

# Introduction to Computation and Complexity

## Exercise Sheet 4

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### Exercise 1

Prove that the function  $n \rightarrow n \cdot \lceil \log_2(n) \rceil$  is a time-constructible function.

**Hint:** if  $2^n < x \leq 2^{n+1}$ , then  $\lceil \log_2(x) \rceil = n + 1$ .

### Exercise 2

Prove the following theorem:

**Theorem 1 (Padding property)** *Given a language  $L$ , let  $L' = \{w \# 0^{2^{|w|}} \mid w \in L\}$ . Then  $L \in \text{EXP}$  if and only if  $L' \in \text{P}$ .*

**Hint 1:** keep in mind that  $n \rightarrow 2^n$  is time-constructible.

**Hint 2:** intuitively, we are merely cheating the definition of time complexity by padding the input with superfluous information to make it bigger, then claiming that the time complexity is polynomial with regards to the size of the inflated input.