Practical "Paritizing" of Emerson-Lei Automata

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Why?

Why?

$$\text{car} + \text{problem} = \text{accident}$$

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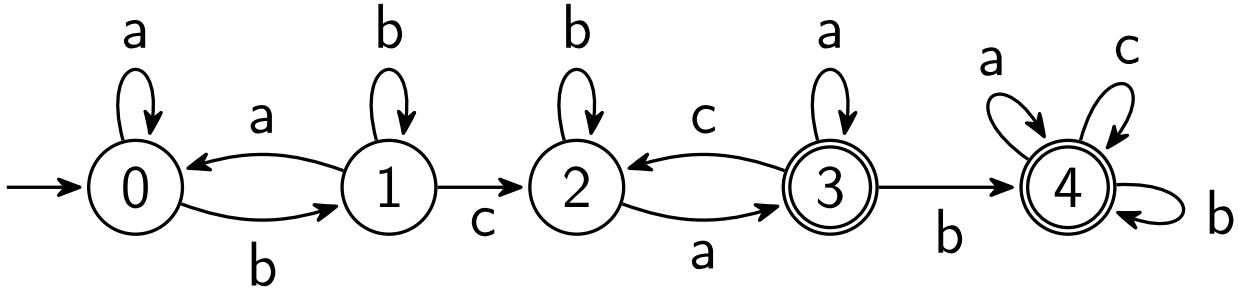
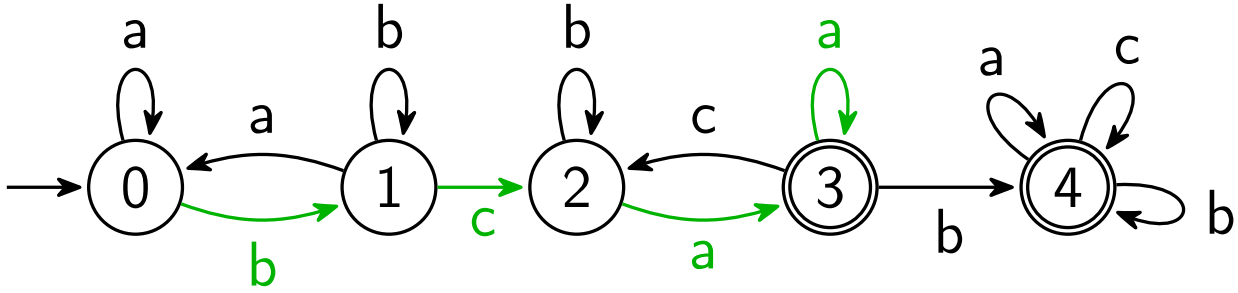
$$\text{specifications (LTL formula)} \rightarrow \text{circuit} = \text{ }$$

$$\text{Specification} \longrightarrow \omega\text{-automaton} \longrightarrow \text{circuit}$$

ω-automata



ω-automata



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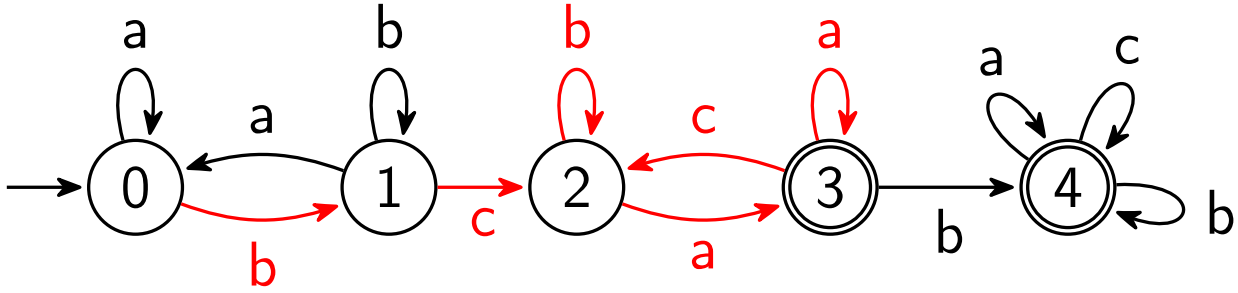
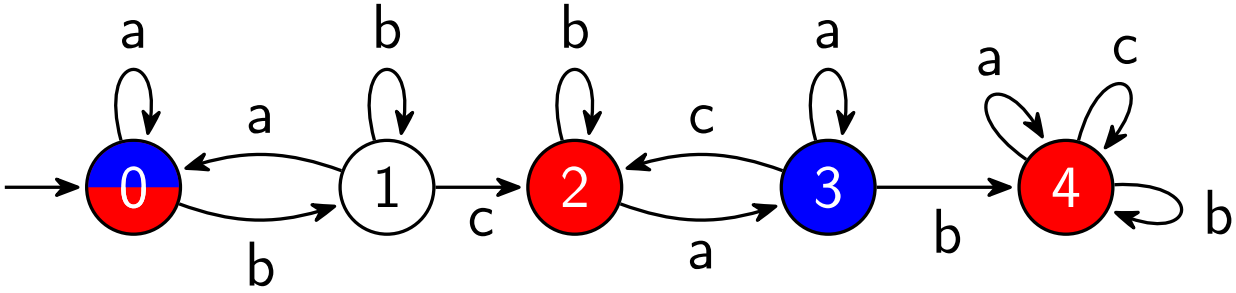
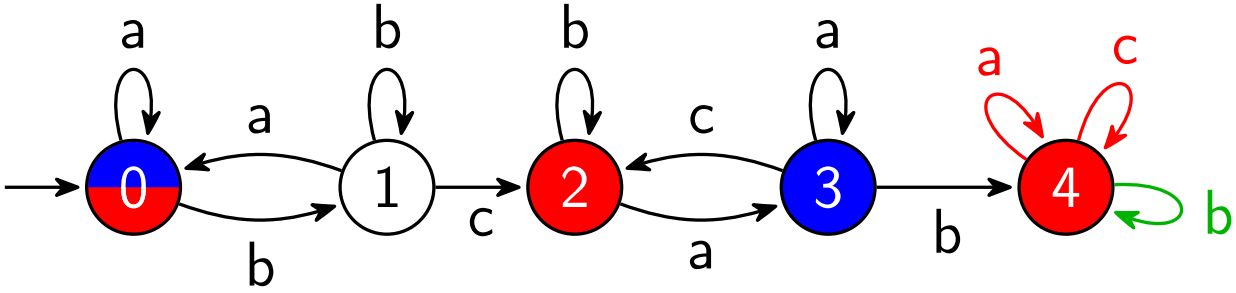


Emerson-Lei automata



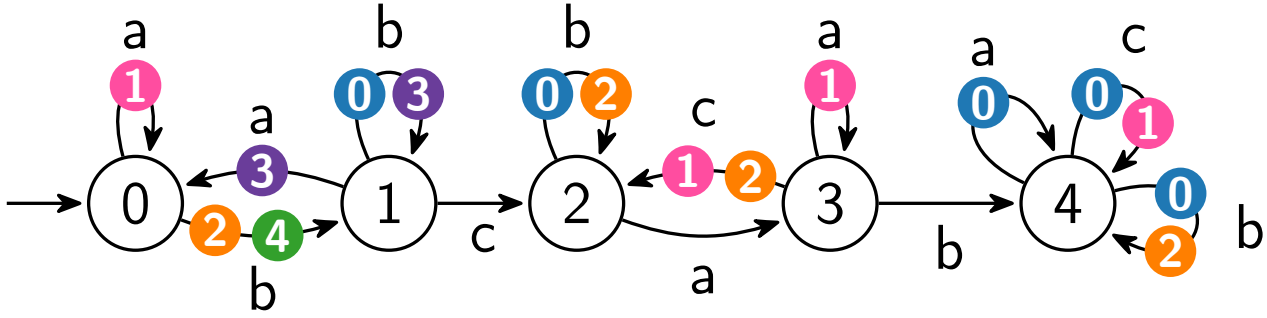
$$(\mathrm{Fin}(\bullet) \wedge \mathrm{Inf}(\bullet)) \vee (\mathrm{Inf}(\bullet) \wedge \mathrm{Fin}(\bullet))$$

Emerson-Lei automata



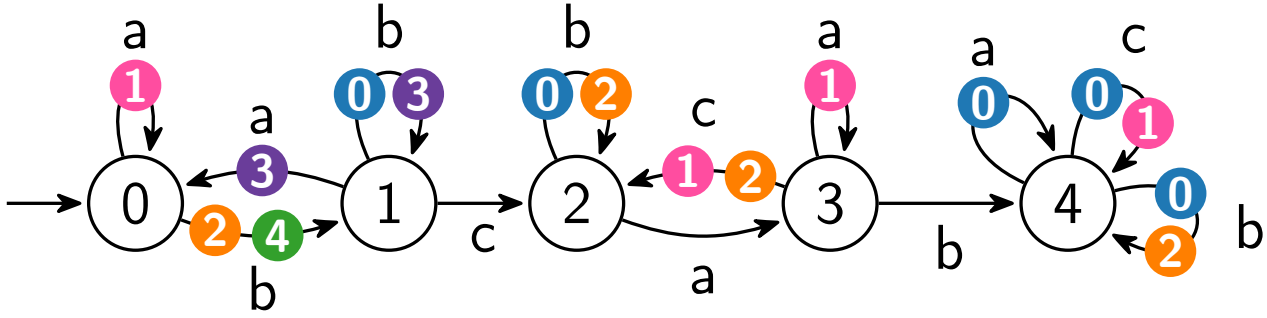
$$(\mathrm{Fin}(\bullet) \wedge \mathrm{Inf}(\bullet)) \vee (\mathrm{Inf}(\bullet) \wedge \mathrm{Fin}(\bullet))$$

TELA: Transition-based Emerson-Lei Automata



$$\big(\mathsf{Inf}(2)\wedge\mathsf{Fin}(1)\big)\vee\big((\mathsf{Inf}(4)\vee\mathsf{Inf}(1))\wedge \mathsf{Fin}(0)\wedge(\mathsf{Fin}(1)\vee\mathsf{Fin}(3))\big)$$

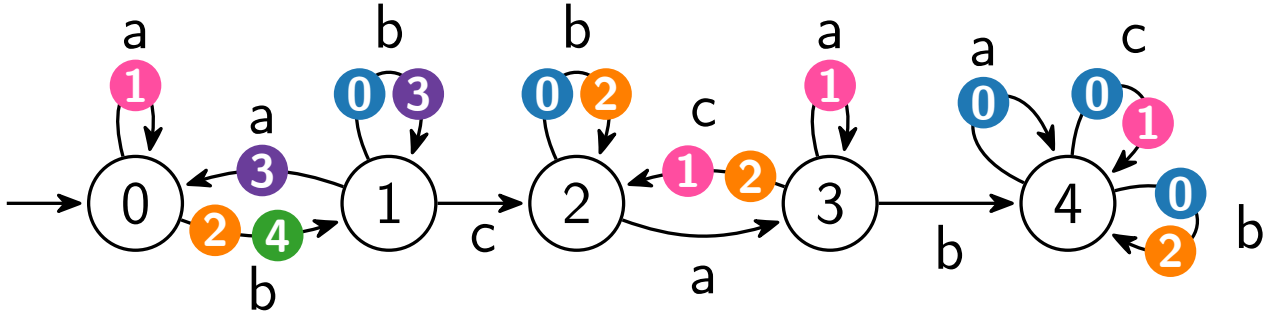
TELA: Transition-based Emerson-Lei Automata



$$(\mathrm{Inf}(2) \wedge \mathrm{Fin}(1)) \vee ((\mathrm{Inf}(4) \vee \mathrm{Inf}(1)) \wedge \mathrm{Fin}(0) \wedge (\mathrm{Fin}(1) \vee \mathrm{Fin}(3)))$$

Büchi: $\mathrm{Inf}(0)$

TELA: Transition-based Emerson-Lei Automata



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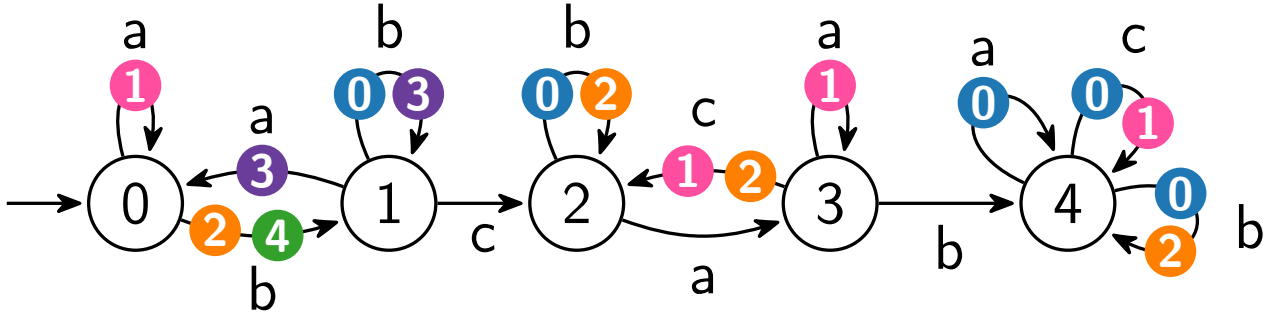
Büchi: $\mathrm{Inf}(0)$

Generalized Büchi: $\mathrm{Inf}(0) \wedge \mathrm{Inf}(1) \wedge \ldots$

TELA: Transition-based Emerson-Lei Automata



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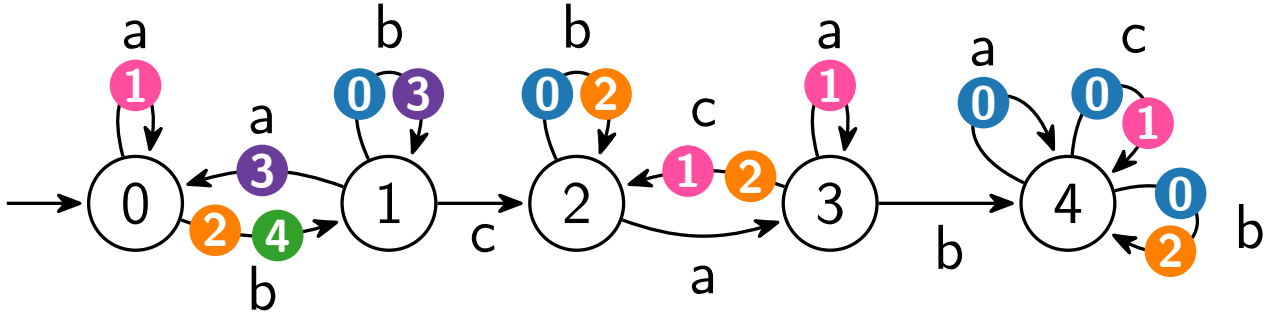
Generalized Büchi: $\mathrm{Inf}(0) \wedge \mathrm{Inf}(1) \wedge \ldots$

Rabin: $(\mathrm{Fin}(0) \wedge \mathrm{Inf}(2)) \vee (\mathrm{Fin}(1) \wedge \mathrm{Inf}(0)) \vee \ldots$

Streett: $(\mathrm{Inf}(0) \vee \mathrm{Fin}(2)) \wedge (\mathrm{Inf}(1) \vee \mathrm{Fin}(0)) \wedge \ldots$

TELA: Transition-based Emerson-Lei Automata



$$(\mathrm{Inf}(2) \wedge \mathrm{Fin}(1)) \vee ((\mathrm{Inf}(4) \vee \mathrm{Inf}(1)) \wedge \mathrm{Fin}(0) \wedge (\mathrm{Fin}(1) \vee \mathrm{Fin}(3)))$$

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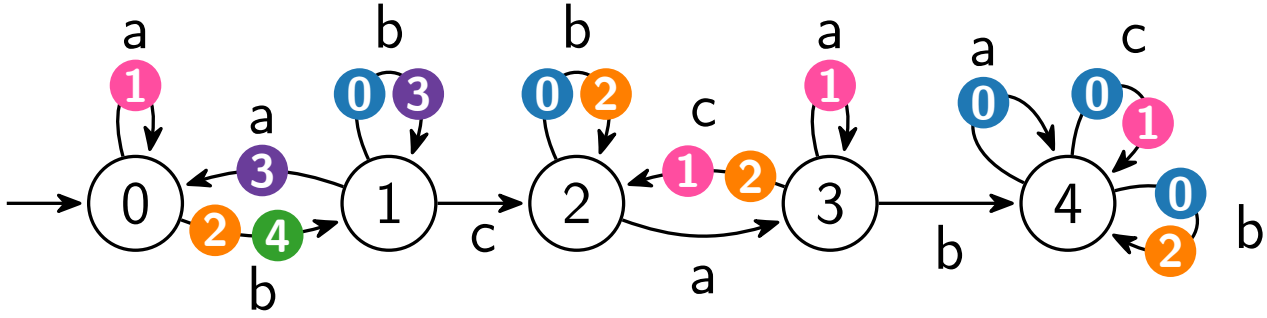
Streett: $(\mathrm{Inf}(0) \vee \mathrm{Fin}(2)) \wedge (\mathrm{Inf}(1) \vee \mathrm{Fin}(0)) \wedge \ldots$

Parity max even: $\ldots \vee (\mathrm{Fin}(3) \wedge (\mathrm{Inf}(2) \vee (\mathrm{Fin}(1) \wedge \mathrm{Inf}(0))))$

TELA: Transition-based Emerson-Lei Automata



$$(\mathrm{Inf}(2) \wedge \mathrm{Fin}(1)) \vee ((\mathrm{Inf}(4) \vee \mathrm{Inf}(1)) \wedge \mathrm{Fin}(0) \wedge (\mathrm{Fin}(1) \vee \mathrm{Fin}(3)))$$

paritization

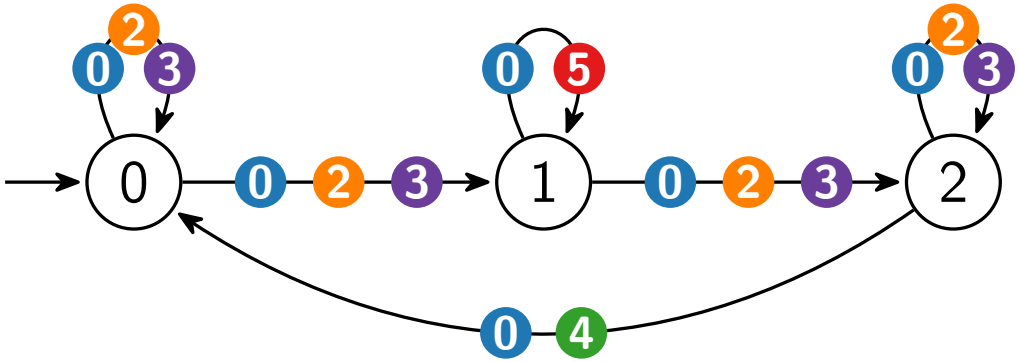
- Büchi: $\mathrm{Inf}(0)$
- Generalized Büchi: $\mathrm{Inf}(0) \wedge \mathrm{Inf}(1) \wedge \ldots$
- Rabin: $(\mathrm{Fin}(0) \wedge \mathrm{Inf}(2)) \vee (\mathrm{Fin}(1) \wedge \mathrm{Inf}(0)) \vee \ldots$
- Streett: $(\mathrm{Inf}(0) \vee \mathrm{Fin}(2)) \wedge (\mathrm{Inf}(1) \vee \mathrm{Fin}(0)) \wedge \ldots$
- Parity max even: $\ldots \vee (\mathrm{Fin}(3) \wedge (\mathrm{Inf}(2) \vee (\mathrm{Fin}(1) \wedge \mathrm{Inf}(0))))$

Outline

1. Direct simplifications of TELA
2. Converting TELA to TPA
   - CAR
   - IAR
3. Improvements
   - Degeneralization
   - Partial degeneralization
   - Propagation of colors
   - TELA Büchi-type
4. General algorithm
   - Example
   - Experimental evaluation
5. Reactive synthesis from LTL specifications
   - Description
   - Experimental evaluation
6. Conclusion

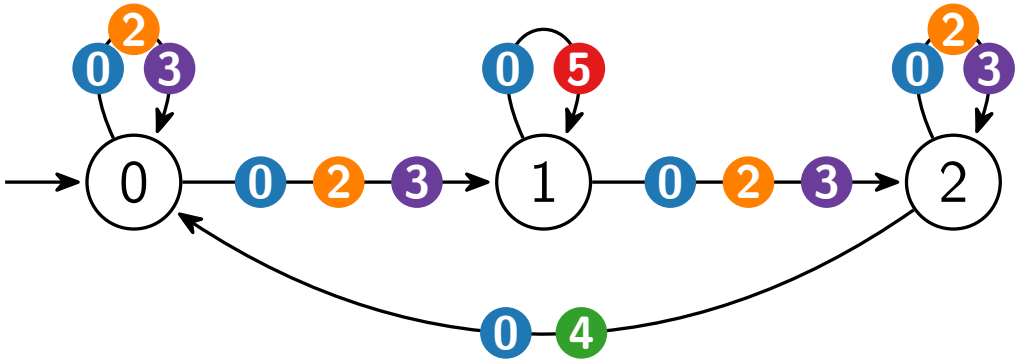
Direct simplifications of TELA

Acceptance simplifications



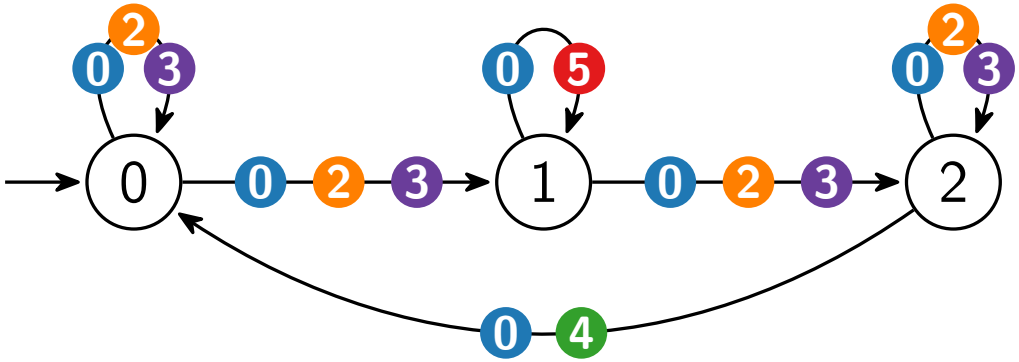
$$(\mathsf{Fin}(0) \vee (\mathsf{Fin}(1) \wedge \mathsf{Fin}(2)) \vee (\mathsf{Inf}(2) \wedge \mathsf{Fin}(4)) \vee \mathsf{Fin}(3)) \wedge \mathsf{Inf}(5)$$

Acceptance simplifications



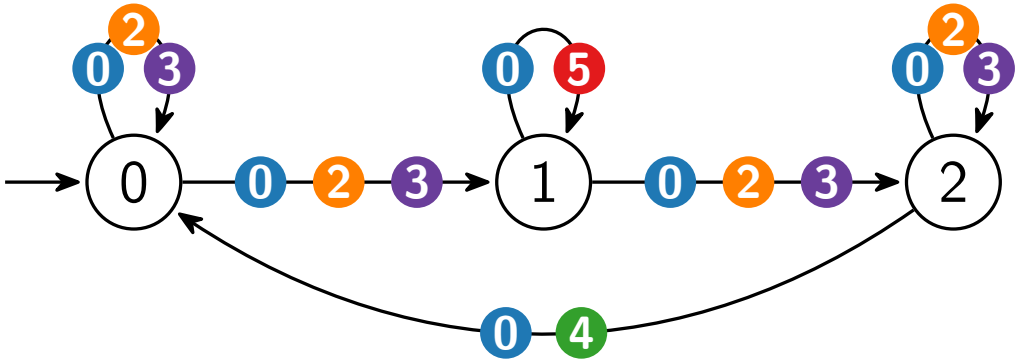
$$(\mathsf{Fin}(0) \vee (\mathsf{Fin}(1) \wedge \mathsf{Fin}(2)) \vee (\mathsf{Inf}(2) \wedge \mathsf{Fin}(4)) \vee \mathsf{Fin}(3)) \wedge \mathsf{Inf}(5)$$
$$= (\bot \vee (\top \wedge \mathsf{Fin}(2)) \vee (\mathsf{Inf}(2) \wedge \mathsf{Fin}(4)) \vee \mathsf{Fin}(3)) \wedge \mathsf{Inf}(5)$$

Acceptance simplifications



$$(\mathsf{Fin}(0) \vee (\mathsf{Fin}(1) \wedge \mathsf{Fin}(2)) \vee (\mathsf{Inf}(2) \wedge \mathsf{Fin}(4)) \vee \mathsf{Fin}(3)) \wedge \mathsf{Inf}(5)$$
$$= (\mathsf{Fin}(2) \vee (\mathsf{Inf}(2) \wedge \mathsf{Fin}(4)) \vee \mathsf{Fin}(3)) \wedge \mathsf{Inf}(5)$$

Acceptance simplifications



$$
\begin{aligned}
& (\mathsf{Fin}(0) \vee (\mathsf{Fin}(1) \wedge \mathsf{Fin}(2)) \vee (\mathsf{Inf}(2) \wedge \mathsf{Fin}(4)) \vee \mathsf{Fin}(3)) \wedge \mathsf{Inf}(5) \\
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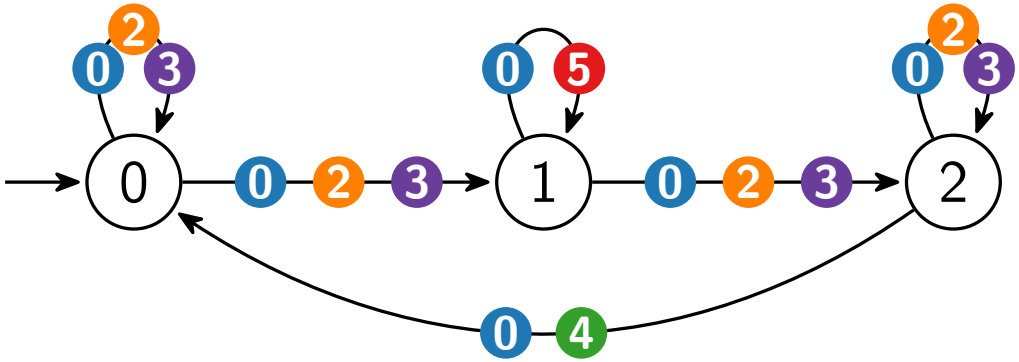
\end{aligned}
$$

Acceptance simplifications



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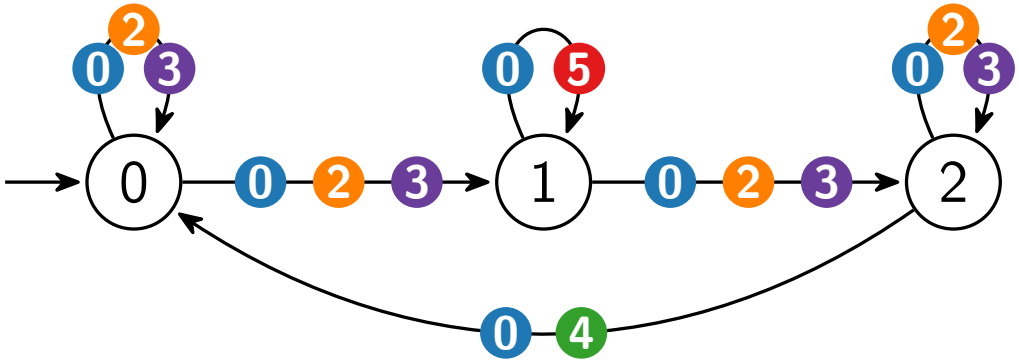

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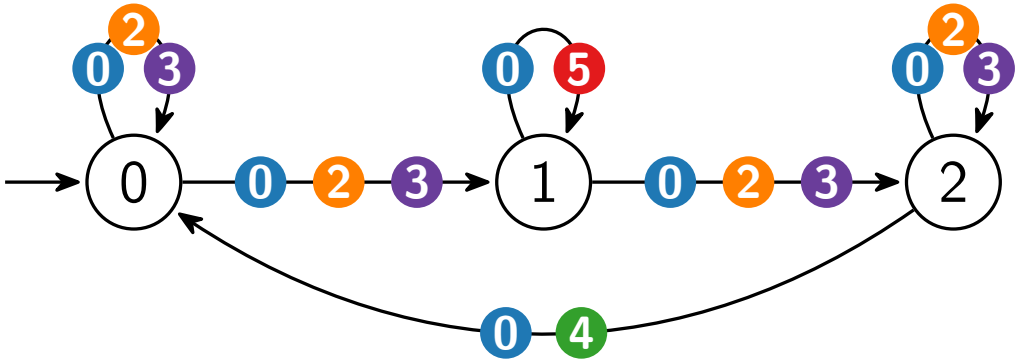

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= \; & (\mathsf{Fin}(2) \vee (\overline{\mathsf{Fin}}(2) \wedge \mathsf{Fin}(4))) \wedge \mathsf{Inf}(5) \\
= \; & (\mathsf{Fin}(2) \vee (\top \wedge \mathsf{Fin}(4))) \wedge \mathsf{Inf}(5)
\end{aligned}
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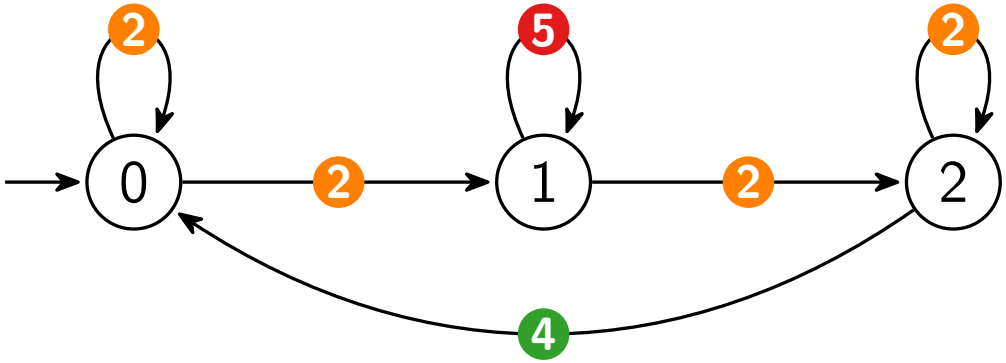
Acceptance simplifications



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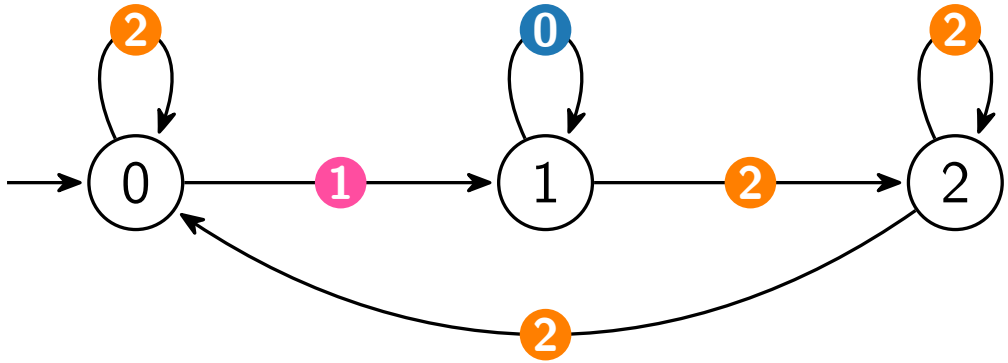
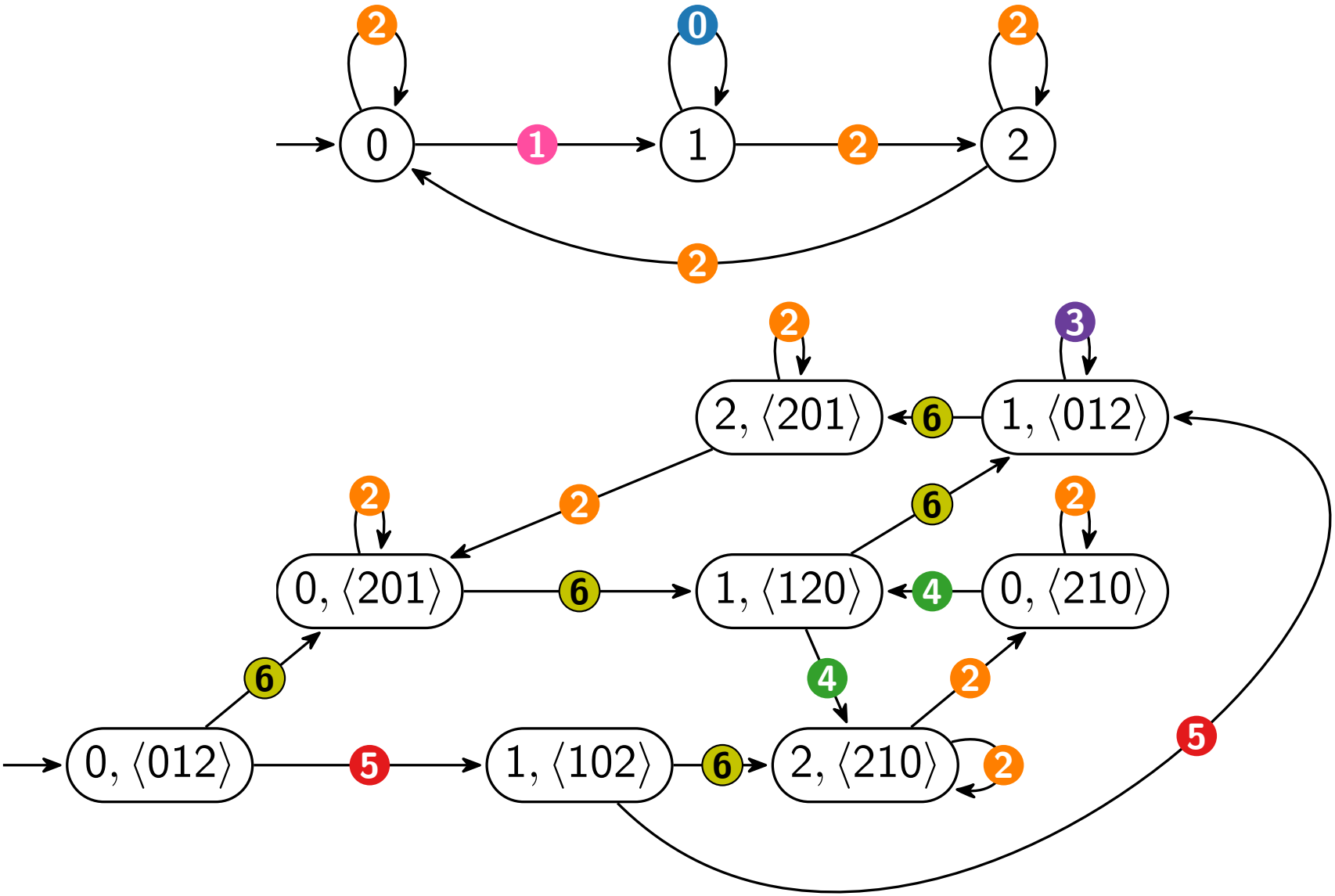
\end{aligned}
$$

Converting TELA to TPA

CAR: color appearance record



CAR: color appearance record



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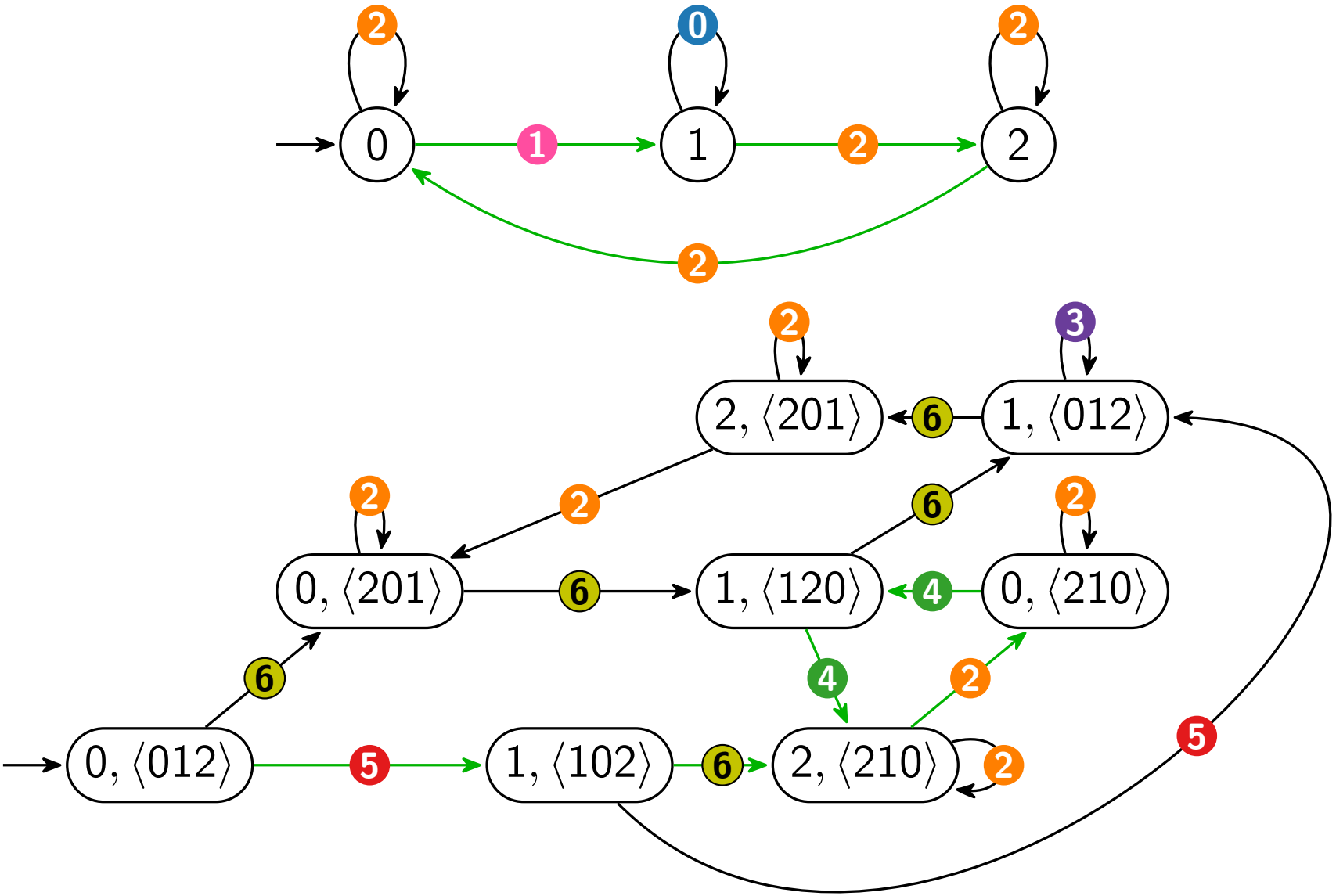

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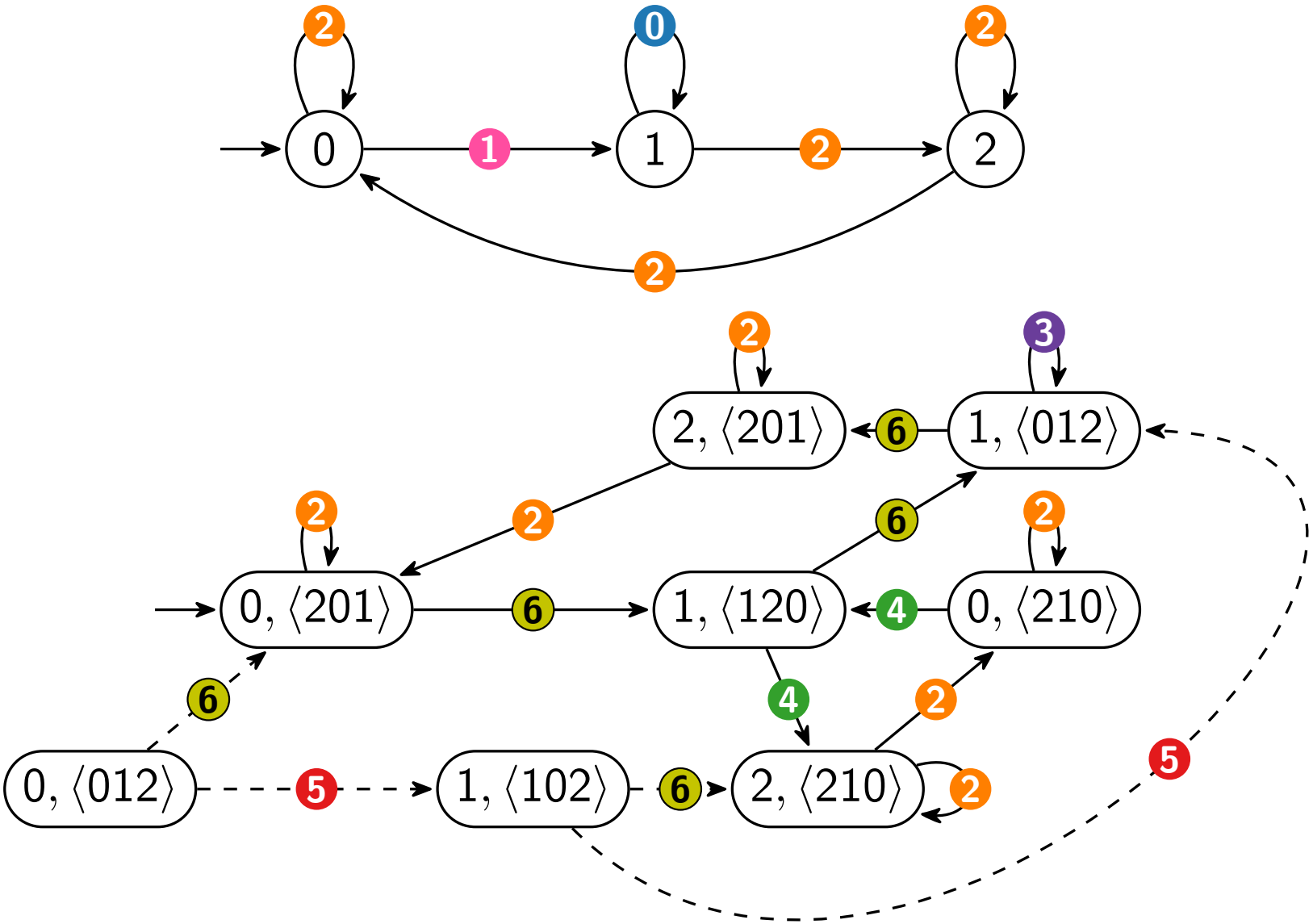

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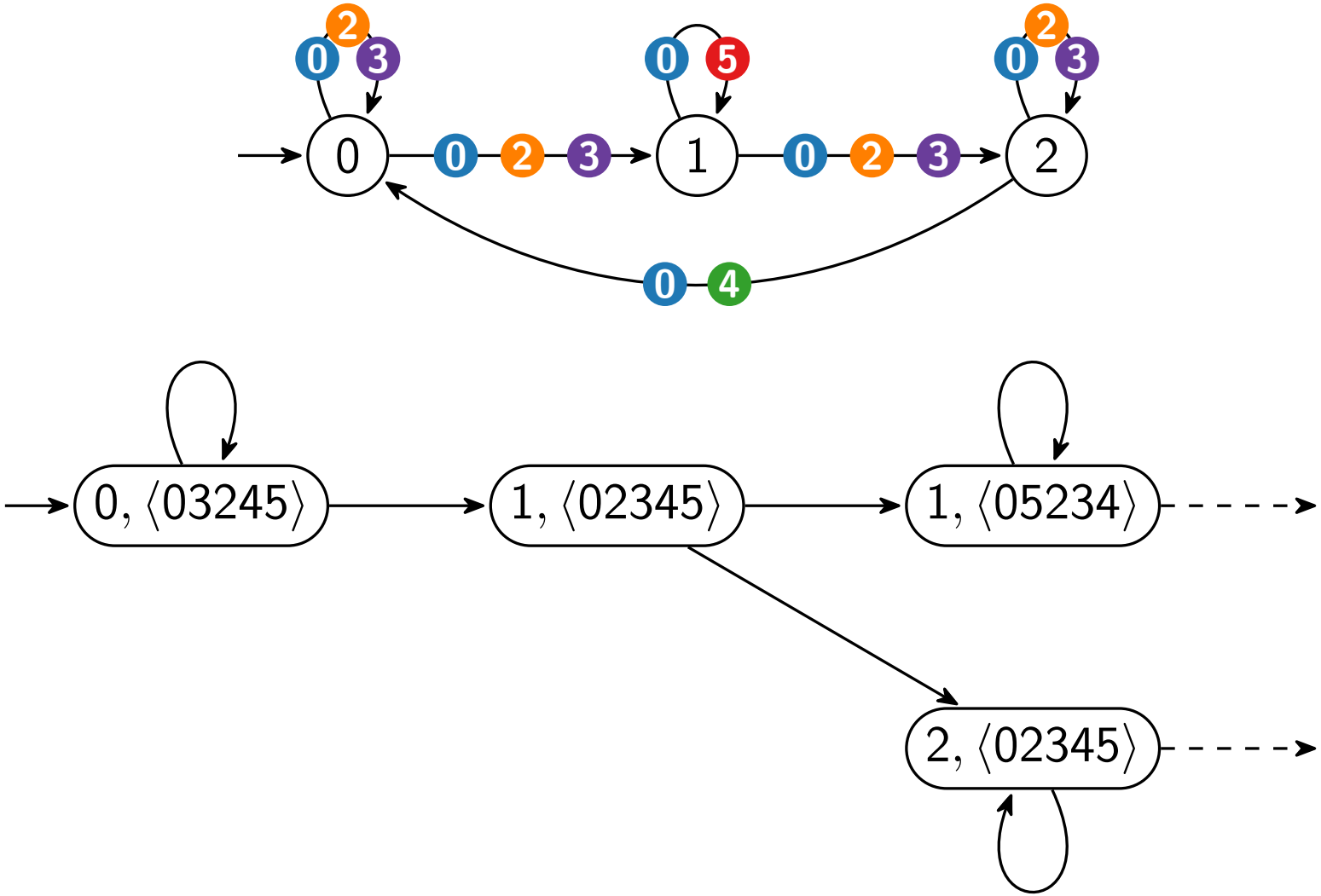

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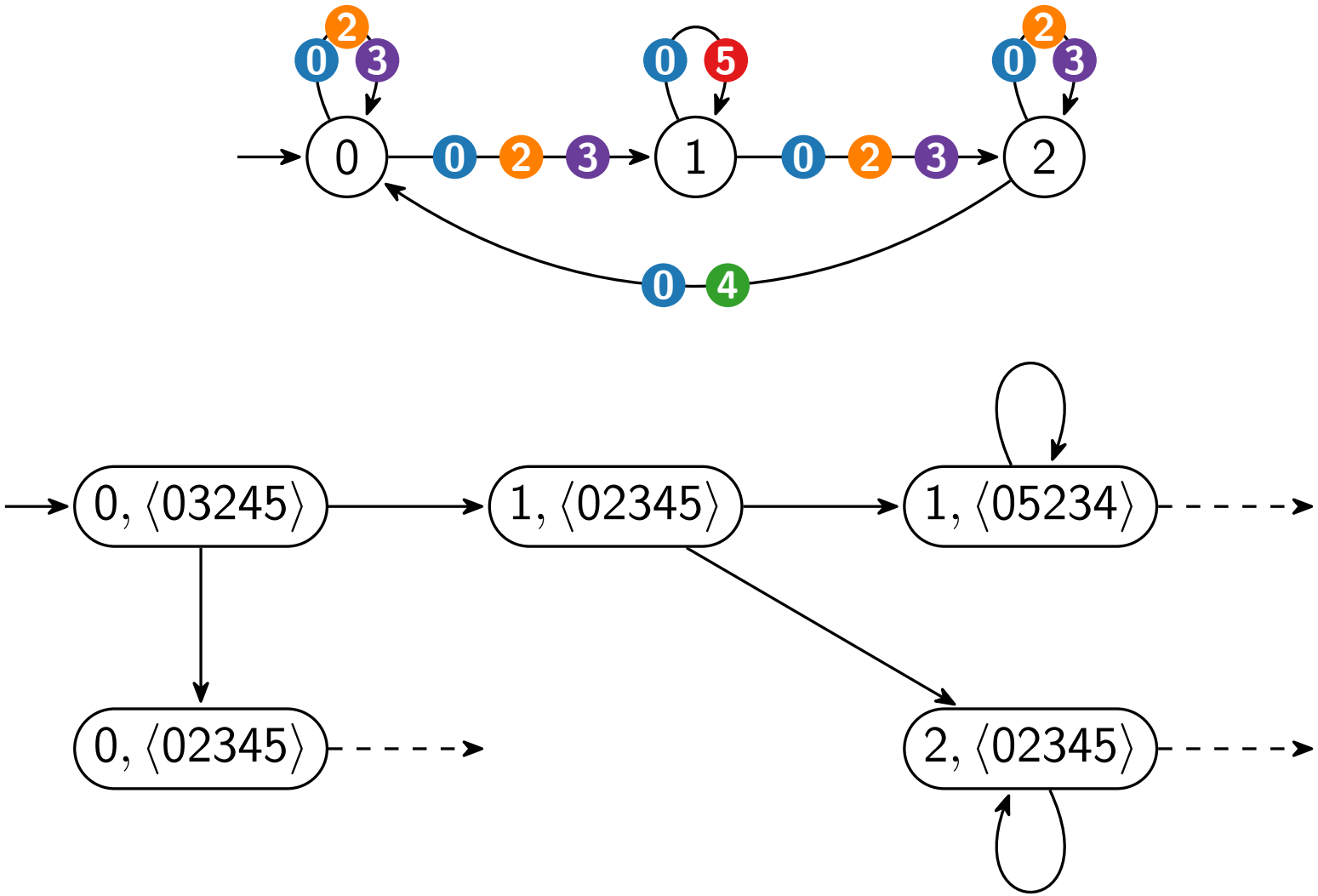
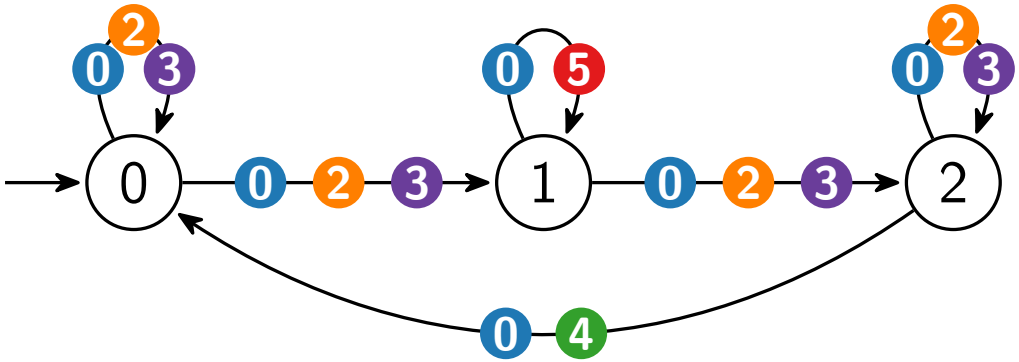
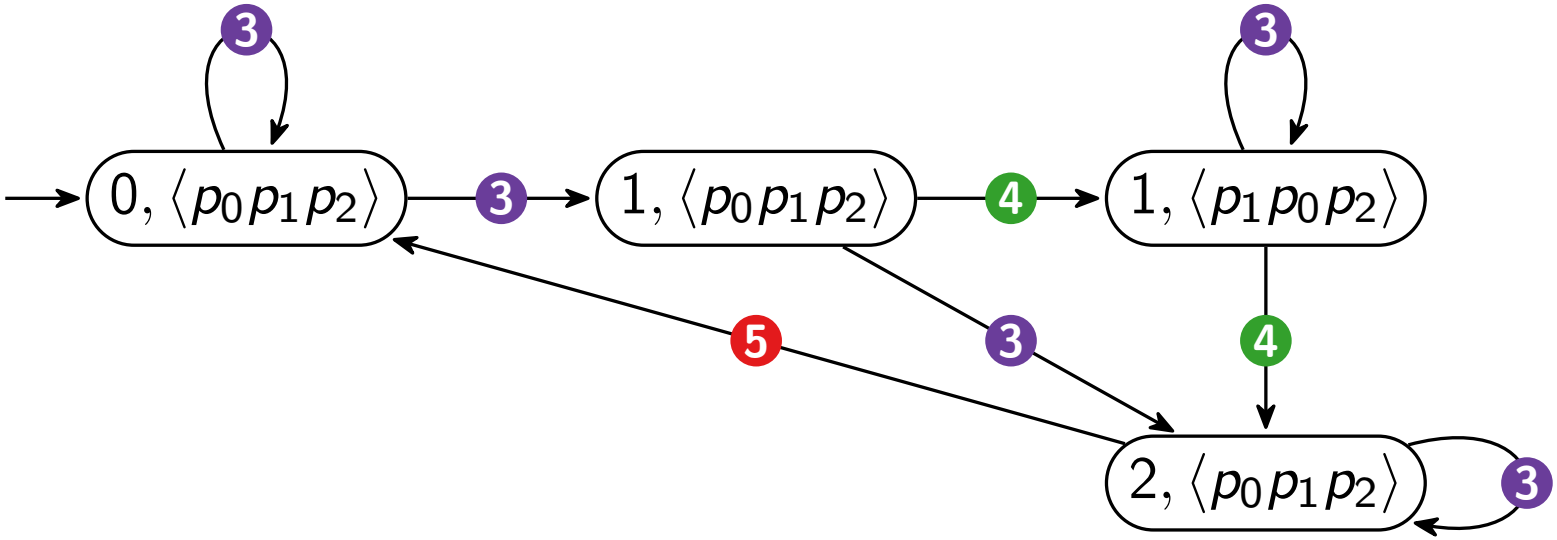


IAR: index appearance record

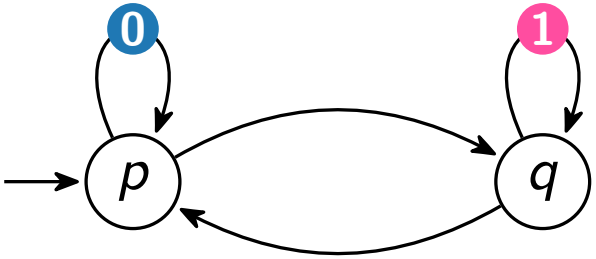


$$\underbrace{(\mathrm{Inf}(0) \wedge \mathrm{Fin}(2))}_{\text{Rabin pair } p_0} \vee \underbrace{(\mathrm{Inf}(3) \wedge \mathrm{Fin}(5))}_{\text{Rabin pair } p_1} \vee \underbrace{\mathrm{Inf}(4)}_{\text{Rabin pair } p_2}$$



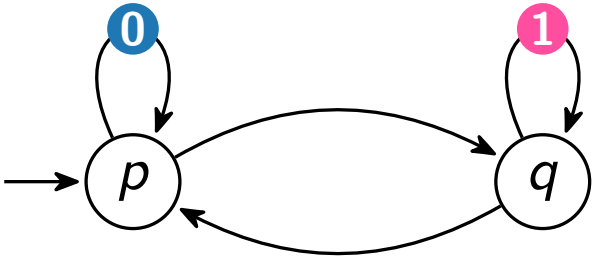
Improvements

Degeneralization

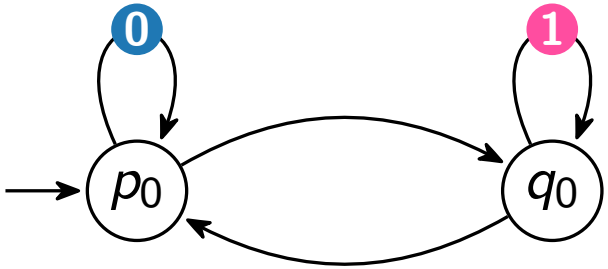


$\mathsf{Inf}(0) \wedge \mathsf{Inf}(1)$

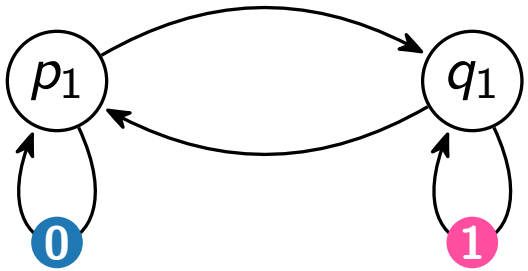
Degeneralization



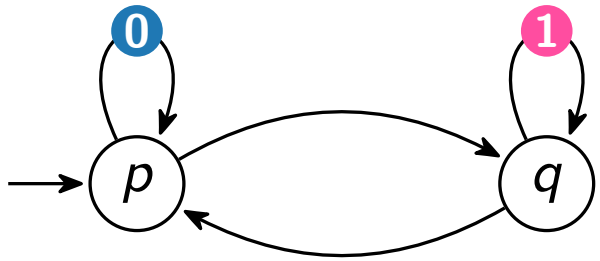
$\mathsf{Inf}(0) \wedge \mathsf{Inf}(1)$



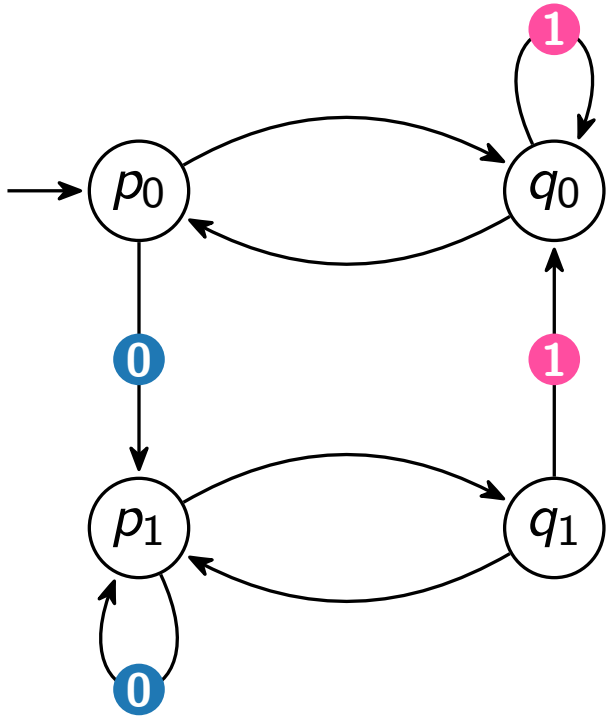
$\mathsf{Inf}(0) \wedge \mathsf{Inf}(1)$



Degeneralization

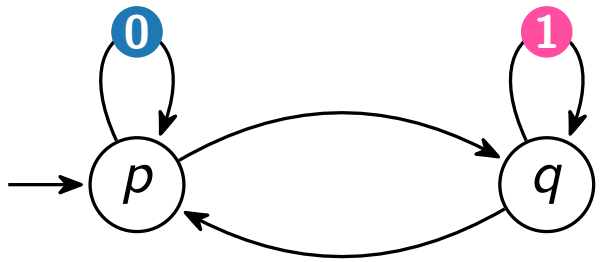


$\mathsf{Inf}(0) \wedge \mathsf{Inf}(1)$

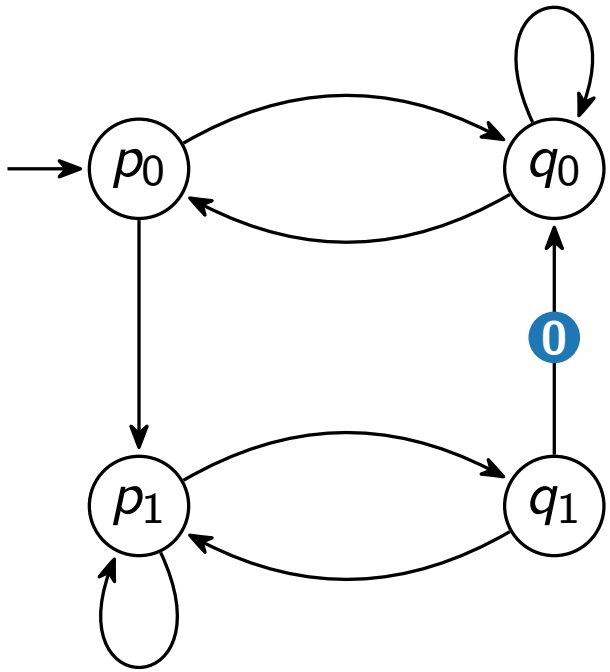


$\mathsf{Inf}(0) \wedge \mathsf{Inf}(1)$

Degeneralization

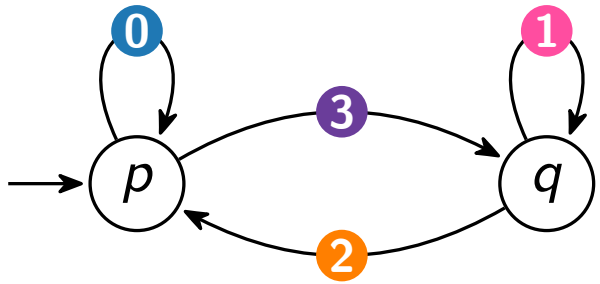


$\mathsf{Inf}(0) \wedge \mathsf{Inf}(1)$

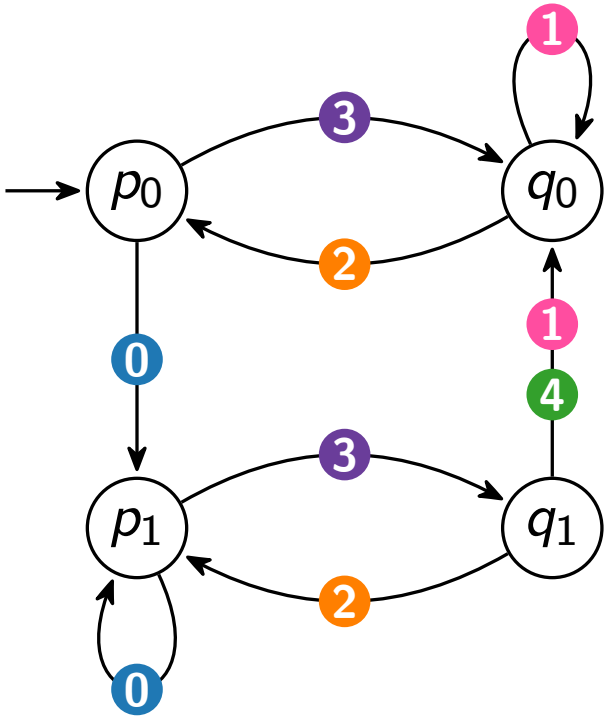


$\mathsf{Inf}(0)$

Partial degeneralization

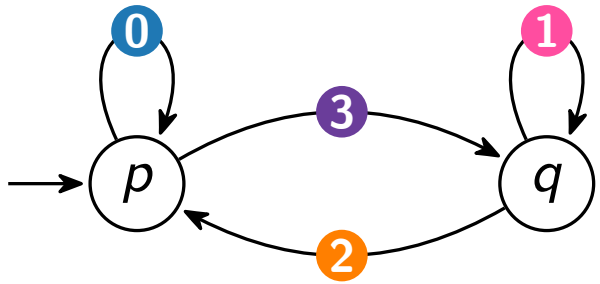


$$\mathrm{Inf}(2) \vee (\mathrm{Fin}(3) \wedge \mathrm{Inf}(0) \wedge \mathrm{Inf}(1))$$

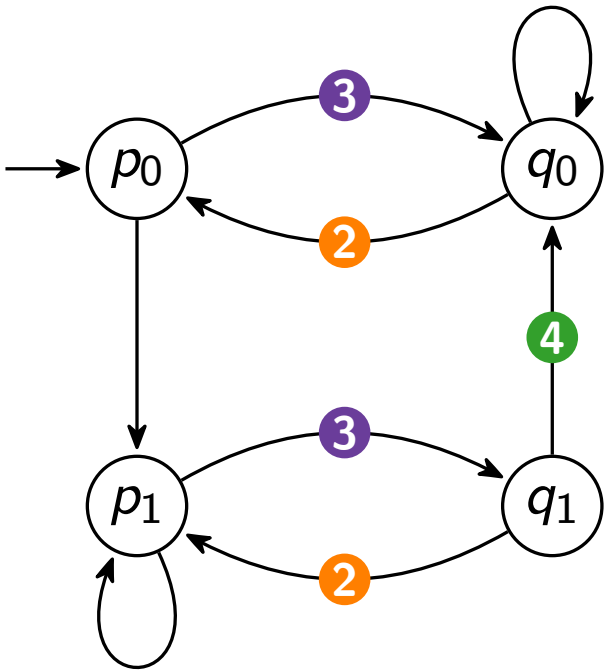


$$\mathrm{Inf}(2) \vee (\mathrm{Fin}(3) \wedge \mathrm{Inf}(4))$$

Partial degeneralization

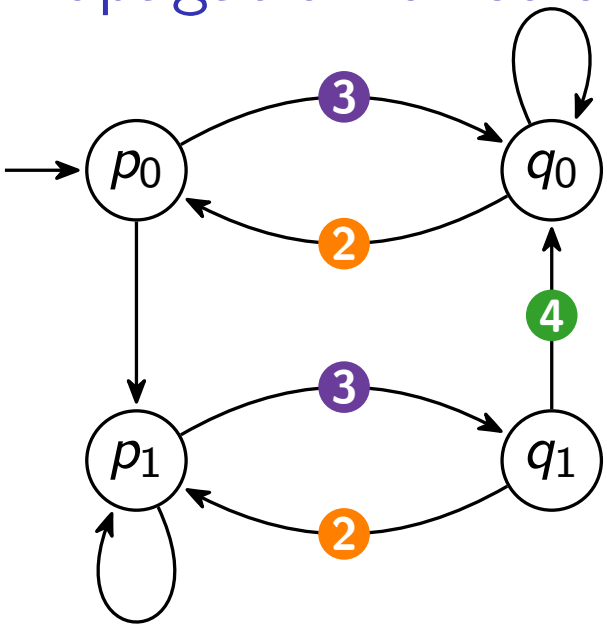


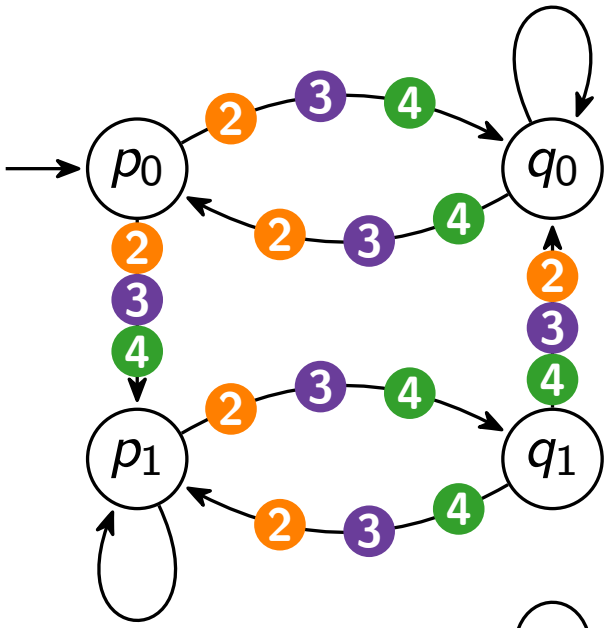
$$\mathrm{Inf}(2) \vee (\mathrm{Fin}(3) \wedge \mathrm{Inf}(0) \wedge \mathrm{Inf}(1))$$



$$\mathrm{Inf}(2) \vee (\mathrm{Fin}(3) \wedge \mathrm{Inf}(4))$$

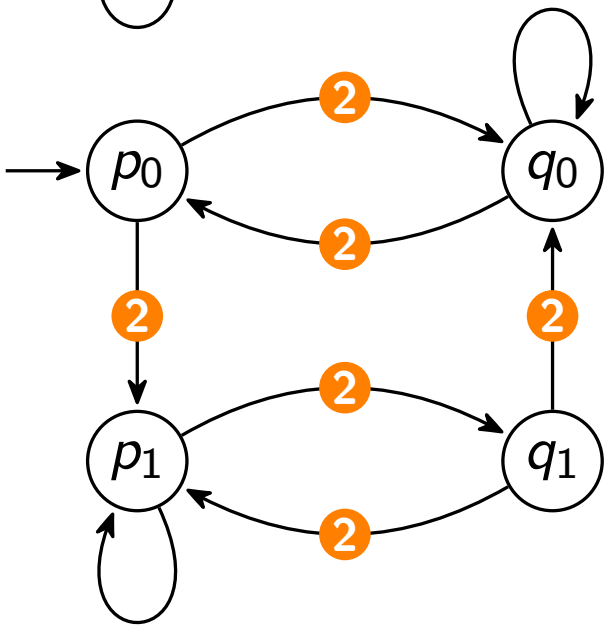
Propagation of colors





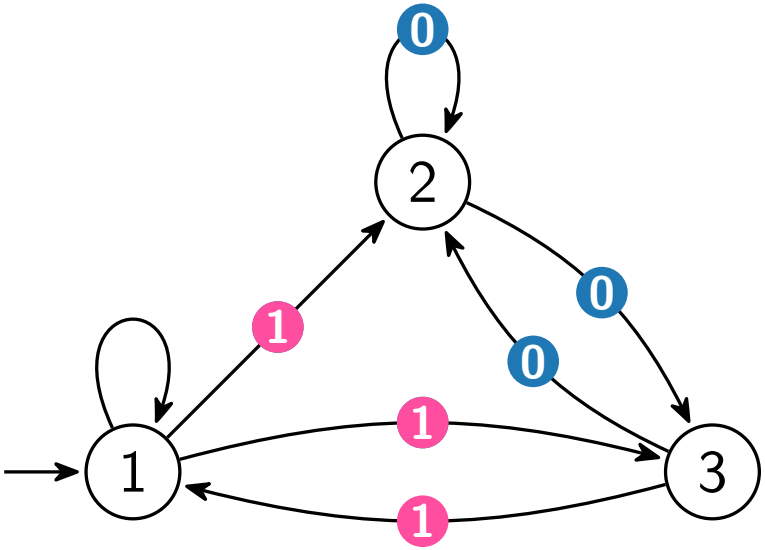
$$\mathrm{Inf}(2) \quad \vee \quad (\mathrm{Fin}(3) \quad \wedge \quad \mathrm{Inf}(4))$$

A color common to incoming transitions (excepted loops) can be added to outgoing transitions and vice versa

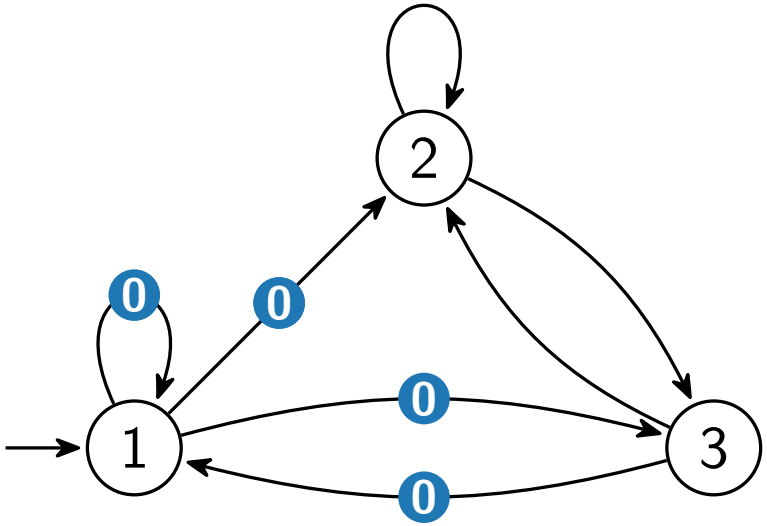


$$\mathrm{Inf}(2)$$

Büchi-type



$\mathsf{Inf}(1) \vee \mathsf{Fin}(0)$



$\mathsf{Inf}(0)$

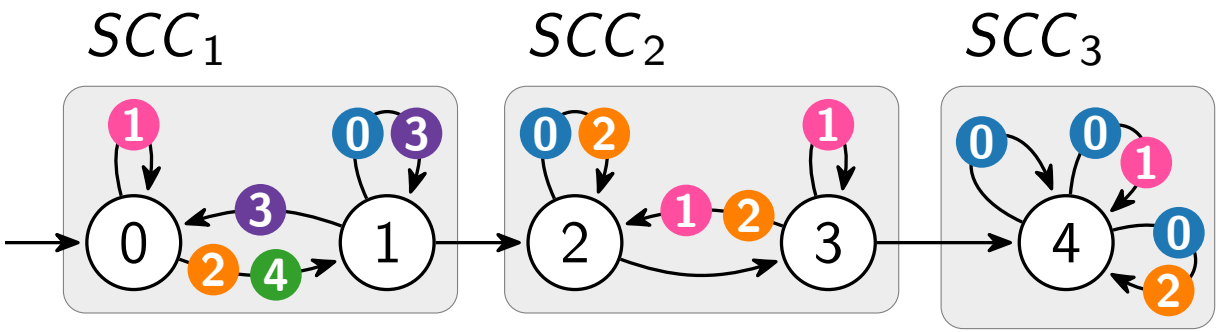
General algorithm

Worst case

From an automaton with n states, k colors, ℓ Rabin/Streett pairs, we have:

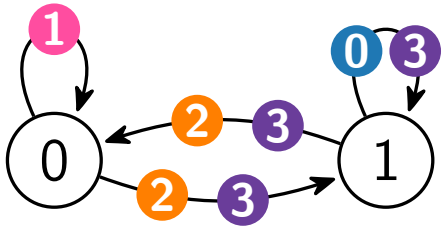
- CAR: $n \cdot k!$ states, $2k + 1$ colors
- IAR: $n \cdot \ell!$ states, $2\ell + 1$ colors
- Partial degeneralization: $k \cdot n$ states, $k + 1$ colors
- Büchi-type: n states, 1 color if possible

Example



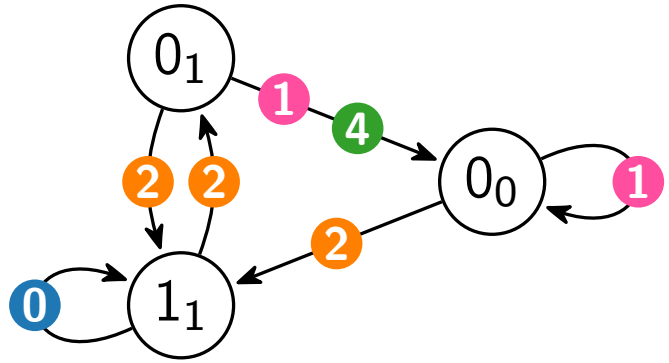
$$(\mathsf{Inf}(2) \wedge \mathsf{Fin}(1)) \vee ((\mathsf{Inf}(4) \vee \mathsf{Inf}(1)) \wedge \mathsf{Fin}(0) \wedge (\mathsf{Fin}(1) \vee \mathsf{Fin}(3)))$$

S+P →



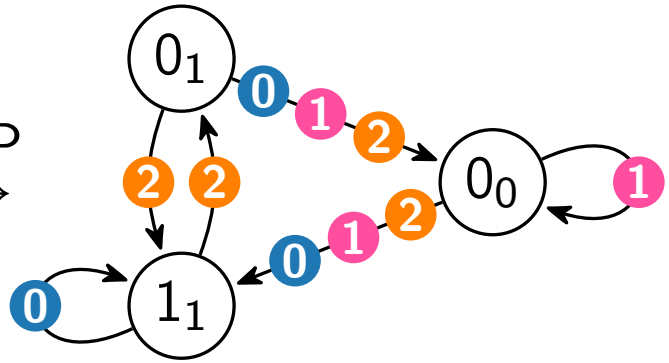
$$(\mathrm{Inf}(2) \wedge \mathrm{Fin}(1)) \vee ((\mathrm{Inf}(2) \vee \mathrm{Inf}(1)) \wedge \mathrm{Fin}(0) \wedge (\mathrm{Fin}(1) \vee \mathrm{Fin}(3)))$$

$\mathrm{PD}_{\{1,3\}}$ →



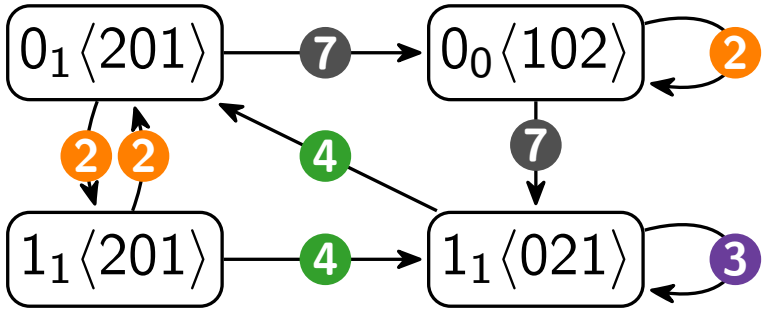
$$(\mathrm{Inf}(2) \wedge \mathrm{Fin}(1)) \vee ((\mathrm{Inf}(2) \vee \mathrm{Inf}(1)) \wedge \mathrm{Fin}(0) \wedge \mathrm{Fin}(4))$$

S+P →

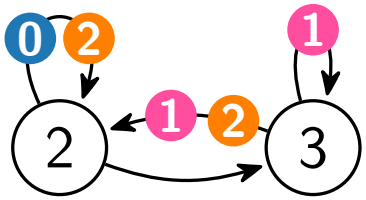


$$(\mathrm{Inf}(2) \wedge \mathrm{Fin}(1)) \vee ((\mathrm{Inf}(2) \vee \mathrm{Inf}(1)) \wedge \mathrm{Fin}(0))$$

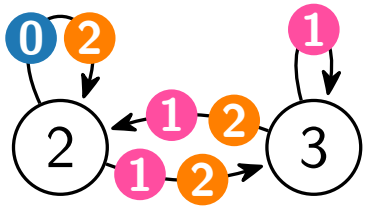
CAR →



parity max even



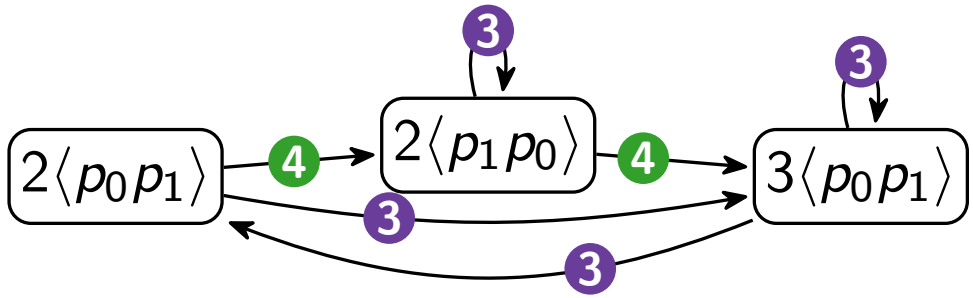
S+P $\longrightarrow$



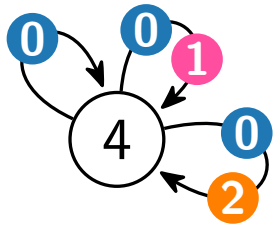
$$(\mathrm{Inf}(2) \wedge \mathrm{Fin}(1)) \vee ((\mathrm{Inf}(4) \vee \mathrm{Inf}(1)) \wedge \mathrm{Fin}(0) \wedge (\mathrm{Fin}(1) \vee \mathrm{Fin}(3)))$$

$$\underbrace{(\mathrm{Inf}(2) \wedge \mathrm{Fin}(1))}_{\text{Rabin pair } p_0} \vee \underbrace{(\mathrm{Inf}(1) \wedge \mathrm{Fin}(0))}_{\text{Rabin pair } p_1}$$

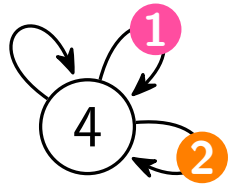
IAR $\longrightarrow$



parity max odd



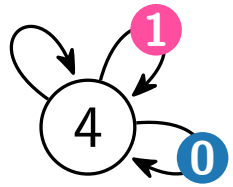
S+P $\longrightarrow$



$(\mathsf{Inf}(2) \wedge \mathsf{Fin}(1)) \vee$
$((\mathsf{Inf}(4) \vee \mathsf{Inf}(1)) \wedge$
$\mathsf{Fin}(0) \wedge (\mathsf{Fin}(1) \vee \mathsf{Fin}(3)))$

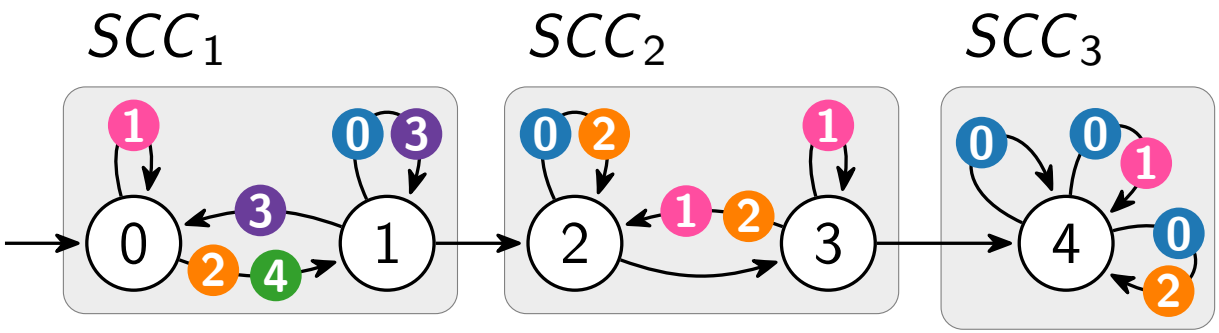
$\mathsf{Inf}(2) \wedge \mathsf{Fin}(1)$

renumber $\longrightarrow$



$\mathsf{Fin}(1) \wedge \mathsf{Inf}(0)$

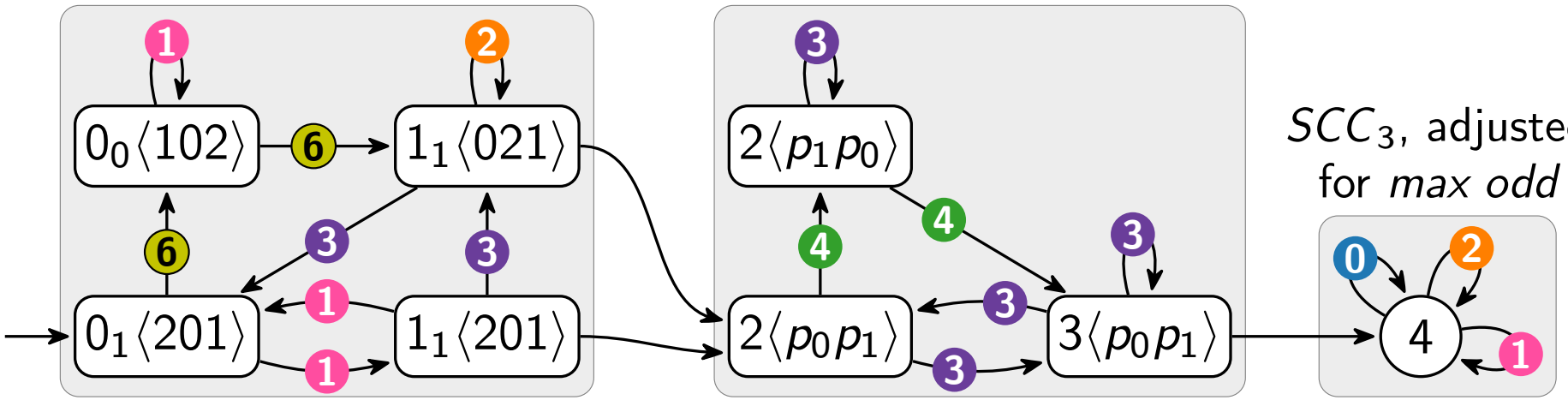
parity max even



$$(\mathrm{Inf}(2) \wedge \mathrm{Fin}(1)) \vee ((\mathrm{Inf}(4) \vee \mathrm{Inf}(1)) \wedge \mathrm{Fin}(0) \wedge (\mathrm{Fin}(1) \vee \mathrm{Fin}(3)))$$

CAR of SCC_1, adjusted for *max odd*

IAR of SCC_2



SCC_3, adjusted for *max odd*

$$\mathrm{Fin}(6) \wedge (\mathrm{Inf}(5) \vee (\mathrm{Fin}(4) \wedge (\mathrm{Inf}(3) \vee (\mathrm{Fin}(2) \wedge (\mathrm{Inf}(1) \vee \mathrm{Fin}(0))))))$$

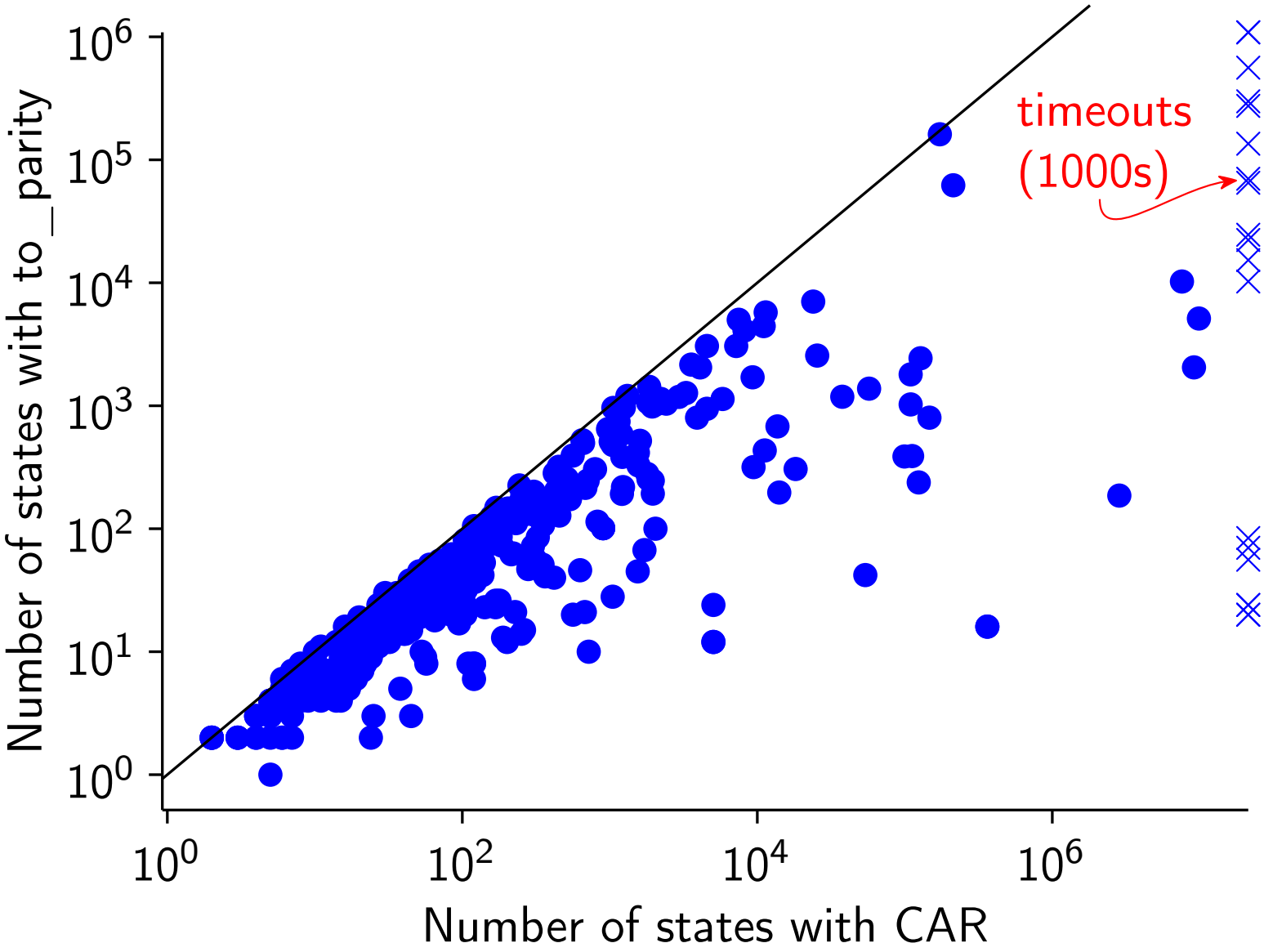
(parity max odd)

Datas

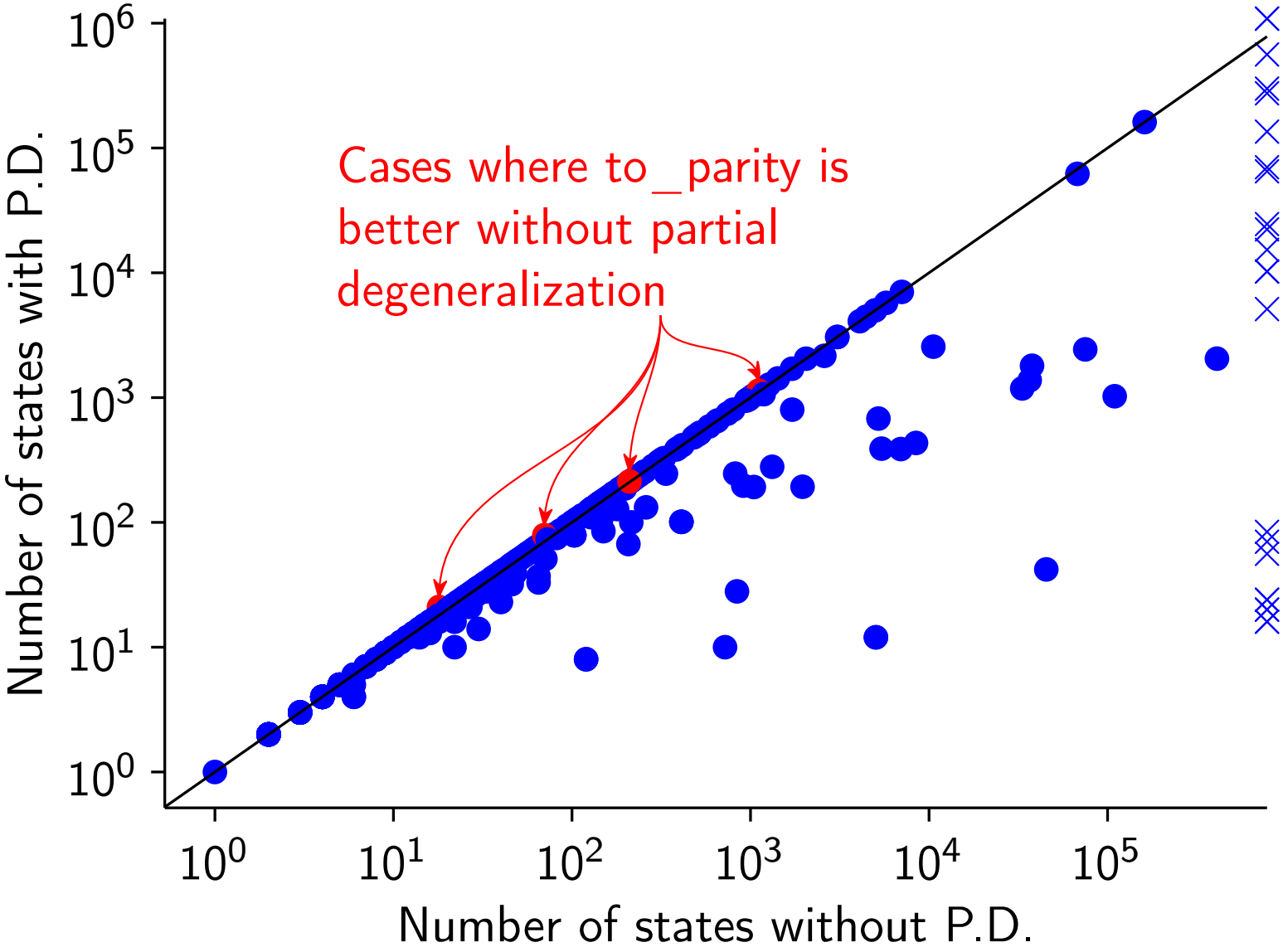
`randaut` (Spot)

SYNTCOMP20 + `ltl2tgba` (Spot)

CAR vs to_parity



Effect of partial degeneralization



Effect of optimizations

configuration	amean	gmean
all	48.71	14.43
unoptimized CAR	5375.02	45.16

Effect of optimizations

configuration	amean	gmean
all	48.71	14.43
all − partial degen	2165.50	20.20
unoptimized CAR	5375.02	45.16

Effect of optimizations

configuration	amean	gmean
all	48.71	14.43
all − propagate colors	55.69	16.91
all − partial degen	2165.50	20.20
unoptimized CAR	5375.02	45.16

Effect of optimizations

configuration	amean	gmean
all	48.71	14.43
all − hist. reuse	51.01	15.18
all − reuse latest	51.05	15.29
all − propagate colors	55.69	16.91
all − partial degen	2165.50	20.20
unoptimized CAR	5375.02	45.16

Effect of optimizations

configuration	amean	gmean
all	48.71	14.43
all − simplify acc	49.32	15.07
all − hist. reuse	51.01	15.18
all − reuse latest	51.05	15.29
all − propagate colors	55.69	16.91
all − partial degen	2165.50	20.20
unoptimized CAR	5375.02	45.16

Effect of optimizations

configuration	amean	gmean
all	48.71	14.43
all − parity prefix	48.97	14.54
all − simplify acc	49.32	15.07
all − hist. reuse	51.01	15.18
all − reuse latest	51.05	15.29
all − propagate colors	55.69	16.91
all − partial degen	2165.50	20.20
unoptimized CAR	5375.02	45.16

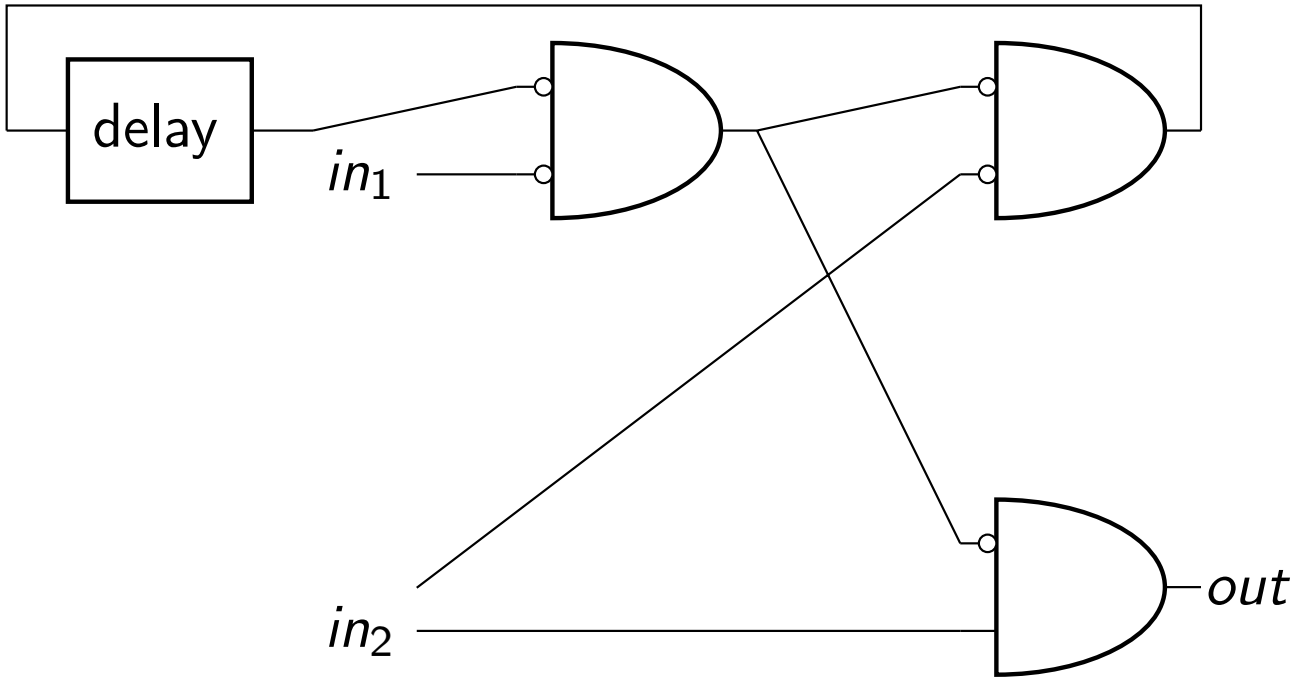
Effect of optimizations

configuration	amean	gmean
all	48.71	14.43
all – Rabin to Büchi	48.72	14.45
all – parity prefix	48.97	14.54
all – simplify acc	49.32	15.07
all – hist. reuse	51.01	15.18
all – reuse latest	51.05	15.29
all – propagate colors	55.69	16.91
all – partial degen	2165.50	20.20
unoptimized CAR	5375.02	45.16

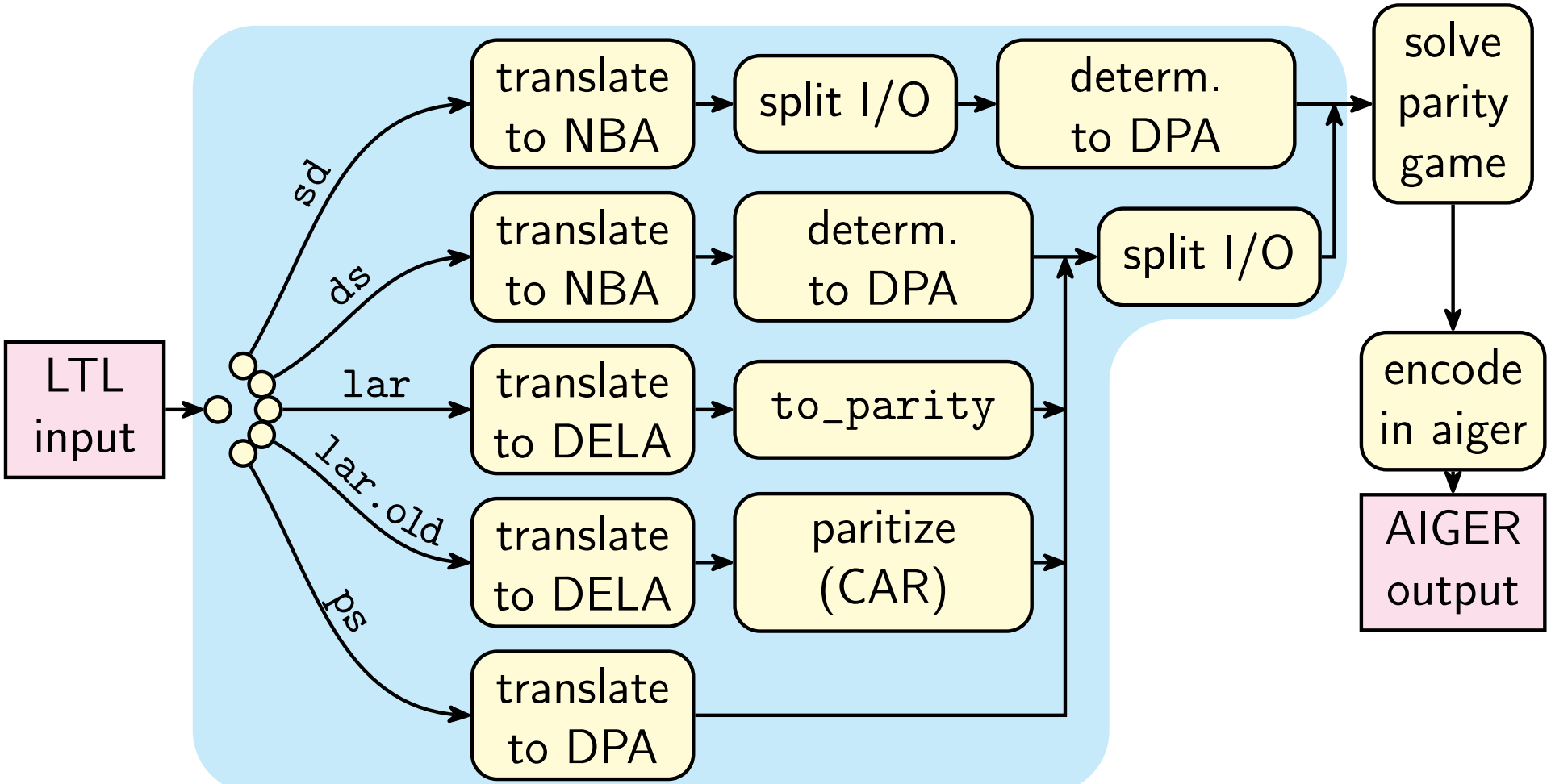
Reactive synthesis from LTL specifications

LTL Synthesis

$$(GFin_1 \wedge GFin_2) \leftrightarrow GFout$$



ltlsynt


LTL input
sd
ds
lar
lar.old
ps
translate to NBA
translate to NBA
translate to DELA
translate to DELA
translate to DPA
split I/O
determ. to DPA
determ. to DPA
to_parity
paritize (CAR)
split I/O
solve parity game
encode in aiger
AIGER output

SYNTCOMP

Compare tools (`Strix`, `ltlsynt`)

SYNTCOMP

Compare tools (`Strix`, `ltlsynt`)

2 categories:

- Synthesis (sequential)
- Realizability (sequential)

SYNTCOMP

Compare tools (`Strix`, `ltlsynt`)

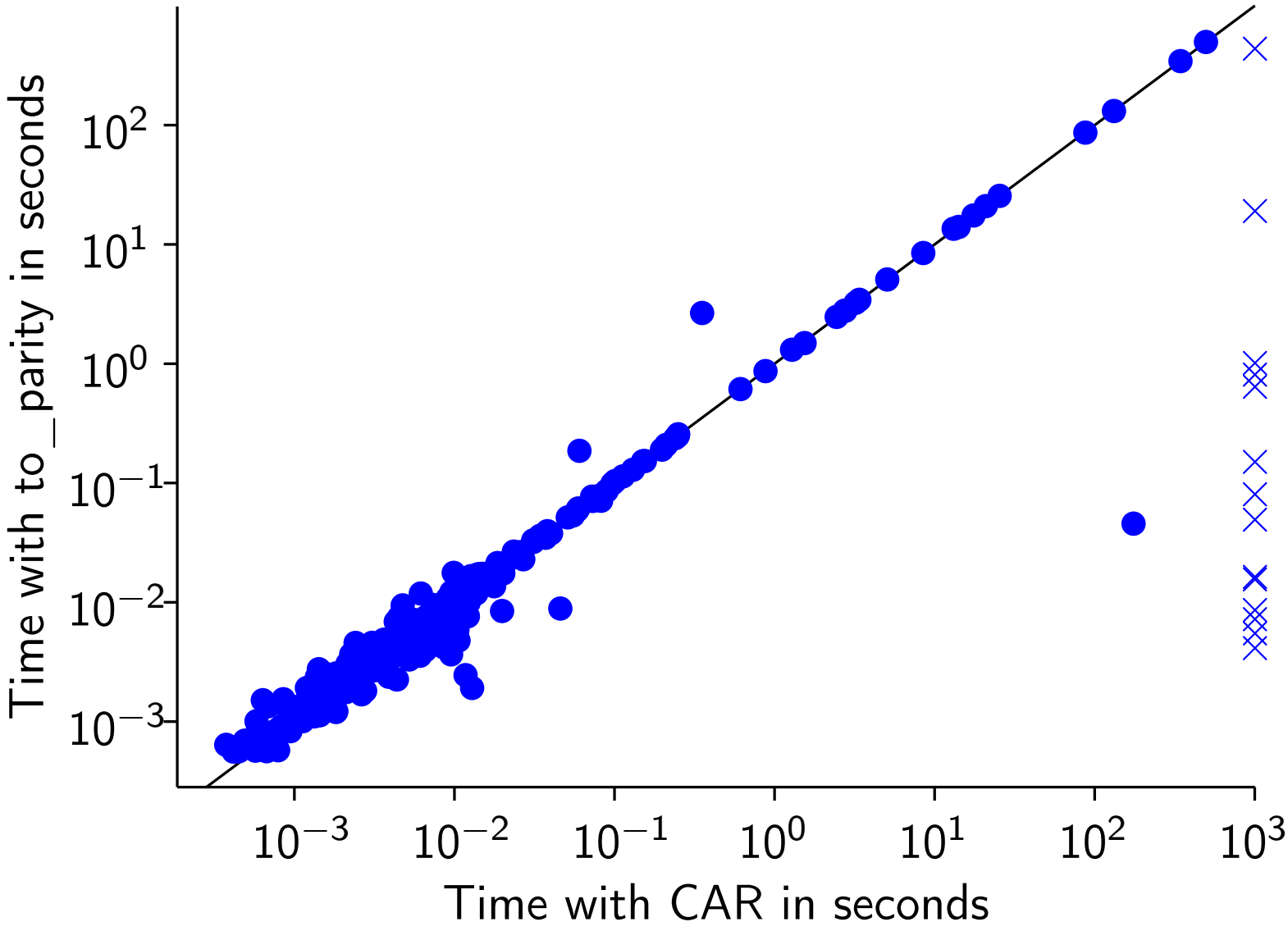
2 categories:

- Synthesis (sequential)
- Realizability (sequential)

2 criteria:

- Number of instances
- Quality

Results


Time with to_parity in seconds
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Time with CAR in seconds

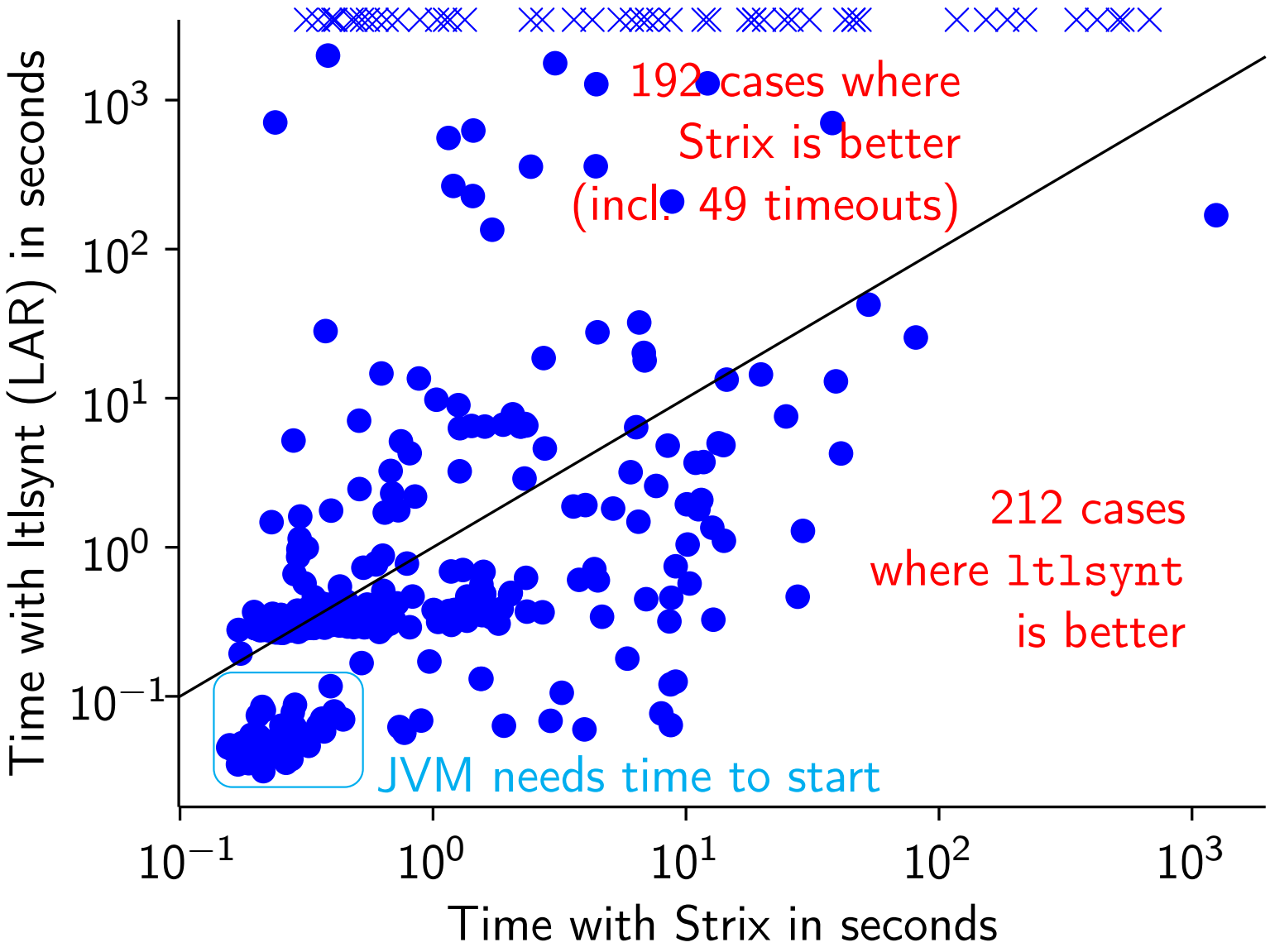
Results

Tool		ltlsynt		
Configuration		lar	ds	sd
Number of solved instances		355	341	341
Quality		395.15	360.10	350.16
Average time of the 334 common cases	arithm.	25.088	37.422	21.290
	géom.	0.473	0.565	0.512

Results

Tool		ltlsynt			Strix
Configuration		lar	ds	sd	bfs
Number of solved instances		355	341	341	404
Quality		395.15	360.10	350.16	599.59
Average time of the	arithm.	25.088	37.422	21.290	2.566
334 common cases	geom.	0.473	0.565	0.512	0.692

SYNTCOMP


192 cases where
Strix is better
(incl. 49 timeouts)
212 cases
where ltlsynt
is better
JVM needs time to start
Time with ltlsynt (LAR) in seconds
Time with Strix in seconds
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

Conclusion

Conclusion

Contributions

Applied optimizations

Conclusion

Contributions

Applied optimizations

Used other algorithms as preprocessing

Conclusion

Contributions

Applied optimizations

Used other algorithms as preprocessing

Implemented in Spot 2.9

Conclusion

Contributions

- Applied optimizations
- Used other algorithms as preprocessing
- Implemented in Spot 2.9
- Showed that our implementation is better than CAR

Conclusion

Contributions

- Applied optimizations
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- Applied our work to synthesis

Conclusion

Contributions

- Applied optimizations
- Used other algorithms as preprocessing
- Implemented in Spot 2.9
- Showed that our implementation is better than CAR
- Applied our work to synthesis
- Compared ltlsynt to Strix during the SYNTCOMP

Conclusion

Future work

Comparison with use of generalized Rabin automata (avoid CAR)

Conclusion

Future work

Comparison with use of generalized Rabin automata (avoid CAR)

Search a heuristic for partial degeneralization

Conclusion

Future work

Comparison with use of generalized Rabin automata (avoid CAR)

Search a heuristic for partial degeneralization

Try to remove colors by searching accepting and rejecting transitions