Fairness

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Fairness

- Hypotheses on the system to verify.
- You may consider fairness hypothesis anytime you have to deal with a repeated choice.
  - Messages sent over a lossy channel will be delivered after a finite number of retries (the repeated choice is whether the channel will lose the message)
  - Starvation-free resource allocator: anybody requesting the resource eventually gets it.
  - Two independent processes running on the same host get a "run slice" infinitely often (the repeated choice is done by the scheduler).
  - Finite wait: a process blocked until it is granted a resource will eventually get it.

Is there any difference between the last three examples?

Fairness hypotheses & the Emptiness Check

These hypotheses can be seen as constraints for the emptiness check.

If two independent processes running on the same host should get a "run slice" infinitely often:

- We want a counterexample where both processes are progressing infinitely often.
- We can ignore runs where one process is stuck.

Do we really need to modify the emptiness check algorithm?

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$$\mathcal{A}_M \otimes \mathcal{A}_{\mathit{fairness} \wedge \neg \mathit{prop}}$$
$$\mathcal{A}_M \otimes \mathcal{A}_{\mathit{fairness}} \otimes \mathcal{A}_{\neg \mathit{prop}}$$

Added complexity depends on A_{fairness}.

Kinds of Fairness Hypotheses

unconditional fairness Something will happen infinitely often.

weak fairness If something happens continuously, something else will happen infinitely often.

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We have

$$\begin{aligned}\mathbf{F}\,\mathbf{G}\,e \rightarrow \mathbf{G}\,\mathbf{F}\,e' &= \neg\,\mathbf{F}\,\mathbf{G}\,e \vee \mathbf{G}\,\mathbf{F}\,e' \\ &= \mathbf{G}\,\mathbf{F}\,\neg e \vee \mathbf{G}\,\mathbf{F}\,e' \\ &= \mathbf{G}\,\mathbf{F}(\neg e \vee e')\end{aligned}$$

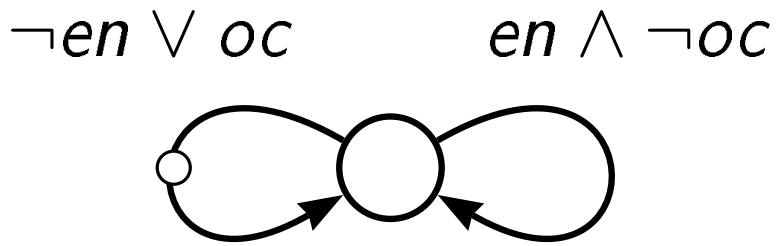
therefore *weak fairness* can be expressed as *unconditional fairness*.

Size of $A_{fairness}$ (Generalized Büchi)

We may want to apply several fairness hypotheses.

	weak fairness	strong fairness
n	$\bigwedge_{i=1}^{n} \mathbf{G}\,\mathbf{F}(\neg en_i \vee oc_i)$	$\bigwedge_{i=1}^{n} \mathbf{G}\,\mathbf{F}\,en_i \rightarrow \mathbf{G}\,\mathbf{F}\,oc_i$
1	1 state	4 states
2	1 state	10 states
3	1 state	28 states
4	1 state	82 states
⋮	⋮	⋮
n	1 state	$3^n + 1$ states
	deterministic	non-deterministic

Look of $A_{\mathbf{G}\,\mathbf{F}(\neg en \vee oc)}$



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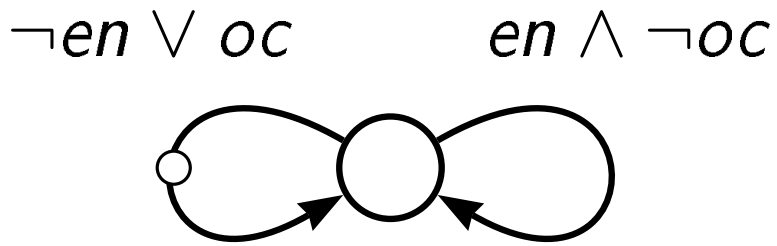
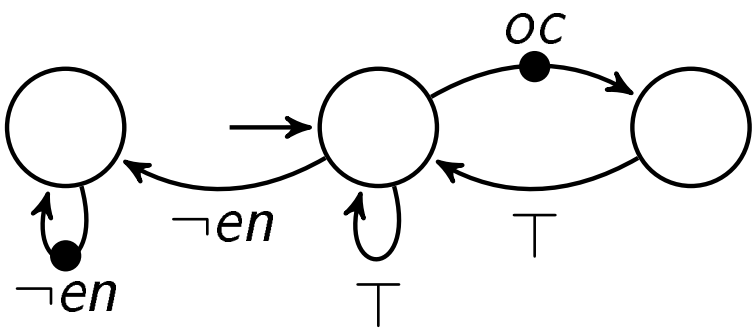


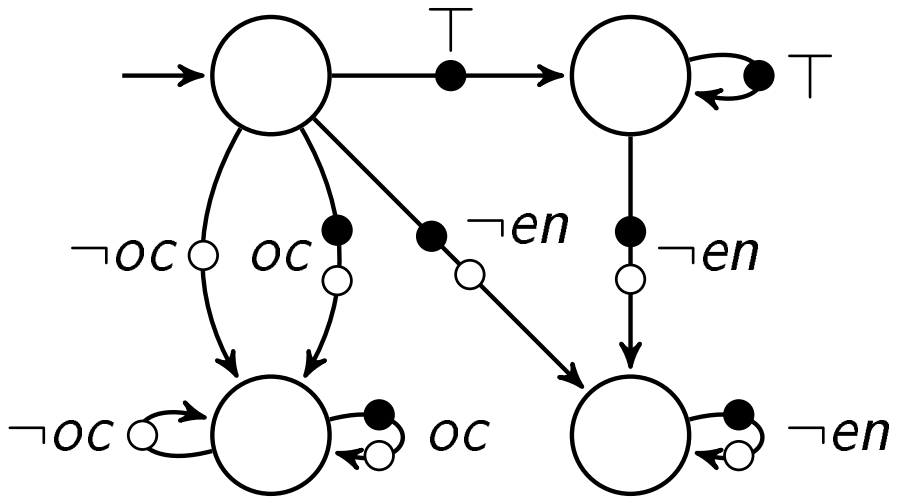
Büchi acceptance conditions correspond to formulæ such as $\mathbf{G}\,\mathbf{F}\,a$.

$$A_M \otimes A_{fairness} \otimes A_{\neg prop}$$

The system can be labeled with the appropriate acceptance conditions as the state space is explored.
Therefore weak fairness is *trivial* if you have a generalized emptiness check.

Look of $A_{\mathbf{G}\,\mathbf{F}\,en \rightarrow \mathbf{G}\,\mathbf{F}\,oc}$





Note: I don't know of any LTL translator who is able to create the left automaton!

With n fairness hypotheses, the left automaton reaches 3^n states and the right automaton reaches $3^n + 1$ states.

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- Weak fairness: $\bigwedge_{i=1}^{n} \mathbf{G}\,\mathbf{F}(\neg en_i \vee oc_i)$

- Strong fairness: $\bigwedge_{i=1}^{n} \mathbf{G}\,\mathbf{F}\,en_i \rightarrow \mathbf{G}\,\mathbf{F}\,oc_i$

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  - Streett automata always 1 state, deterministic.

Streett Automata

- Differ from Büchi automata only in acceptance conditions.
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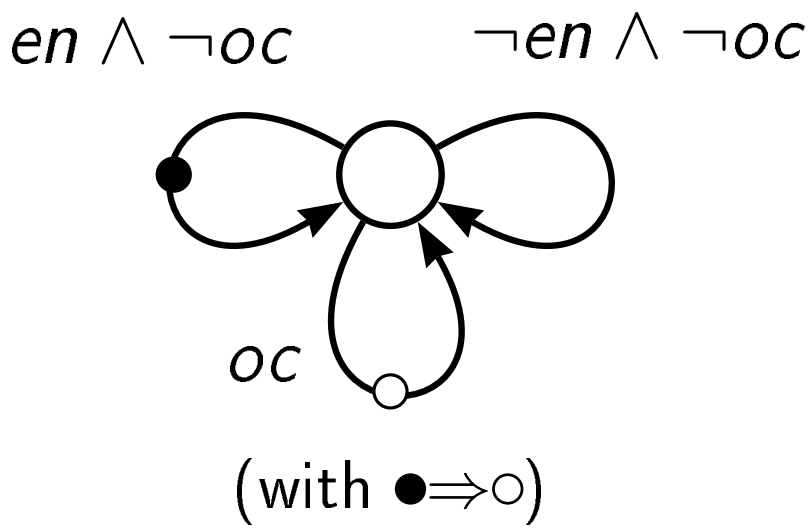
Streett Automata

- Differ from Büchi automata only in acceptance conditions.
- Acceptance conditions look like "if a run sees ● infinitely often, then it will see ○ infinitely often" (can be generalized to more pairs of colors)
- Exactly what is needed to recognize $\mathbf{G}\,\mathbf{F}\,en \rightarrow \mathbf{G}\,\mathbf{F}\,oc$:



Emptiness Check for Streett Automata

- For Büchi, we look for SCCs whose acceptance conditions verify $\bullet \wedge \circ \wedge \bullet \wedge \bullet$.
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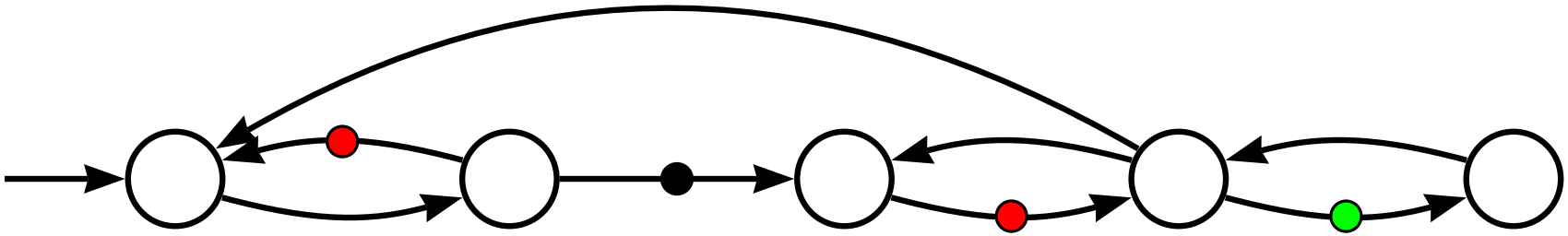

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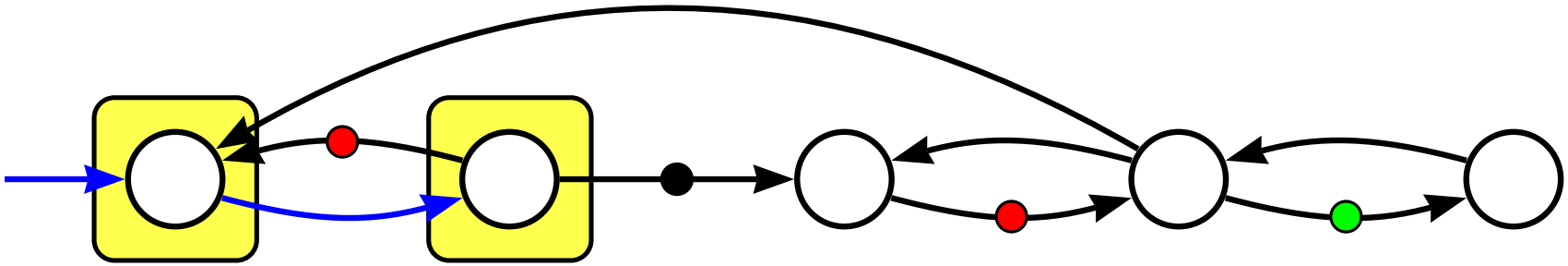

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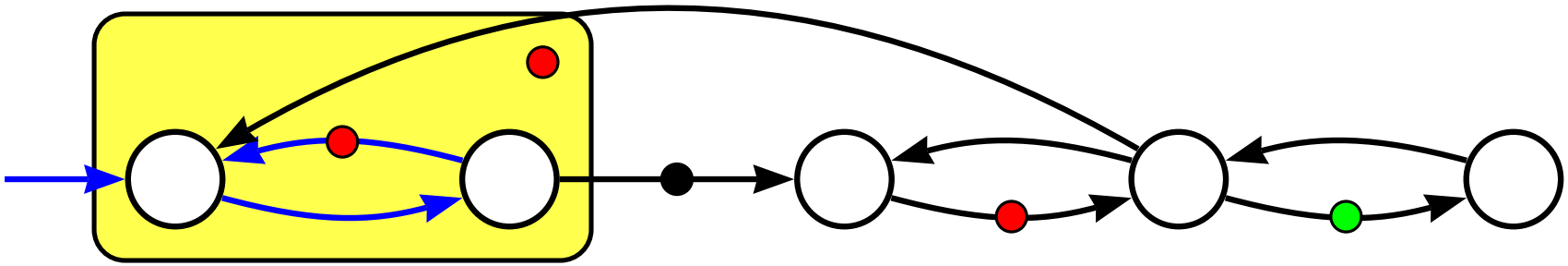

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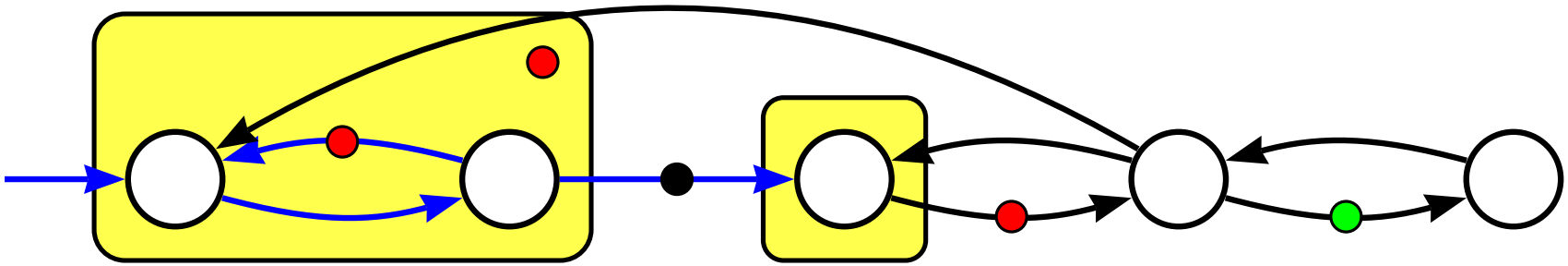

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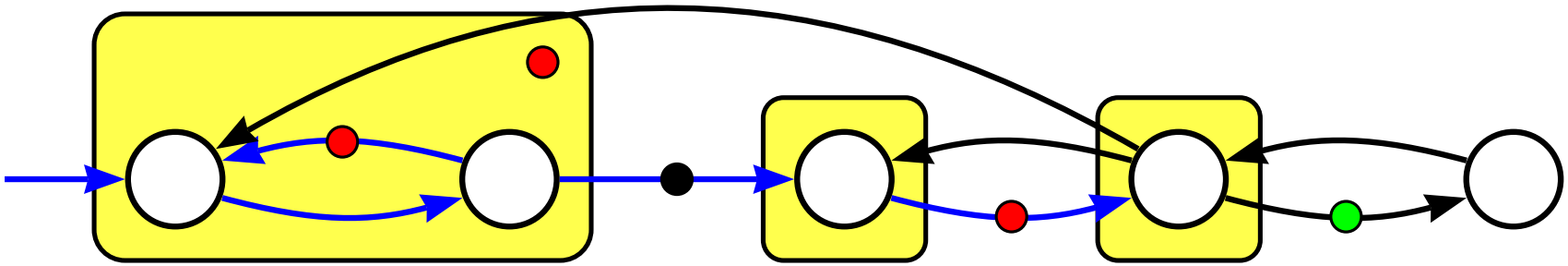

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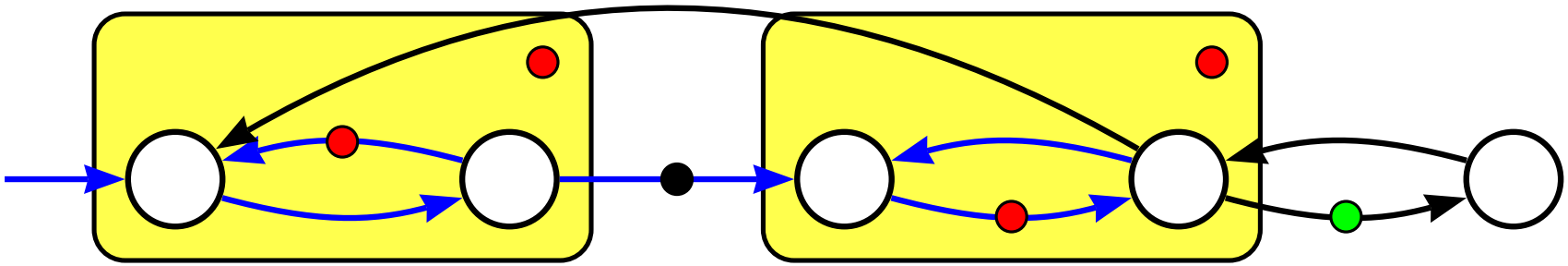

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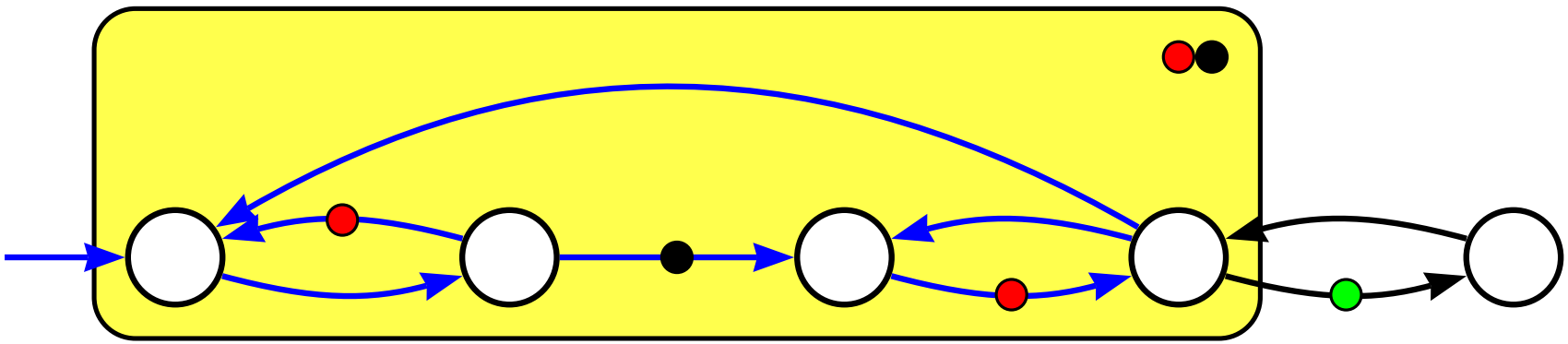

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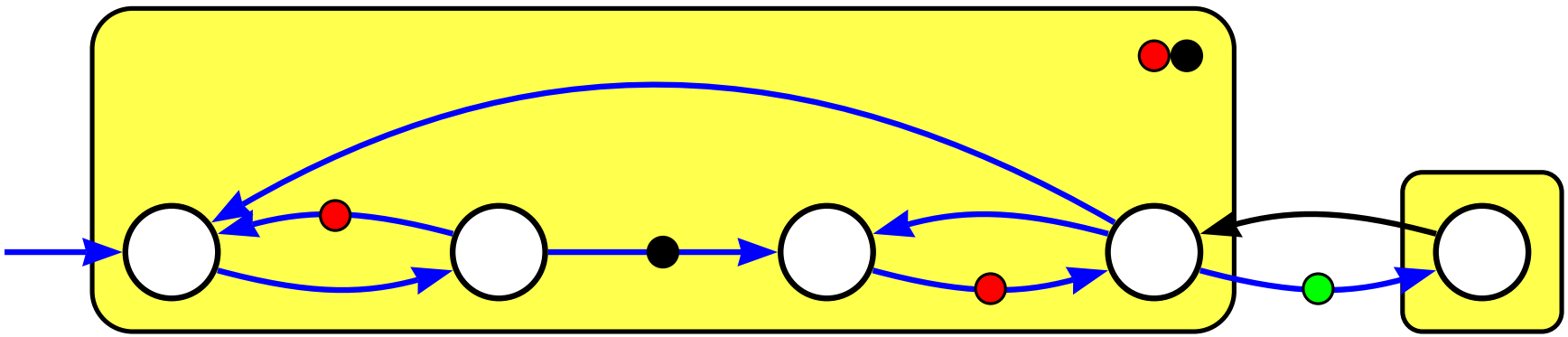

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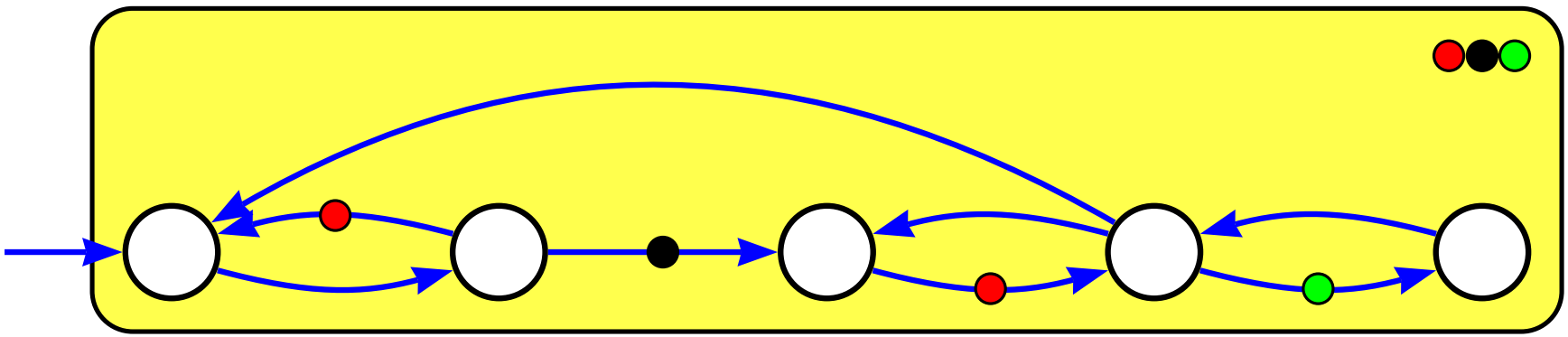

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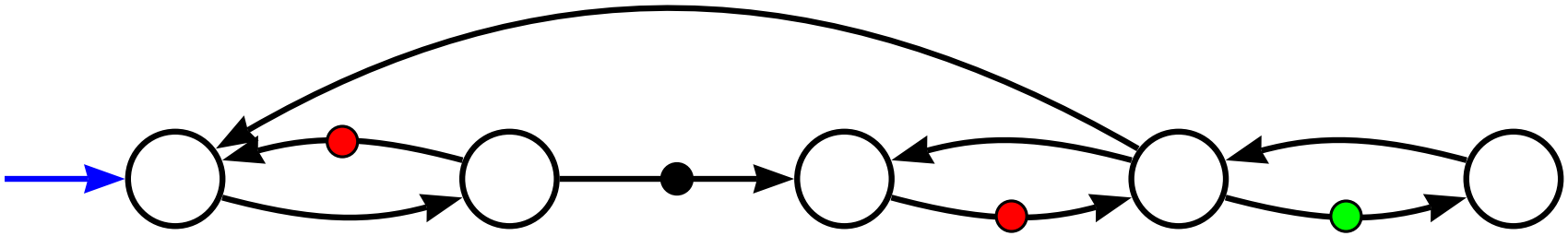

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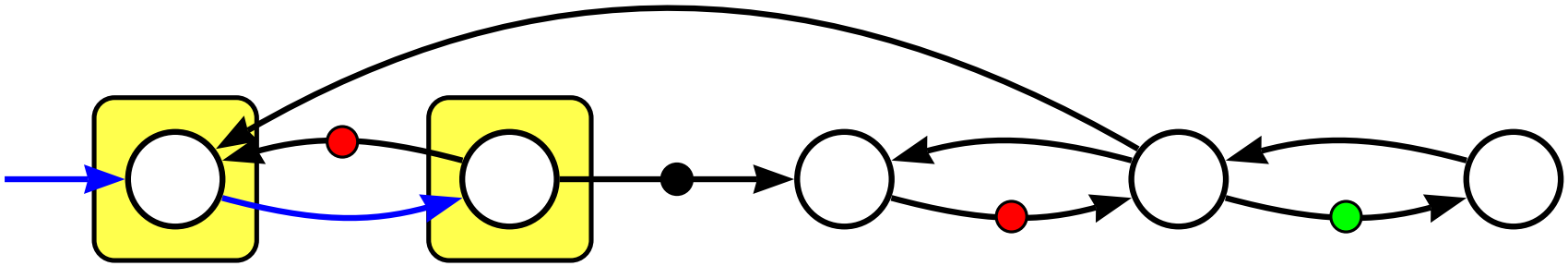

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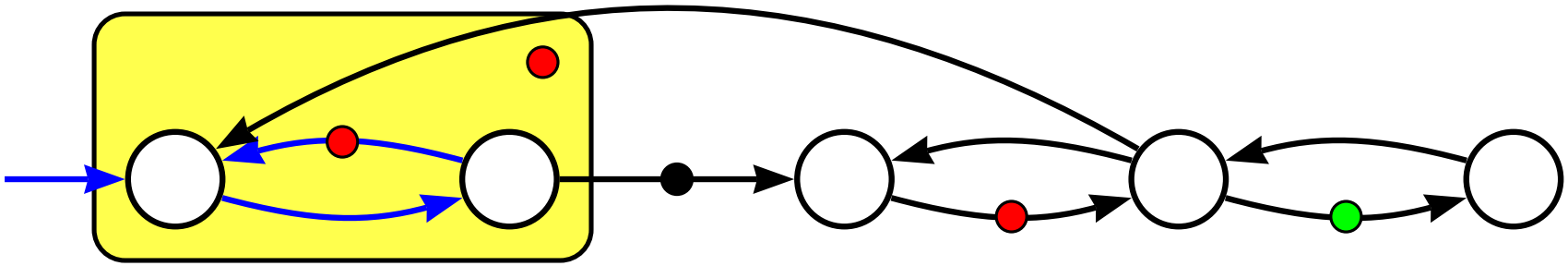

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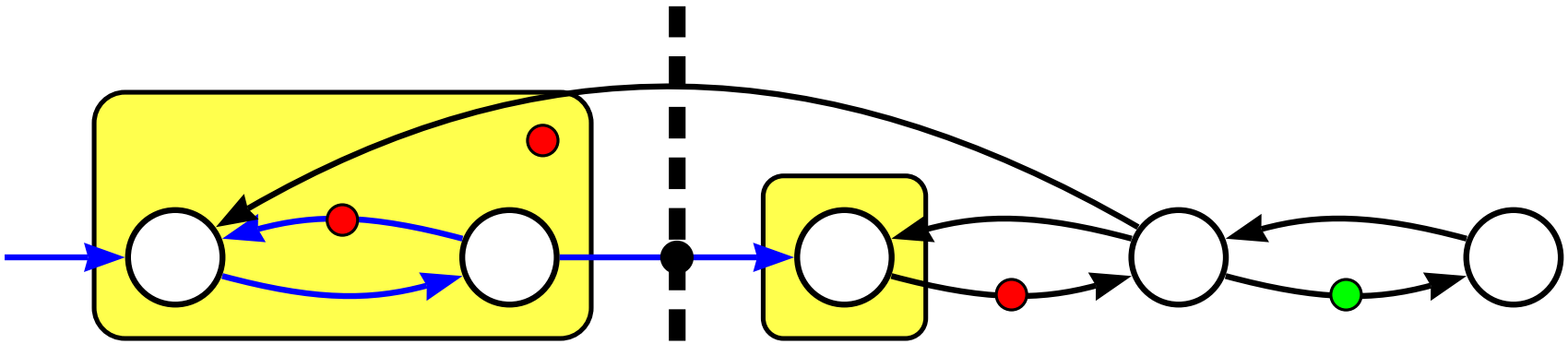

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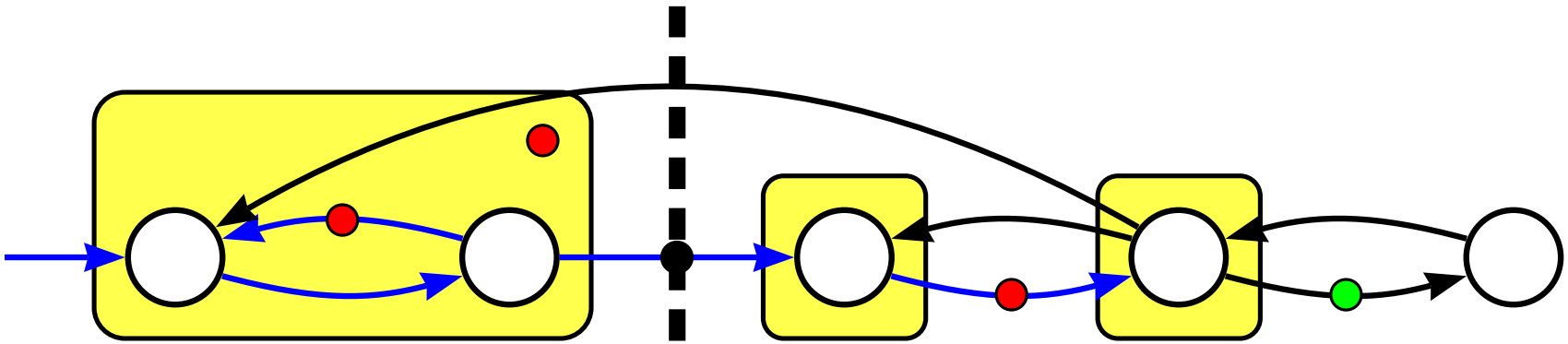

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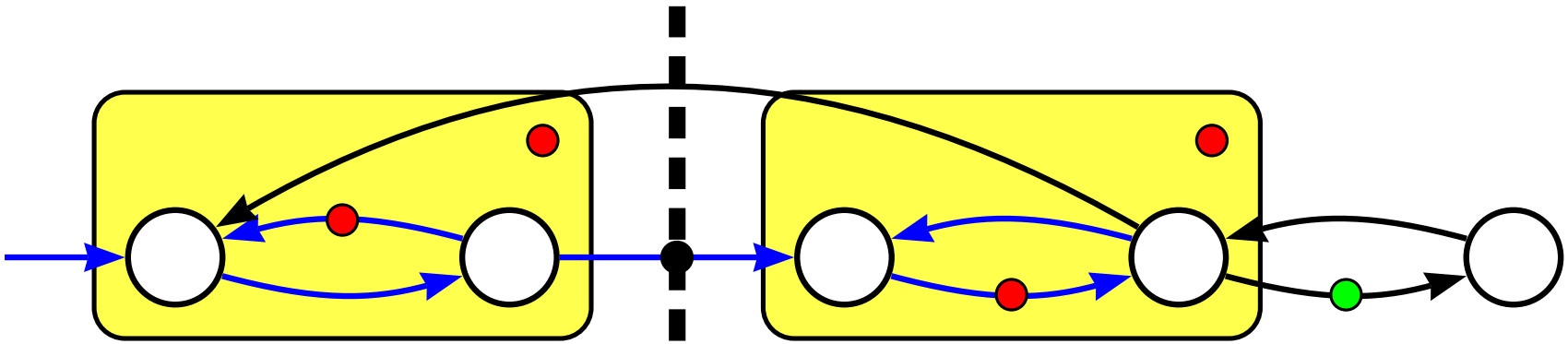

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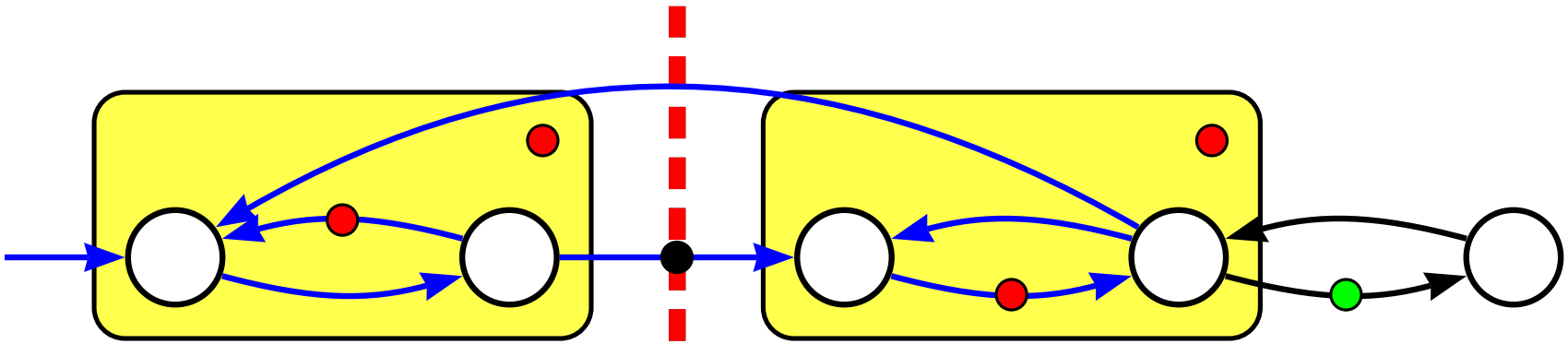

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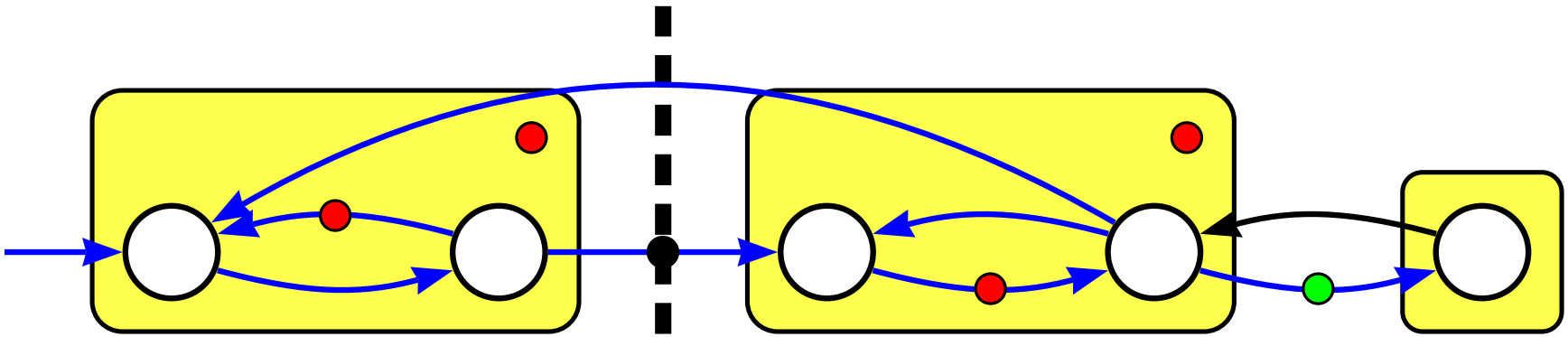

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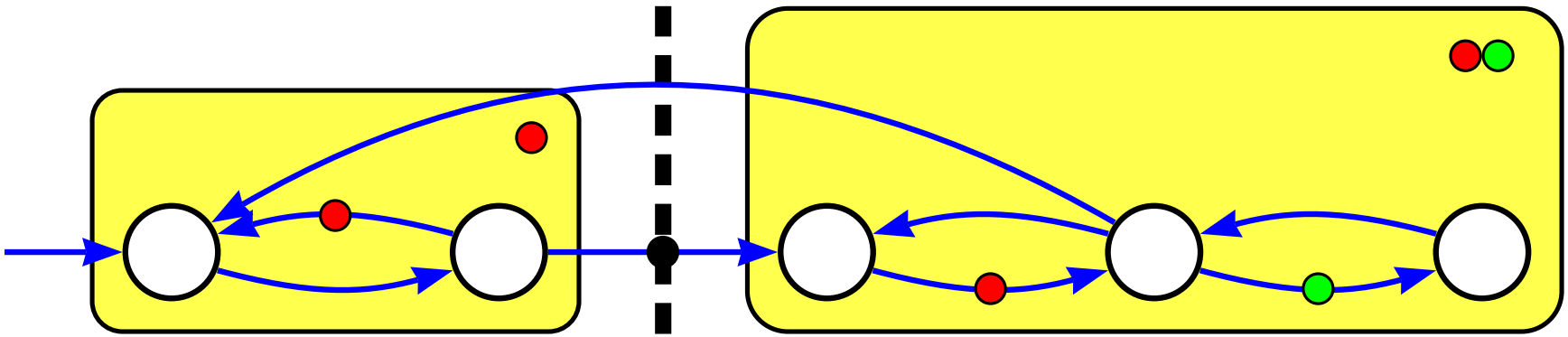

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Finally

- To be correct, has to be combined with a variant of heuristic H2 (ordering successors SCC-wise)
- "Barriers" must be crossed only after all normal successors have been visited.

Slowdown (w.r.t. generalized Büchi emptiness check):

- SCC are revisited at worst m times if m pairs of acceptance conditions.
- Compare with the $3^m + 1$ states of the Büchi automaton for the corresponding LTL formula...

This algorithm makes it possible to handle strong fairness with a linear slowdown instead of an exponential slowdown.

Büchi and Streett automata

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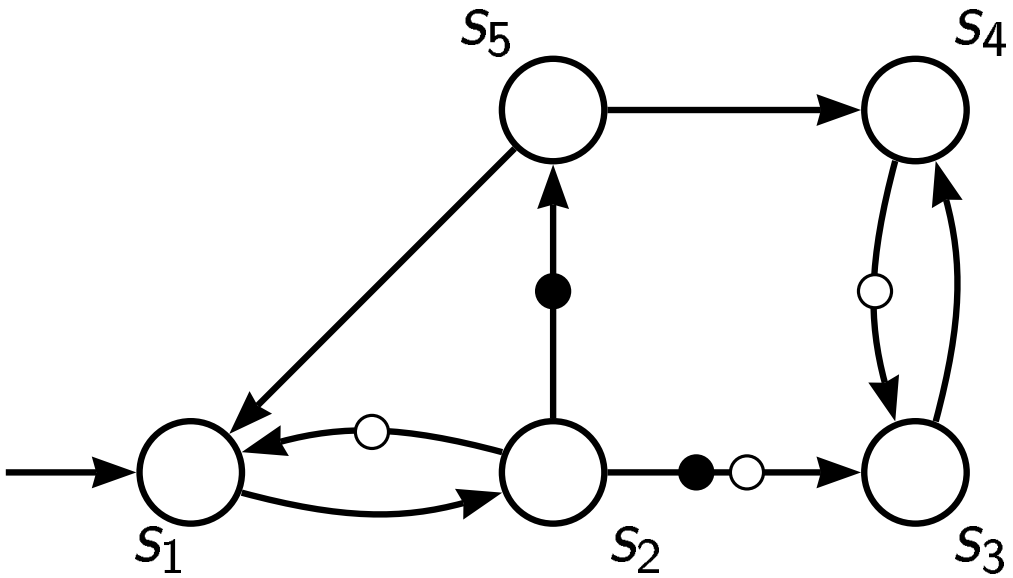

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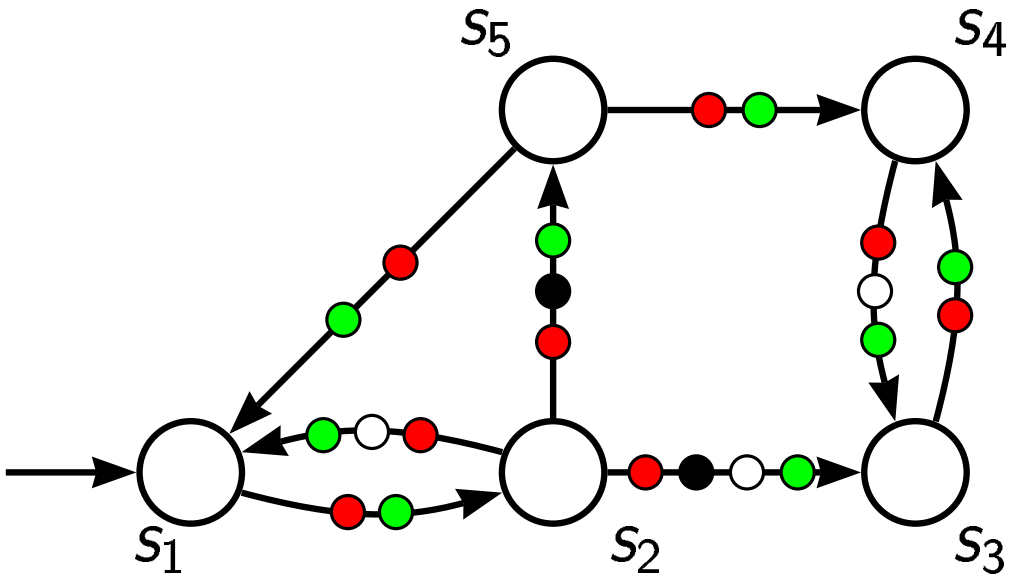

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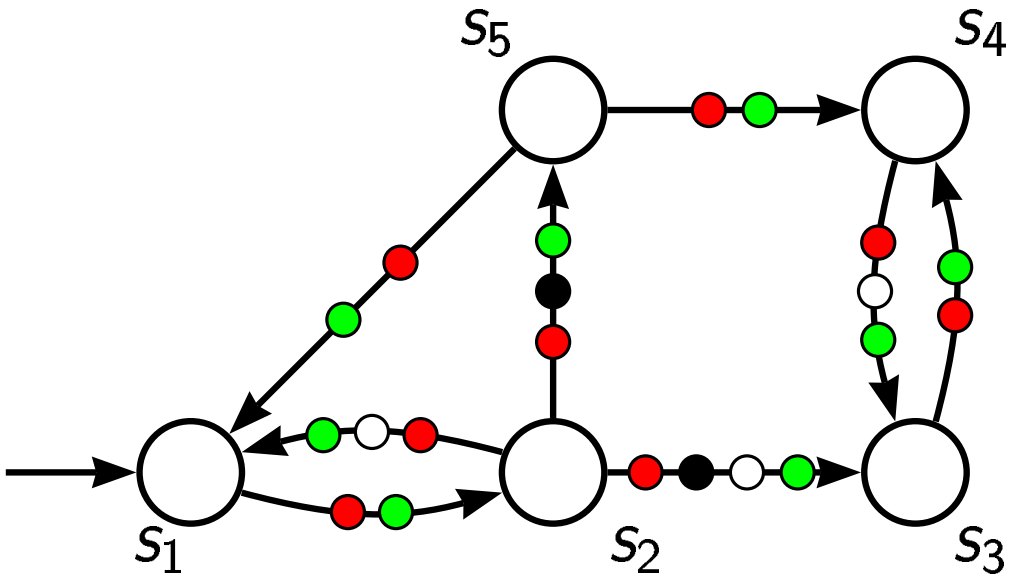

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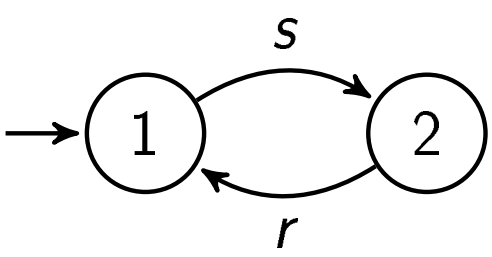
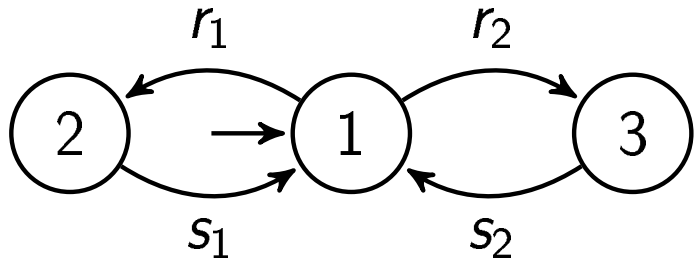
- Converting Streett automata into (generalized) Büchi automata? Exponential construction. No polynomial algorithm exist.

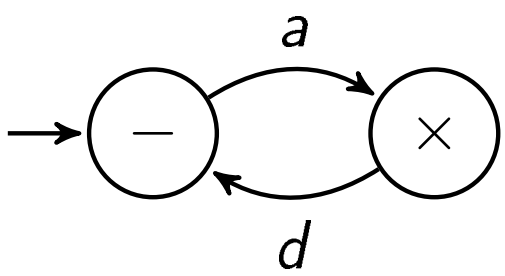
Back to the client/server example



Client C



Server S



Channel B

Synchronization rules for the system $\langle C, C, S, B, B, B, B \rangle$:

$$
\begin{array}{ll}
(1) & \langle\ s\ ,\ .\ ,\ .\ ,\ .\ ,\ .\ ,\ .\ ,\ a\ ,\ .\ \rangle \\
(2) & \langle\ .\ ,\ s\ ,\ .\ ,\ .\ ,\ .\ ,\ .\ ,\ .\ ,\ a\ \rangle \\
(3) & \langle\ r\ ,\ .\ ,\ .\ ,\ .\ ,\ d\ ,\ .\ ,\ .\ ,\ .\ \rangle \\
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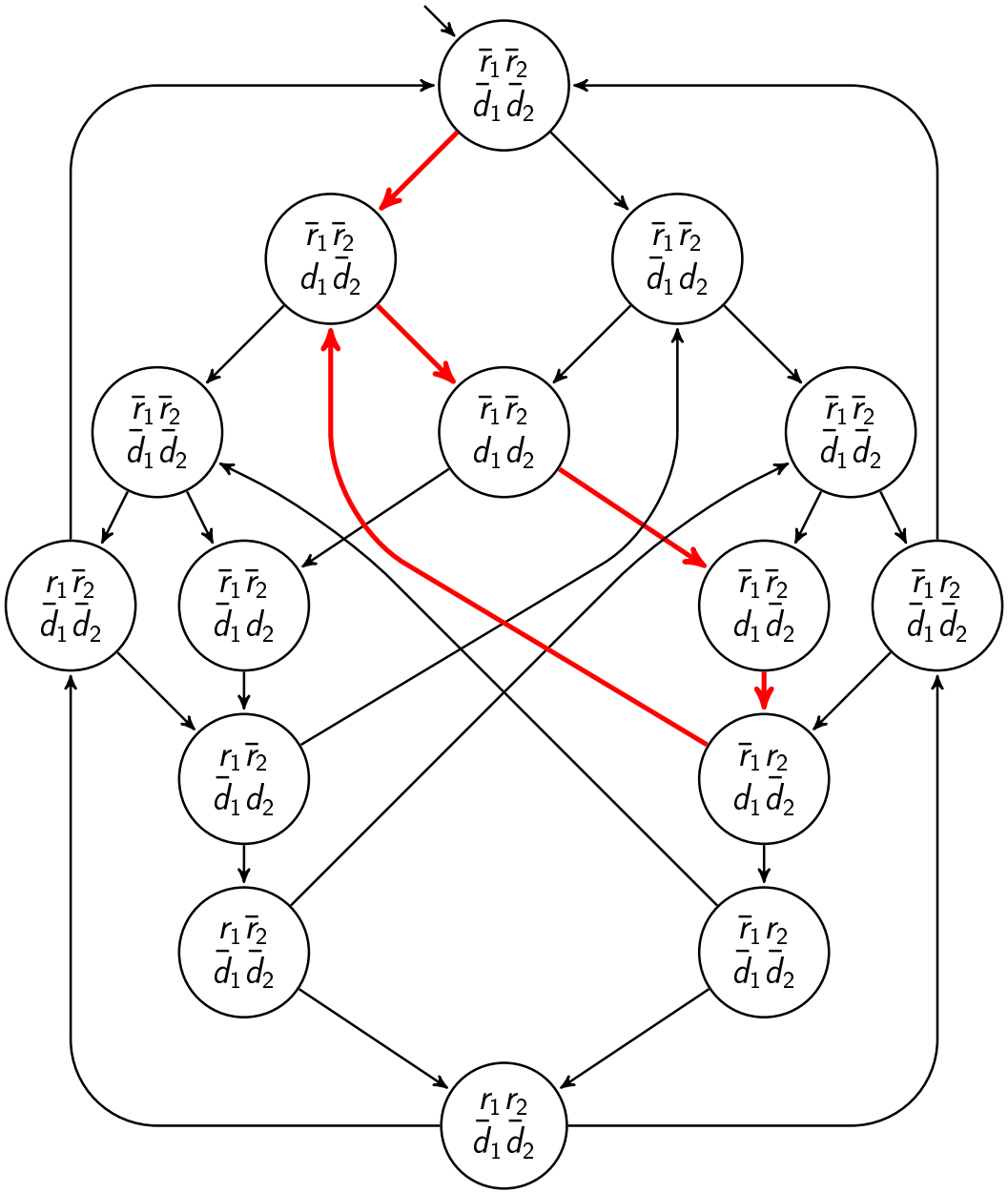
(5) & \langle\ .\ ,\ .\ ,\ .\ ,\ r_1\ ,\ .\ ,\ .\ ,\ .\ ,\ d\ ,\ .\ \rangle \\
(6) & \langle\ .\ ,\ .\ ,\ .\ ,\ s_1\ ,\ a\ ,\ .\ ,\ .\ ,\ .\ ,\ .\ \rangle \\
(7) & \langle\ .\ ,\ .\ ,\ .\ ,\ r_2\ ,\ .\ ,\ .\ ,\ .\ ,\ .\ ,\ d\ \rangle \\
(8) & \langle\ .\ ,\ .\ ,\ .\ ,\ s_2\ ,\ .\ ,\ a\ ,\ .\ ,\ .\ ,\ .\ \rangle
\end{array}
$$

If a client sends a request, will it eventually get an answer?

(unfair) Kripke structure



How to we make it fair?

$$
\begin{array}{ll}
(1) & \langle\, s, ., ., ., ., ., a, . \,\rangle \\
(2) & \langle\, ., s, ., ., ., ., ., a \,\rangle \\
(3) & \langle\, r, ., ., ., d, ., ., ., . \,\rangle \\
(4) & \langle\, ., r, ., ., ., d, ., ., . \,\rangle \\
(5) & \langle\, ., ., ., r_1, ., ., ., d, . \,\rangle \\
(6) & \langle\, ., ., ., s_1, a, ., ., ., . \,\rangle \\
(7) & \langle\, ., ., ., r_2, ., ., ., ., d \,\rangle \\
(8) & \langle\, ., ., ., s_2, ., ., a, ., ., . \,\rangle
\end{array}
$$

How to we make it fair?

$$
\begin{array}{ll}
(1) & \langle\, s\,,\,.\,,\,.\,,\,.\,,\,.\,,\,a\,,\,.\,\rangle \\
(2) & \langle\,.\,,\,s\,,\,.\,,\,.\,,\,.\,,\,.\,,\,a\,\rangle \\
(3) & \langle\, r\,,\,.\,,\,.\,,\,d\,,\,.\,,\,.\,,\,.\,\rangle \\
(4) & \langle\,.\,,\,r\,,\,.\,,\,.\,,\,d\,,\,.\,,\,.\,\rangle \\
(5) & \langle\,.\,,\,.\,,\,r_1\,,\,.\,,\,.\,,\,d\,,\,.\,\rangle \\
(6) & \langle\,.\,,\,.\,,\,s_1\,,\,a\,,\,.\,,\,.\,,\,.\,\rangle \\
(7) & \langle\,.\,,\,.\,,\,r_2\,,\,.\,,\,.\,,\,.\,,\,d\,\rangle \\
(8) & \langle\,.\,,\,.\,,\,s_2\,,\,.\,,\,a\,,\,.\,,\,.\,\rangle
\end{array}
$$

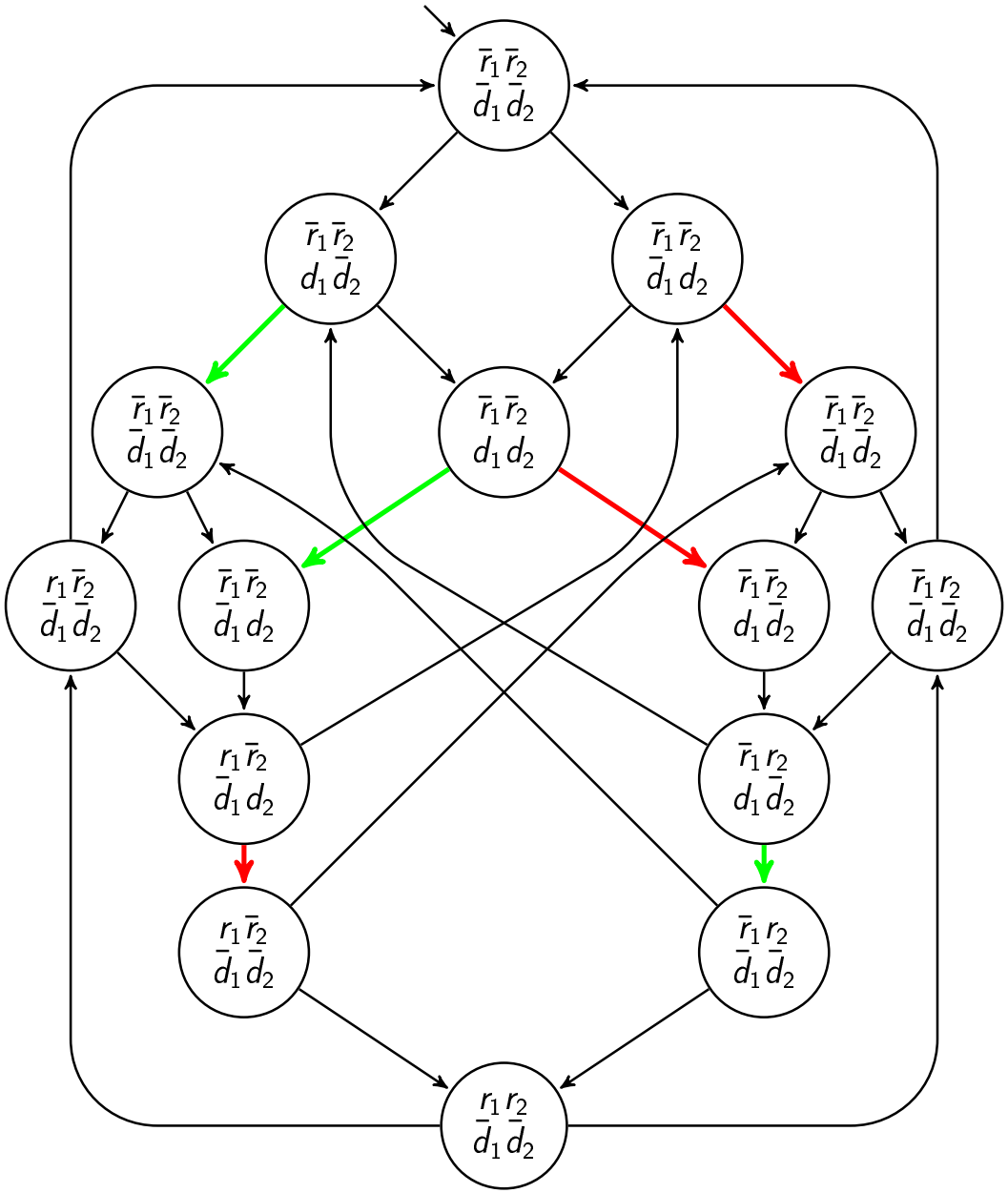
We want the choice between transition (5) and (6) to be fair.
Do we want strong or weak fairness?

How to we make it fair?

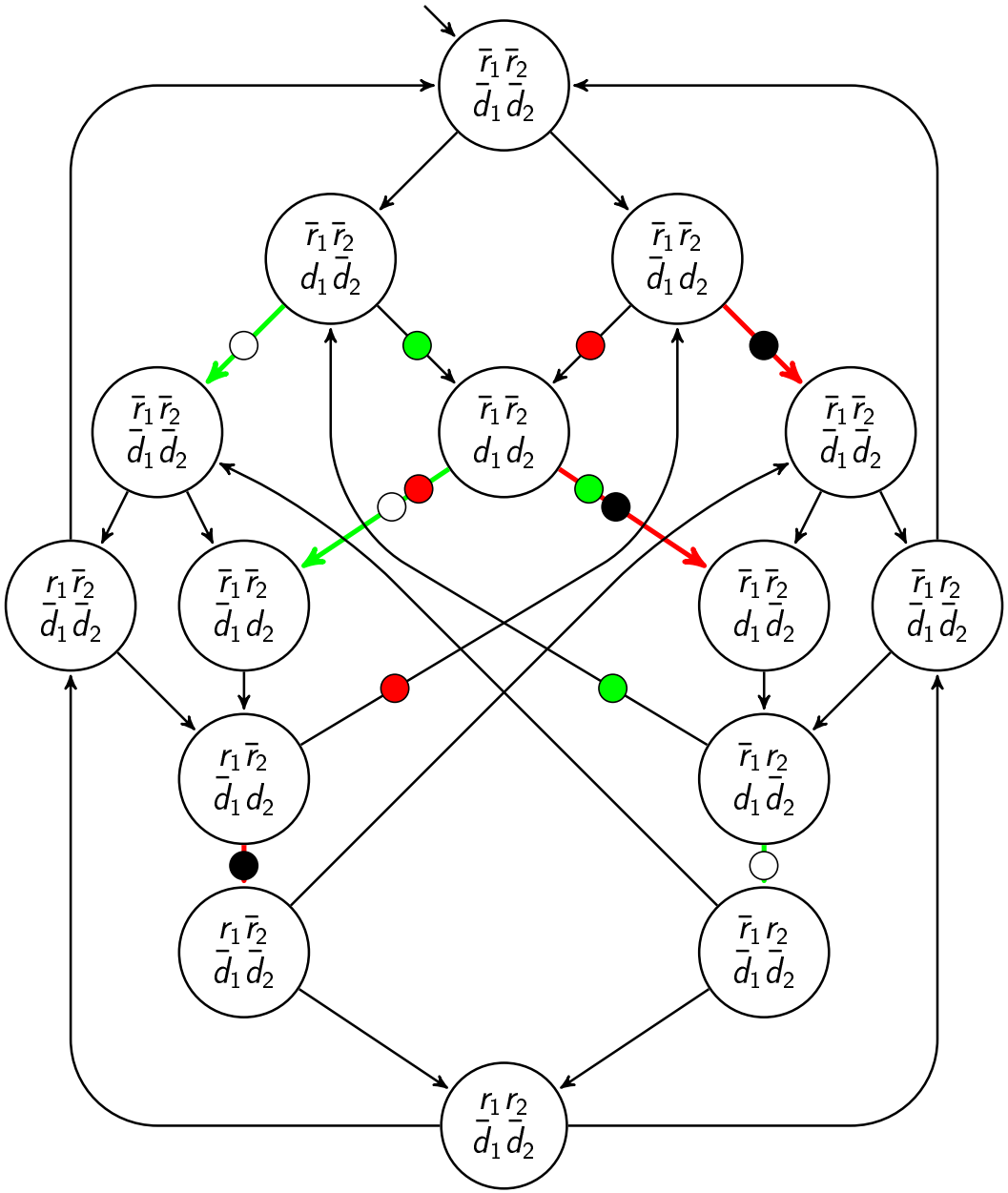
$$
\begin{array}{ll}
(1) & \langle\, s\,,\,.\,,\,.\,,\,.\,,\,.\,,\,a\,,\,.\,\rangle \\
(2) & \langle\, .\,,\,s\,,\,.\,,\,.\,,\,.\,,\,.\,,\,a\,\rangle \\
(3) & \langle\, r\,,\,.\,,\,.\,,\,d\,,\,.\,,\,.\,,\,.\,\rangle \\
(4) & \langle\, .\,,\,r\,,\,.\,,\,.\,,\,d\,,\,.\,,\,.\,\rangle \\
(5) & \langle\, .\,,\,.\,,\,r_1\,,\,.\,,\,.\,,\,d\,,\,.\,\rangle \\
(6) & \langle\, .\,,\,.\,,\,s_1\,,\,a\,,\,.\,,\,.\,,\,.\,\rangle \\
(7) & \langle\, .\,,\,.\,,\,r_2\,,\,.\,,\,.\,,\,.\,,\,d\,\rangle \\
(8) & \langle\, .\,,\,.\,,\,s_2\,,\,.\,,\,a\,,\,.\,,\,.\,\rangle
\end{array}
$$

We want the choice between transition (5) and (6) to be fair.
Do we want strong or weak fairness?
If both (5) and (7) are enabled (= can occur) and we pick (5), then transition (7) will not be enabled until (6) occurs. Therefore if we always pick (5), (7) will not be enabled continuously: it will only be enabled infinitely often.
We need strong fairness!

Making the Kripke Structure fair



Making the Kripke Structure fair ($\bullet \Rightarrow \circ \wedge \bullet \Rightarrow \bullet$)



Another kind of fairness

The previous example still allows scenarios where one client never work. What if we want to disallow this?

$$
\begin{array}{ll}
(1) & \langle\, s\,,\,.\,,\,.\,,\,.\,,\,.\,,\,a\,,\,.\,\rangle \\
(2) & \langle\,.\,,\,s\,,\,.\,,\,.\,,\,.\,,\,.\,,\,a\,\rangle \\
(3) & \langle\, r\,,\,.\,,\,.\,,\,d\,,\,.\,,\,.\,,\,.\,\rangle \\
(4) & \langle\,.\,,\,r\,,\,.\,,\,.\,,\,d\,,\,.\,,\,.\,\rangle \\
(5) & \langle\,.\,,\,.\,,\,r_1\,,\,.\,,\,.\,,\,d\,,\,.\,\rangle \\
(6) & \langle\,.\,,\,.\,,\,s_1\,,\,a\,,\,.\,,\,.\,,\,.\,\rangle \\
(7) & \langle\,.\,,\,.\,,\,r_2\,,\,.\,,\,.\,,\,.\,,\,d\,\rangle \\
(8) & \langle\,.\,,\,.\,,\,s_2\,,\,.\,,\,a\,,\,.\,,\,.\,\rangle
\end{array}
$$

Another kind of fairness

The previous example still allows scenarios where one client never work. What if we want to disallow this?

$$
\begin{array}{ll}
(1) & \langle\, s, ., ., ., ., ., a, . \,\rangle \\
(2) & \langle\, ., s, ., ., ., ., ., a \,\rangle \\
(3) & \langle\, r, ., ., ., d, ., ., ., . \,\rangle \\
(4) & \langle\, ., r, ., ., ., d, ., ., . \,\rangle \\
(5) & \langle\, ., ., ., r_1, ., ., ., d, . \,\rangle \\
(6) & \langle\, ., ., ., s_1, a, ., ., ., . \,\rangle \\
(7) & \langle\, ., ., ., r_2, ., ., ., ., d \,\rangle \\
(8) & \langle\, ., ., ., s_2, ., ., a, ., ., . \,\rangle
\end{array}
$$

We want (1) or (3) to occur infinitely often (i.e., client 1 progresses), and (2) or (3) to occur infinitely often (i.e., client 2 progresses too).

Strong or weak?

Another kind of fairness

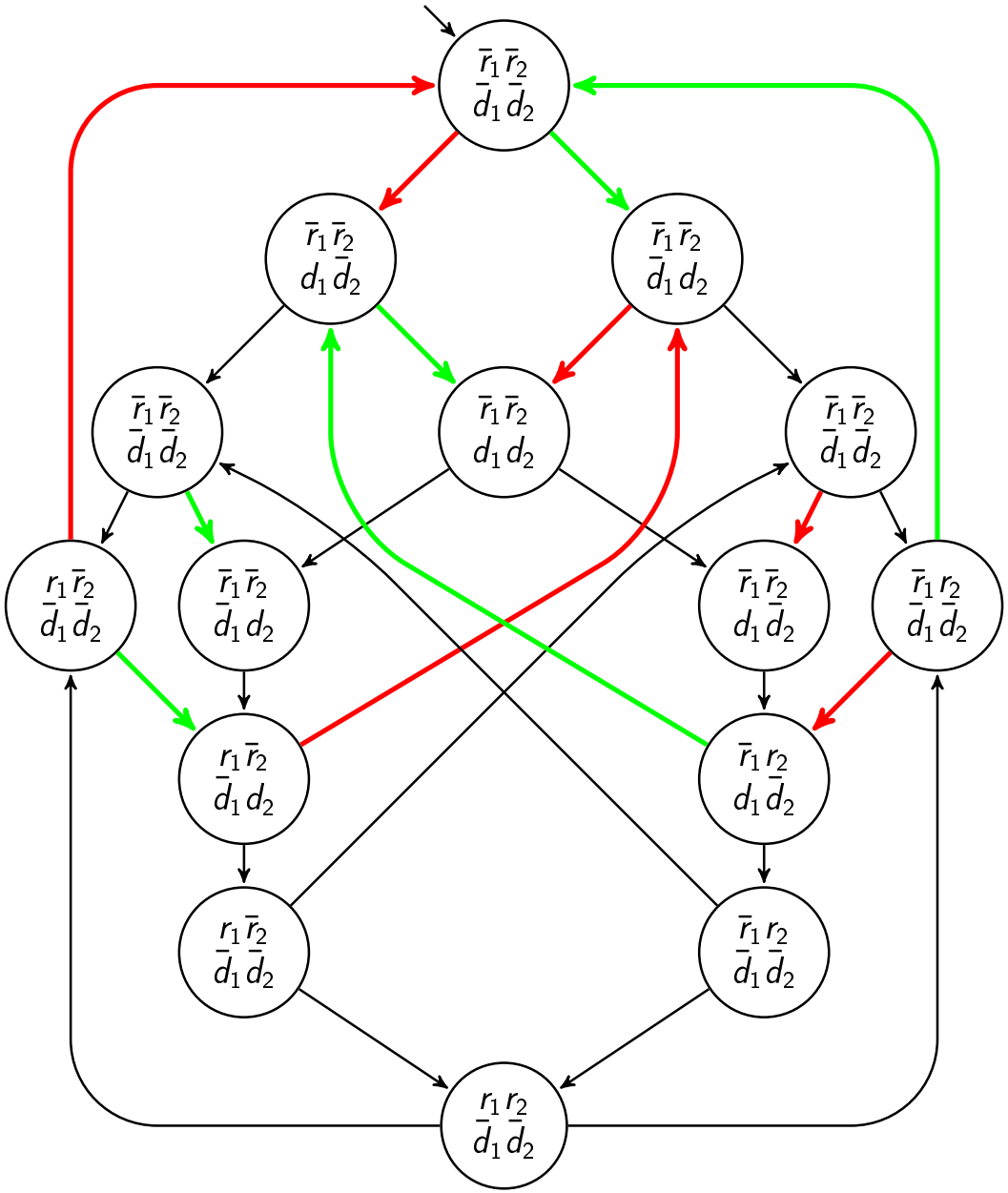
The previous example still allows scenarios where one client never work. What if we want to disallow this?

$$
\begin{array}{ll}
(1) & \langle\, s, ., ., ., ., ., a, . \,\rangle \\
(2) & \langle\, ., s, ., ., ., ., ., a \,\rangle \\
(3) & \langle\, r, ., ., ., d, ., ., ., . \,\rangle \\
(4) & \langle\, ., r, ., ., ., d, ., ., . \,\rangle \\
(5) & \langle\, ., ., ., r_1, ., ., ., d, . \,\rangle \\
(6) & \langle\, ., ., ., s_1, a, ., ., ., . \,\rangle \\
(7) & \langle\, ., ., ., r_2, ., ., ., ., d \,\rangle \\
(8) & \langle\, ., ., ., s_2, ., ., a, ., ., . \,\rangle
\end{array}
$$

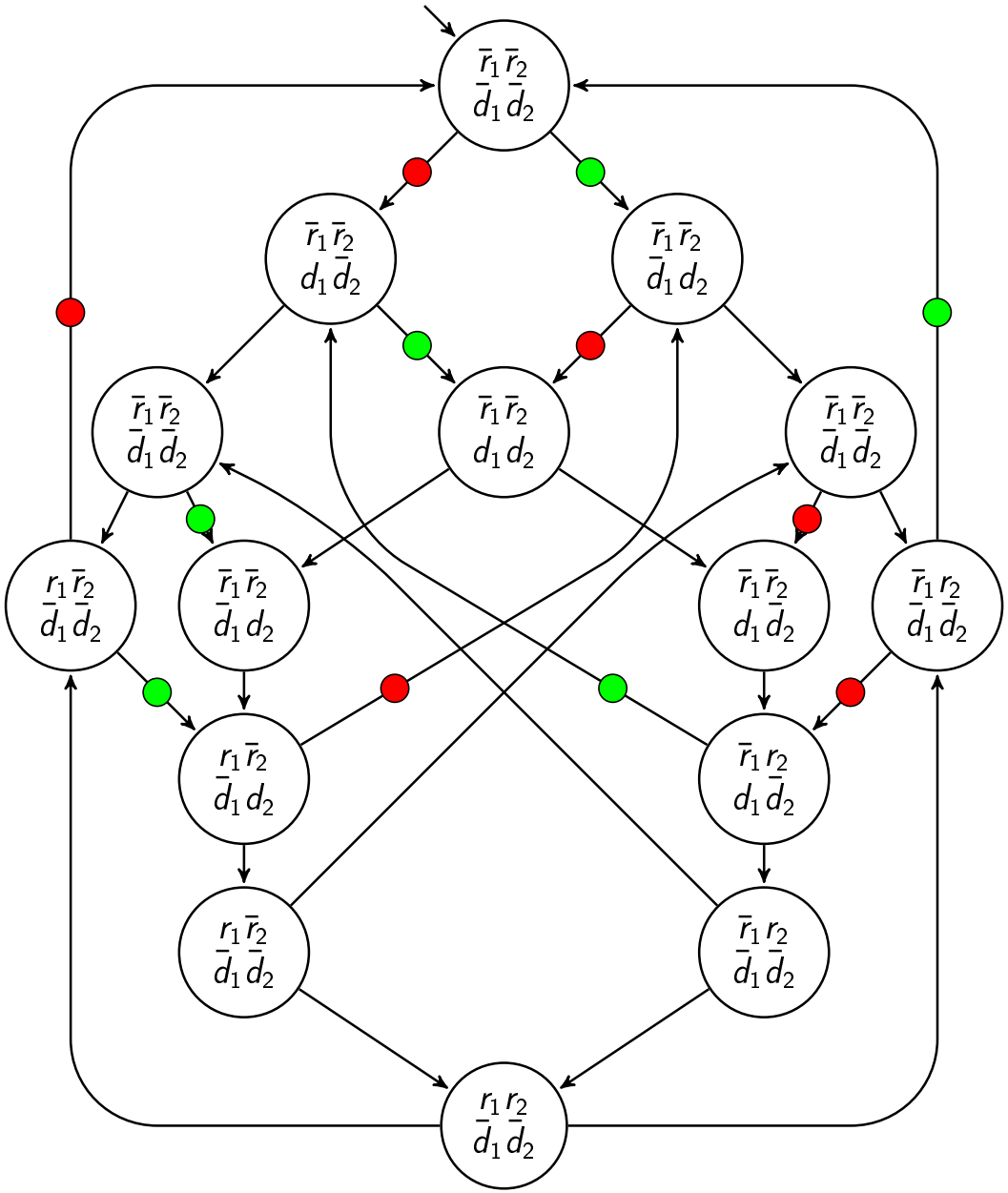
We want (1) or (3) to occur infinitely often (i.e., client 1 progresses), and (2) or (3) to occur infinitely often (i.e., client 2 progresses too).

Strong or weak? Weak. In fact, unconditional fairness.

Making the Kripke Structure fair



Making the Kripke Structure fair (Büchi acc.)



A small LTL formula...

$$\Big((\mathbf{G}\,\mathbf{F}\,p_0 \rightarrow \mathbf{G}\,\mathbf{F}\,p_1) \wedge (\mathbf{G}\,\mathbf{F}\,p_2 \rightarrow \mathbf{G}\,\mathbf{F}\,p_0) \wedge$$
$$(\mathbf{G}\,\mathbf{F}\,p_3 \rightarrow \mathbf{G}\,\mathbf{F}\,p_2) \wedge (\mathbf{G}\,\mathbf{F}\,p_4 \rightarrow \mathbf{G}\,\mathbf{F}\,p_2) \wedge$$
$$(\mathbf{G}\,\mathbf{F}\,p_5 \rightarrow \mathbf{G}\,\mathbf{F}\,p_3) \wedge (\mathbf{G}\,\mathbf{F}\,p_6 \rightarrow \mathbf{G}\,\mathbf{F}(p_5 \vee p_4)) \wedge$$
$$(\mathbf{G}\,\mathbf{F}\,p_7 \rightarrow \mathbf{G}\,\mathbf{F}\,p_6) \wedge (\mathbf{G}\,\mathbf{F}\,p_1 \rightarrow \mathbf{G}\,\mathbf{F}\,p_7)\Big) \rightarrow \mathbf{G}\,\mathbf{F}\,p_8$$

How many states to encode the negation in a Büchi automaton?

A small LTL formula...

$$\Big((\mathbf{G}\,\mathbf{F}\,p_0 \rightarrow \mathbf{G}\,\mathbf{F}\,p_1) \wedge (\mathbf{G}\,\mathbf{F}\,p_2 \rightarrow \mathbf{G}\,\mathbf{F}\,p_0) \wedge$$
$$(\mathbf{G}\,\mathbf{F}\,p_3 \rightarrow \mathbf{G}\,\mathbf{F}\,p_2) \wedge (\mathbf{G}\,\mathbf{F}\,p_4 \rightarrow \mathbf{G}\,\mathbf{F}\,p_2) \wedge$$
$$(\mathbf{G}\,\mathbf{F}\,p_5 \rightarrow \mathbf{G}\,\mathbf{F}\,p_3) \wedge (\mathbf{G}\,\mathbf{F}\,p_6 \rightarrow \mathbf{G}\,\mathbf{F}(p_5 \vee p_4)) \wedge$$
$$(\mathbf{G}\,\mathbf{F}\,p_7 \rightarrow \mathbf{G}\,\mathbf{F}\,p_6) \wedge (\mathbf{G}\,\mathbf{F}\,p_1 \rightarrow \mathbf{G}\,\mathbf{F}\,p_7)\Big) \rightarrow \mathbf{G}\,\mathbf{F}\,p_8$$

How many states to encode the negation in a Büchi automaton?

Spot's LTL to Büchi translation without optimizations : 7291 states.
With optimizations : 1731 states.
Sebastiani et al. dedicated translation : 1281 states.

A small LTL formula...

$$\Big((\mathbf{G}\,\mathbf{F}\,p_0 \rightarrow \mathbf{G}\,\mathbf{F}\,p_1) \wedge (\mathbf{G}\,\mathbf{F}\,p_2 \rightarrow \mathbf{G}\,\mathbf{F}\,p_0) \wedge$$
$$(\mathbf{G}\,\mathbf{F}\,p_3 \rightarrow \mathbf{G}\,\mathbf{F}\,p_2) \wedge (\mathbf{G}\,\mathbf{F}\,p_4 \rightarrow \mathbf{G}\,\mathbf{F}\,p_2) \wedge$$
$$(\mathbf{G}\,\mathbf{F}\,p_5 \rightarrow \mathbf{G}\,\mathbf{F}\,p_3) \wedge (\mathbf{G}\,\mathbf{F}\,p_6 \rightarrow \mathbf{G}\,\mathbf{F}(p_5 \vee p_4)) \wedge$$
$$(\mathbf{G}\,\mathbf{F}\,p_7 \rightarrow \mathbf{G}\,\mathbf{F}\,p_6) \wedge (\mathbf{G}\,\mathbf{F}\,p_1 \rightarrow \mathbf{G}\,\mathbf{F}\,p_7)\Big) \rightarrow \mathbf{G}\,\mathbf{F}\,p_8$$

How many states to encode the negation in a Streett automaton?

A small LTL formula...

$$\Big((\mathbf{G}\,\mathbf{F}\,p_0 \to \mathbf{G}\,\mathbf{F}\,p_1) \wedge (\mathbf{G}\,\mathbf{F}\,p_2 \to \mathbf{G}\,\mathbf{F}\,p_0) \wedge$$
$$(\mathbf{G}\,\mathbf{F}\,p_3 \to \mathbf{G}\,\mathbf{F}\,p_2) \wedge (\mathbf{G}\,\mathbf{F}\,p_4 \to \mathbf{G}\,\mathbf{F}\,p_2) \wedge$$
$$(\mathbf{G}\,\mathbf{F}\,p_5 \to \mathbf{G}\,\mathbf{F}\,p_3) \wedge (\mathbf{G}\,\mathbf{F}\,p_6 \to \mathbf{G}\,\mathbf{F}(p_5 \vee p_4)) \wedge$$
$$(\mathbf{G}\,\mathbf{F}\,p_7 \to \mathbf{G}\,\mathbf{F}\,p_6) \wedge (\mathbf{G}\,\mathbf{F}\,p_1 \to \mathbf{G}\,\mathbf{F}\,p_7)\Big) \to \mathbf{G}\,\mathbf{F}\,p_8$$

How many states to encode the negation in a Streett automaton?
Formula of the form $\psi \to \varphi$ where ψ is a strong fairness hypothesis.

$$\mathcal{A}_{\neg(\psi\to\varphi)} = \mathcal{A}_{\psi} \otimes \mathcal{A}_{\neg\varphi}$$

$\mathcal{A}_{\psi}$: 1-state deterministic Streett automaton with 8 pairs of acc.cond.
$\mathcal{A}_{\neg\varphi} = \mathcal{A}_{\mathbf{F}\,\mathbf{G}\,\neg p_8}$: 2-state Büchi automaton (w/o acc.cond.)
Therefore $\mathcal{A}_{\neg(\psi\to\varphi)}$ can be represented as a 2-state Streett automaton with 8 pairs of acc.cond.