Introduction to Computation and Complexity
Exercise Sheet 3

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Exercise 1

Prove that the set of Turing machine codes for Turing machines that do not accept binary words containing the symbol 1 is undecidable.

Exercise 2

Prove that the proposition "*Does the language of a Turing machine contain at least two strings?*" is undecidable.

Exercise 3

Prove that the halting problem can be reduced to the complement $^{c}L_{reg}$ of the set L_{reg} of Turing machine codes $\langle M \rangle$ whose language $\mathcal{L}(M)$ is regular. Then prove that L_{reg} is undecidable.

Hint: find a decidable, non-regular language.

Exercise 4

Prove that the set L_n^+ of Turing machine codes for Turing machines accepting at least n words is undecidable and recursively enumerable.

Exercise 5

Prove that the set L_n^- of Turing machine codes for Turing machines accepting less than n words is undecidable and not recursively enumerable.