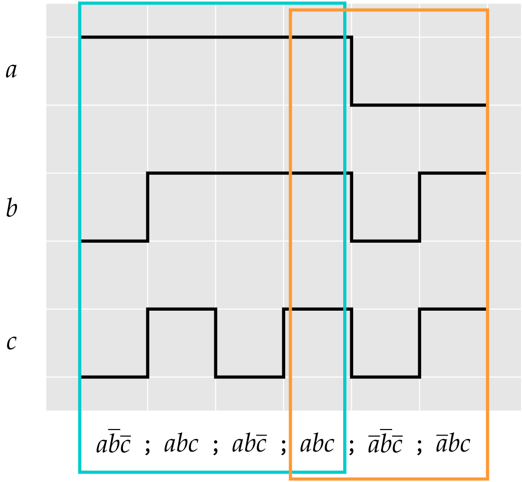
Concatenation: $E \,;\, F$

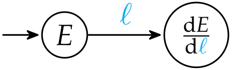


SERE syntax: fusion operator

Fusion: $E : F$



Regex to automaton translation

- Brzozowski, 1964 

$\ell \in \Sigma$

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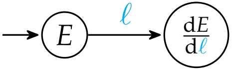
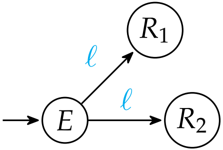


$$\frac{\mathrm{d}E}{\mathrm{d}\ell} = R_1 \vee R_2$$

- Antimirov, 1996



$$\frac{\partial E}{\partial \ell} = \{R_1, R_2\}$$

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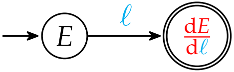
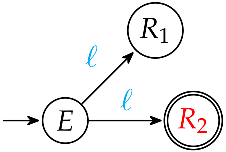
- Brzozowski, 1964 

$\frac{dE}{d\ell} = R_1 \vee R_2$

- Antimirov, 1996 

$\frac{\partial E}{\partial \ell} = \{R_1, R_2\}$, $\varepsilon \models R_2$

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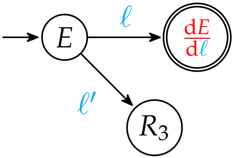
- Brzozowski, 1964 

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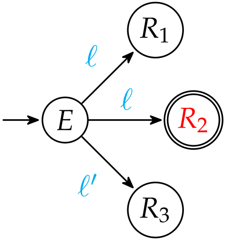
- Antimirov, 1996 

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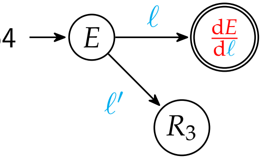

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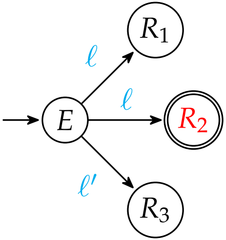

$$\frac{\partial E}{\partial \ell} = \{R_1, R_2\} \;,\; \varepsilon \models R_2$$

$\ell \in \Sigma$

$$\mathsf{LF}(E) = \{(\ell, R_1), (\ell, R_2), (\ell', R_3)\}$$

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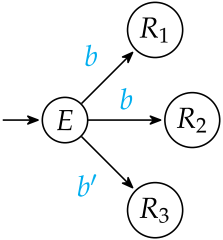

$$\mathsf{LF}(E) = \{(b, R_1), (b, R_2), (b', R_3)\}$$

$b \subseteq 2^{AP}$

Antimirov's Linear Forms

$$\begin{aligned}
\mathsf{LF}(\bot) &= \varnothing \\
\mathsf{LF}(\varepsilon) &= \varnothing \\
\mathsf{LF}(\ell) &= \{(\ell, \varepsilon)\} \\
\mathsf{LF}(r_1 \vee r_2) &= \mathsf{LF}(r_1) \cup \mathsf{LF}(r_2) \\
\mathsf{LF}(r^\star) &= \mathsf{LF}(r) \,;\, r^\star \\
\mathsf{LF}(r_1 \,;\, r_2) &= (\mathsf{LF}(r_1) \,;\, r_2) \cup (\lambda(r_1) \,;\, \mathsf{LF}(r_2))
\end{aligned}$$

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\mathsf{LF}(r_1 \mathbin{;} r_2) &= (\mathsf{LF}(r_1) \mathbin{;} r_2) \cup (\lambda(r_1) \mathbin{;} \mathsf{LF}(r_2)) \\
\mathsf{LF}(r_1 : r_2) &= (\mathsf{LF}(r_1) : r_2) \cup \left\{ (p_i \wedge p_j, s_j) \;\middle|\; \begin{array}{l} (p_i, s_i) \in \mathsf{LF}(r_1),\ \lambda(s_i) = \varepsilon, \\ (p_j, s_j) \in \mathsf{LF}(r_2) \end{array} \right\} \\
\mathsf{LF}(r_1 \wedge r_2) &= \{(p_i \wedge p_j, s_i \wedge s_j) \mid (p_i, s_i) \in \mathsf{LF}(r_1),\ (p_j, s_j) \in \mathsf{LF}(r_2)\} \\
\mathsf{LF}(\mathsf{fm}(r)) &= \{(p_i, \mathsf{fm}(s_i)) \mid (p_i, s_i) \in \mathrm{det}(\mathsf{LF}(r))\}
\end{aligned}
$$

Algorithm 1: Translation

input : A SERE ϕ
output: An automaton $\mathcal{A}$ such that $\mathscr{L}(\mathcal{A}) = \mathscr{L}(\phi)$

```
Q, δ, F ← {φ}, ∅, ∅;
todo.push(φ);
while todo ≠ ∅ do
    f ← todo.pop();
    foreach (p, s) ∈ LF (f) do
        if s ∉ Q then
            Q ← Q ∪ {s};
            todo.push (s);
            if ε ⊨ s then
                F ← F ∪ {s};
        δ ← δ ∪ {f -p-> s};
return ⟨Q, δ, φ, F⟩;
```

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    if $s \notin Q$ **then**
      $Q \leftarrow Q \cup \{s\}$;
      todo.push (s);
      if $\varepsilon \models s$ **then**
        $F \leftarrow F \cup \{s\}$;
    $\delta \leftarrow \delta \cup \{f \xrightarrow{p} s\}$;
return $\langle Q, \delta, \phi, F \rangle$;

Initial state is the input formula ϕ

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Initial state is the input formula ϕ

Loop over LF pairs

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Loop over LF pairs

States are labeled by SEREs

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Identify accepting states

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return $\langle Q, \delta, \phi, F \rangle$;

Initial state is the input formula ϕ

Loop over LF pairs

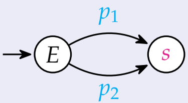
States are labeled by SEREs

Identify accepting states

Two simplifications on linear forms

Unique Suffix

$$\mathsf{LF}(E) = \{(p_1, s), (p_2, s)\}$$



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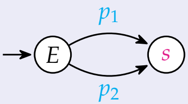
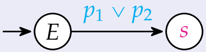


$$\mathsf{US}(\mathsf{LF}(E)) = \{(p_1 \vee p_2, s)\}$$



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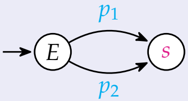

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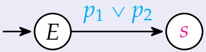

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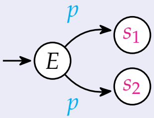

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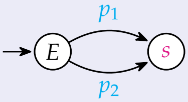

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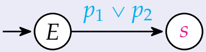

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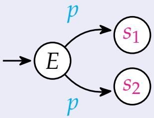
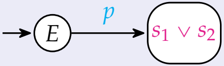


$$\mathsf{UP}(\mathsf{LF}(E)) = \{(p, s_1 \vee s_2)\}$$



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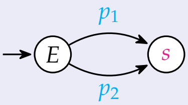

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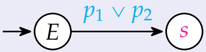

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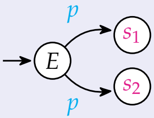

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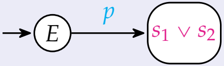


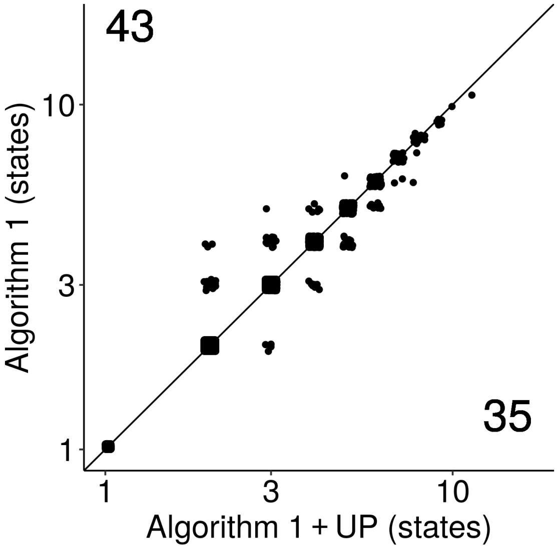
Note that this is not equivalent to a determinization in our case

Benchmarking UP effects

Benchmark dataset

12500 SEREs, randomly generated with Spot [a]. We enforced a certain heterogeneity in the dataset.

[a] https://spot.lre.epita.fr



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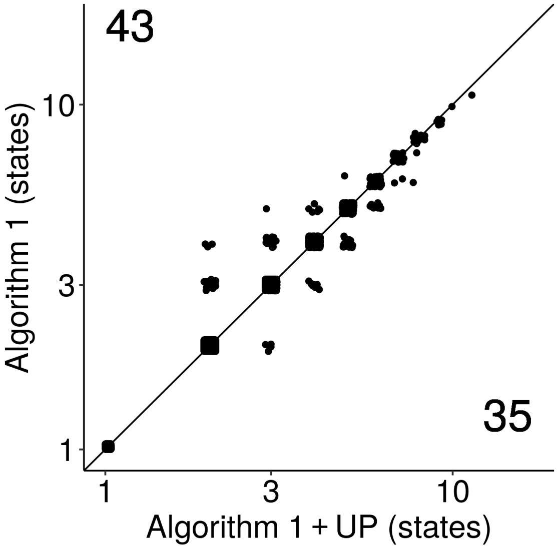
Some jitter was added to the plot



An interesting Linear Form property

Two expressions can have the same linear form:

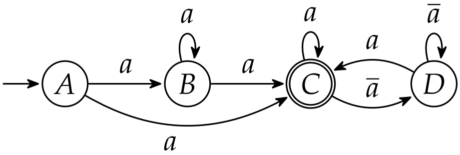
Example

$$\mathsf{LF}(\mathsf{a}*) = \mathsf{LF}(\mathsf{a} \ ; \ \mathsf{a}*) = \{(\mathsf{a}, \mathsf{a}*)\}$$

Property (Antimirov, 1996)

$$\mathsf{LF}(E) = \mathsf{LF}(F) \implies \mathscr{L}(E) \backslash \{\varepsilon\} = \mathscr{L}(F) \backslash \{\varepsilon\}$$

Improving the translation

Algo 1: 

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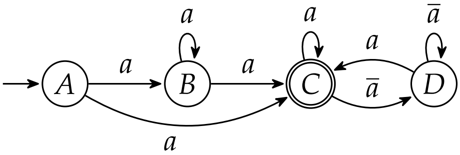

Notice

$$\mathsf{LF}(A) = \mathsf{LF}(B)$$

$$\mathsf{LF}(C) = \mathsf{LF}(D)$$

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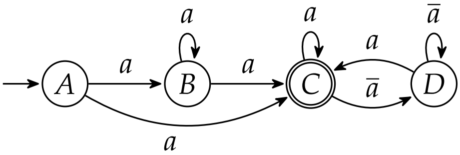
Algo 1: 

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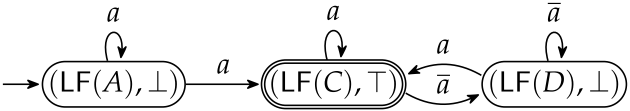
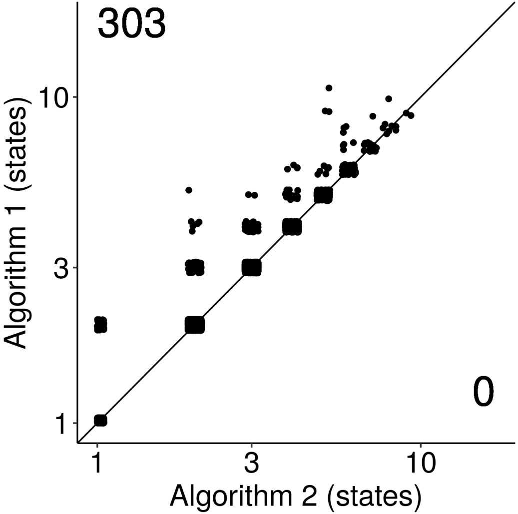
Algo 2: labeling with linear forms 

Algorithm 2 is an improvement



Improving the translation

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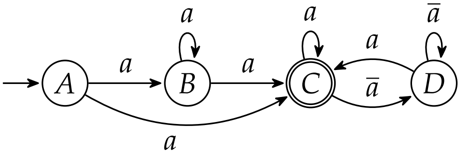

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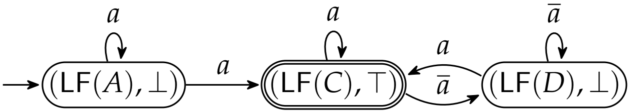

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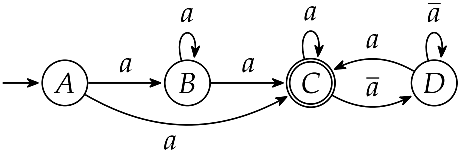
Algo 1: 

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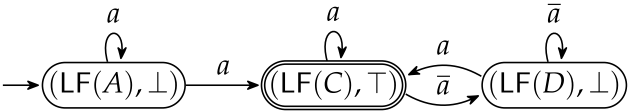
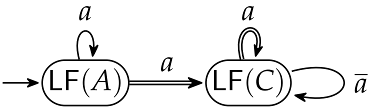
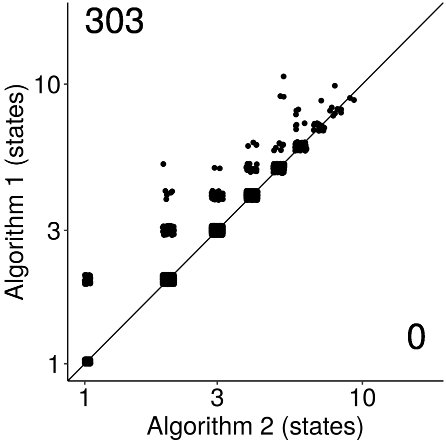
Algo 2: labeling with linear forms 

Algo 3: using transition-based acceptance 

Algorithm 3 is better



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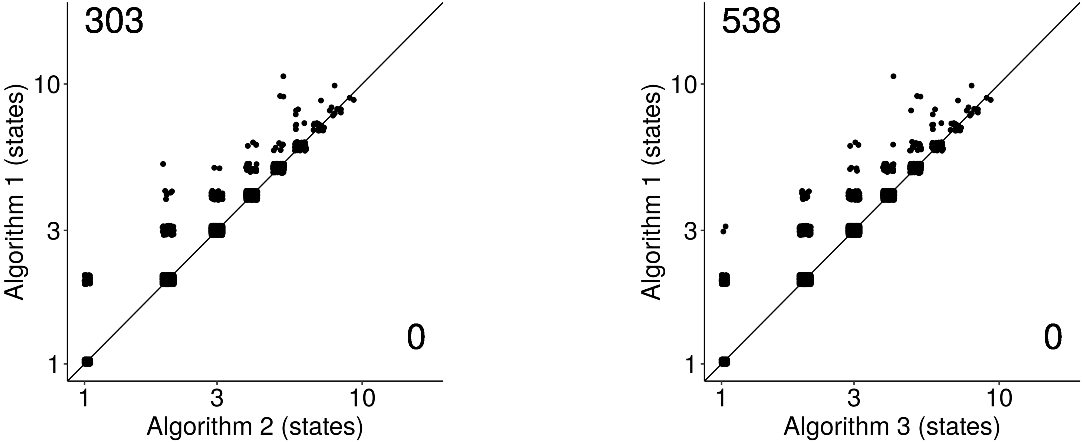
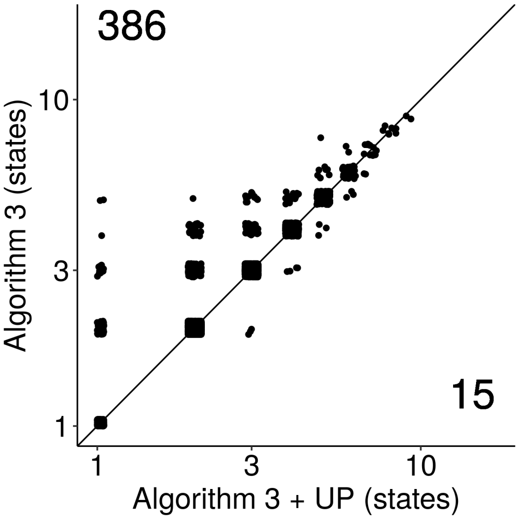


Effect of UP on transition-based acceptance



Wrapping up

Contributions

- Extended Antimirov's linear forms
  - Support alphabet $\Sigma = 2^{AP}$
  - Support SERE operators
- Evaluation of UP / US simplifications' performance
- Labeling of automaton states with linear forms
- Translation using transition-based acceptance

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Future work

Build on this work to translate PSL / SVA to Büchi automata, and evaluate performance of translation in a model checking context.