# Theory of Computation

(CS340)

Closed book exam. Duration: 1 hour

13 September 2010

## Exercise 1

1. Construct a non-deterministic finite automaton $A$ whose language over $\Sigma = \{a, b\}$ satisfies the following two constraints:
   - All words of $\mathscr{L}(A)$ have a length that is divisible by 3.
   - All words of $\mathscr{L}(A)$ start with the letter $a$ and end with the letter $b$.

   **Please justify your construction.**
2. Give a regular grammar that produces $\mathscr{L}(A)$.

## Exercise 2

Consider the regular expression $(a^\star b)^\star a$ defined over $\Sigma = \{a, b\}$.

1. Construct a DFA that recognizes $\mathscr{L}((a^\star b)^\star a)$
2. Explain why there cannot exist a 2-state DFA that recognizes the same language.
3. Construct a regular expression for the **complement** language. (Please show the steps of Brzozowski and McCluskey's algorithm after each elimination of a state.)

## Exercise 3

1. Prove that the language $L_1 = \{a^n b^m c^n \mid n \in \mathbb{N}, m \in \mathbb{N}\}$ is not regular.
2. Is is possible to find two **regular** languages $L_2$ and $L_3$ such that $L_2 \subseteq L_1 \subseteq L_3$?

## Exercise 4

Give context-free grammars for the following three languages defined over $\Sigma = \{a, b\}$.

1. $L_4 = \{a^{3i} b^i \mid i \in \mathbb{N}\}$,
2. $L_5 = \{a^m b^n \mid \forall m, n \text{ such that } m \geq n \geq 0\}$,
3. $L_6 = \{u \in \Sigma^\star \mid \text{the number of } a \text{ in } u \text{ is equal to the number of } b \text{ in } u\}$.