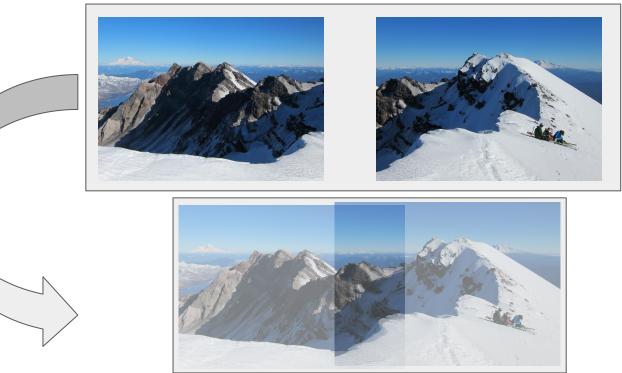
# MLRF Lecture 02 J. Chazalon, LRDE/EPITA, 2021

# Local feature detectors Lecture 02 part 04

How are panorama pictures created from multiple pictures?

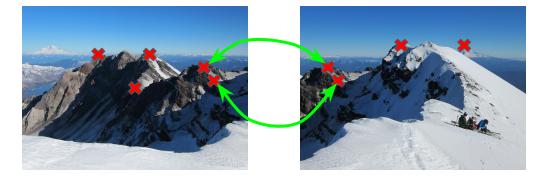


How are panorama pictures created from multiple pictures?



1. Detect small parts invariant under viewpoint change: "Keypoints"

How are panorama pictures created from multiple pictures?



- 1. Detect small parts invariant under viewpoint change: keypoints
- 2. Find pairs of matching keypoints using a **<u>description</u>** of their neighborhood

How are panorama pictures created from multiple pictures?



- 1. Detect small parts invariant under viewpoint change: keypoints
- 2. Find pairs of matching keypoints using a **description** of their neighborhood
- 3. Compute the most likely transformation to blend images together

### The need for local feature detectors

While **dense computation** of local feature descriptors is possible (grid of points), this is **rarely used in practice** (lots of computations, lots of useless features).

**Detection =** Find **anchors** to describe a **feature of interest**.

- Edge / line
- Area around a corner / a stable point
- Blob (area of variable size)

A good feature of interest is **stable over the perturbations** our signal will face:

- Translation, rotation, zoom, perspective
- Illumination changes
- Noise, compression

# Some classical detectors

Edge (gradient detectors)

- Sobel
- Canny

Corner

- Harris & Stephens and variants
- FAST
- Laplacian of Gaussian, Difference of Gaussian, Determinant of Hessian

Blob

- MSER

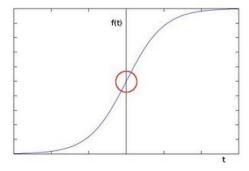
# Edge detectors

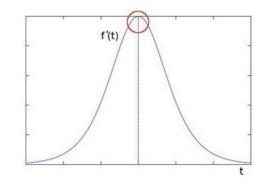
# What's an edge?

Image is a function

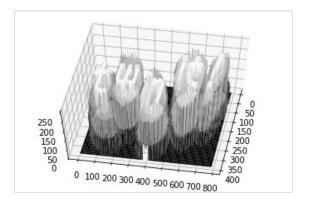
Edges are rapid changes in this function

The derivative of a function exhibits the edges







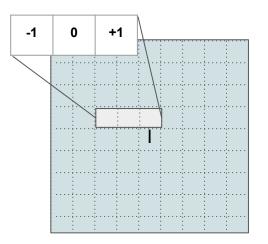


### Image derivatives

Recall: 
$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a-h)}{2h}$$

We don't have an "actual" function, must estimate

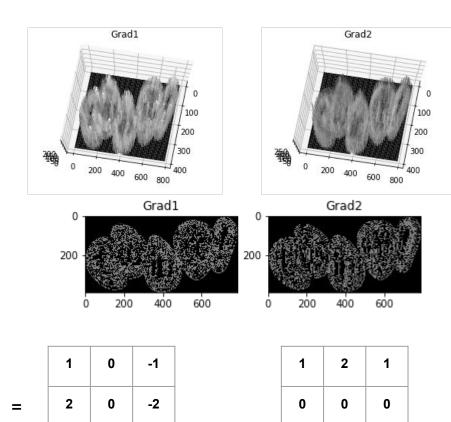
```
Possibility: set h = 1
Apply filter -1 0 +1 to the image
(x gradient)
```

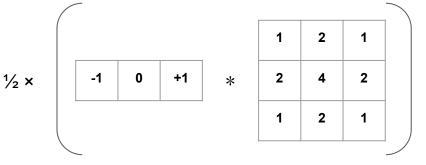


### Image derivatives

We get terribly spiky results, we need to interpolate / smooth. ⇒ Gaussian filter

We get a Sobel filter





Horizontal Sobel

0

-1

1

Vertical Sobel

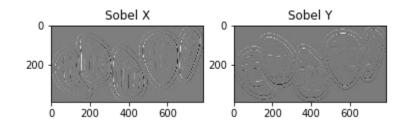
12

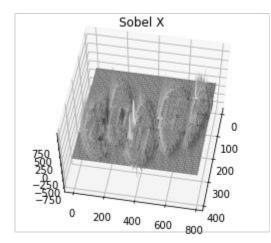
-1

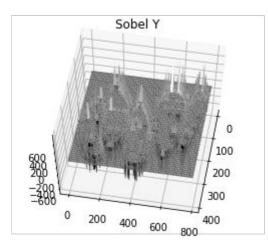
-2

-1

### Sobel filter

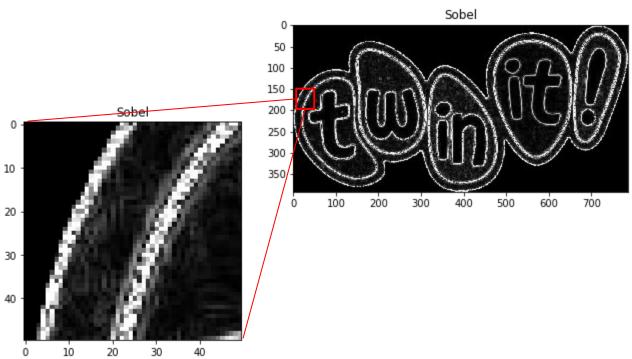






### Gradient magnitude with Sobel

### $sqrt(Sobel_x^2 + Sobel_y^2)$



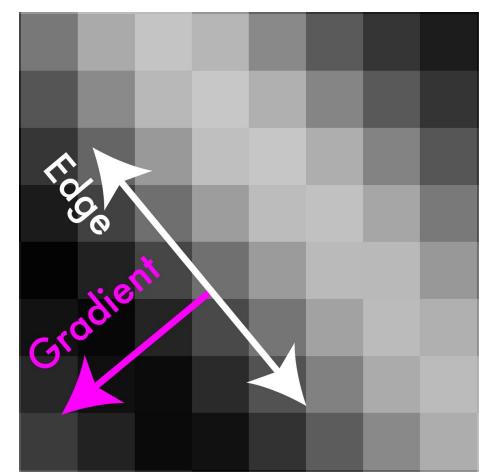
# Canny edge detection

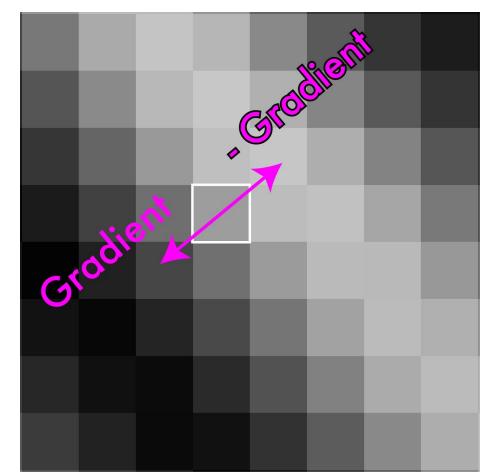
Extract real lines!

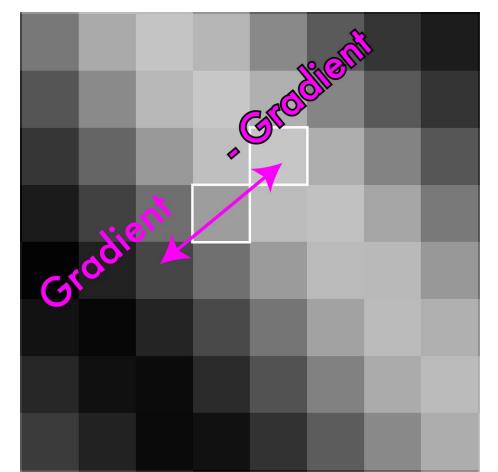
Algorithm:

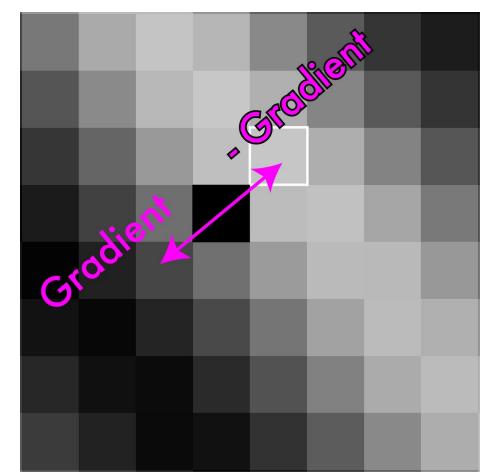
Sobel operator

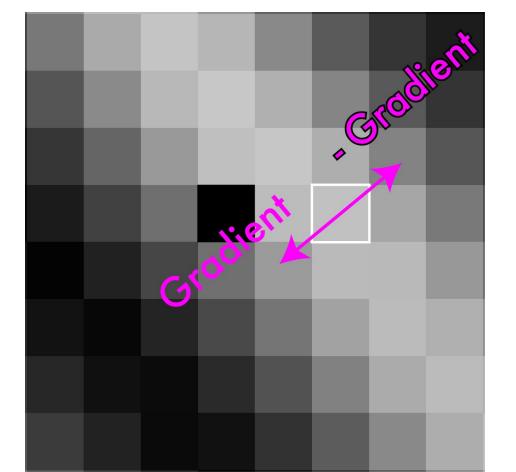
- Smooth image (only want "real" edges, not noise)
- Calculate gradient direction and magnitude
- Non-maximum suppression perpendicular to edge
- Threshold into strong, weak, no edge
- Keep only weak pixels connected to strong ones

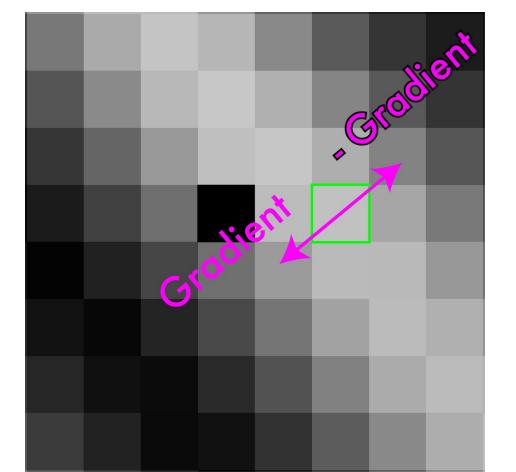


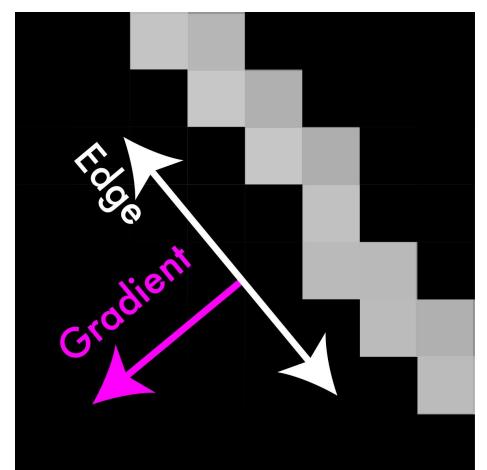


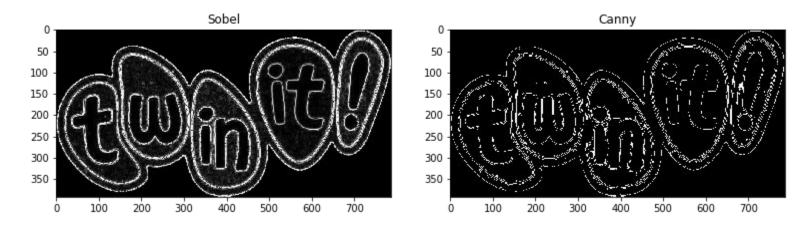


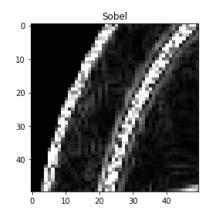


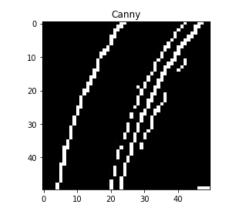












23

# **Canny: finalization**

#### **Threshold edges**

- Still some noise
- Only want strong edges
- 2 thresholds, 3 cases
  - R > T: strong edge
  - R < T but R > t: weak edge
  - R < t: no edge
- Why two thresholds?

#### Connect weak edges to strong edges

- Strong edges are edges!
- Weak edges are edges
   iff they connect to strong
- Look in some neighborhood (usually 8 closest)

# Corner detectors Introduction, Harris detector

# Good features

Reminder:

Good features are unique!

- Can find the "same" feature easily
- Not mistaken for "different" features

Good features are robust under perturbation

- Can detect them under translation, rotation...
- Intensity shift...
- Noise...

How close are two patches?

- Sum squared difference
- Images I, J
- $\Sigma_{\mathbf{x},y} (I(\mathbf{x},y) J(\mathbf{x},y))^2$

Say we are stitching a panorama

Want patches in image to match to other image

Need to only match one spot





### Sky? Bad!

- Very little variation
- Could match any other sky

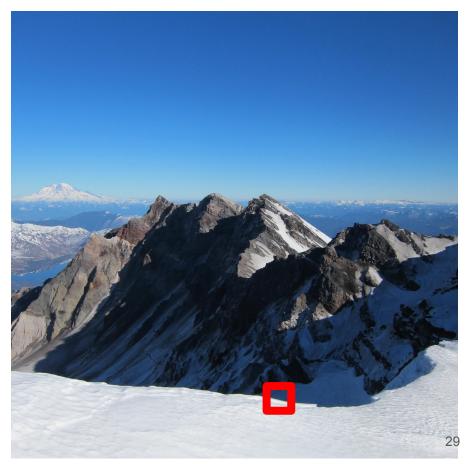


### Sky? Bad!

- Very little variation
- Could match any other sky

### Edge? OK...

- Variation in one direction
- Could match other patches along same edge



### Sky? Bad!

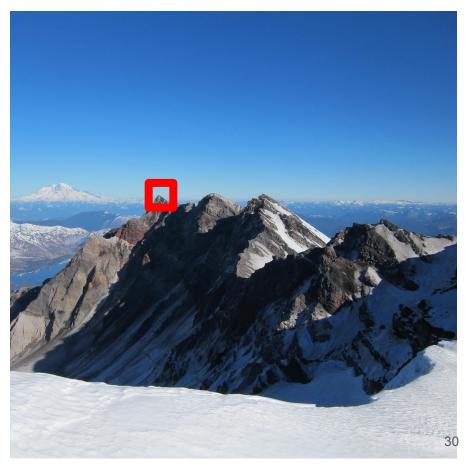
- Very little variation
- Could match any other sky

### Edge? OK...

- Variation in one direction
- Could match other patches along same edge

### Corners? good!

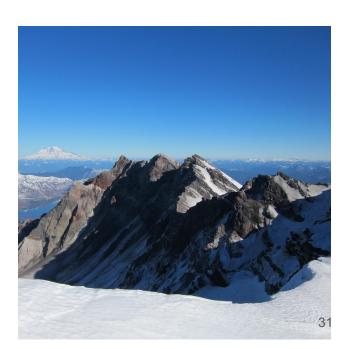
- Only one alignment matches



Want a patch that is unique in the image

Can calculate distance between patch and every other patch, lot of computation





Want a patch that is unique in the image

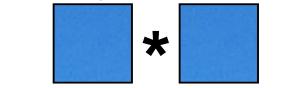
Can calculate distance between patch and every other patch, lot of computation Instead, we could think about auto-correlation:

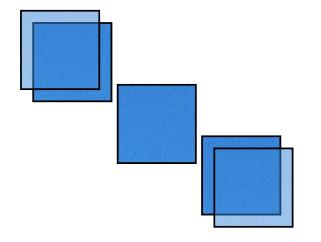
How well does image match shifted version of itself?

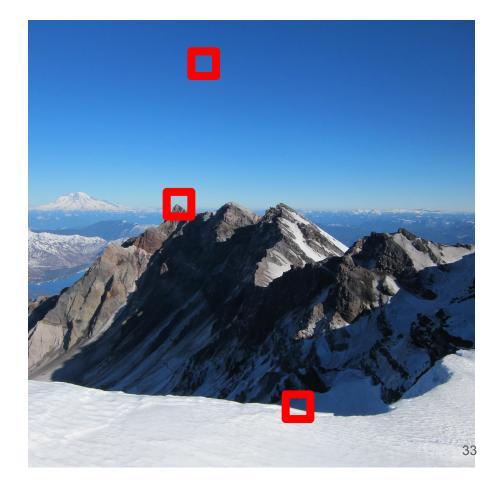
 $\Sigma_{\mathbf{d}}\Sigma_{\mathbf{x},\mathbf{y}} (\mathbf{I}(\mathbf{x},\mathbf{y}) - \mathbf{I}(\mathbf{x}+\mathbf{d}_{\mathbf{x}},\mathbf{y}+\mathbf{d}_{\mathbf{y}}))^2$ 

Measure of self-difference (how am I not myself?)

### Sky: low everywhere

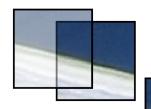






### Edge: low along edge

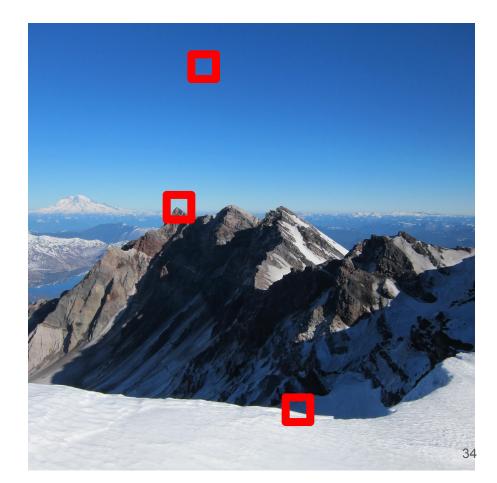






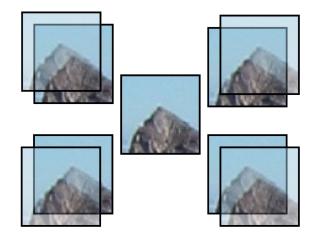


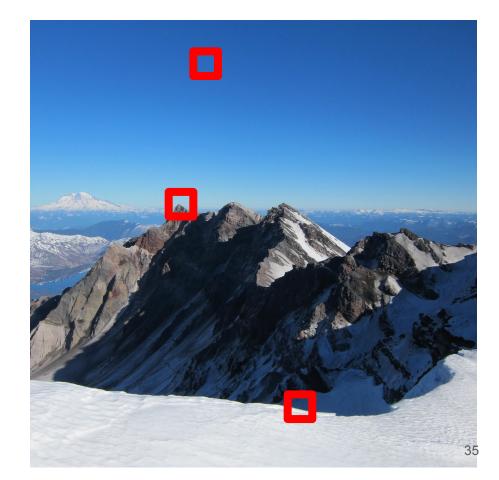




### Corner: mostly high





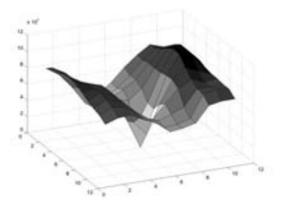


Corner: mostly high

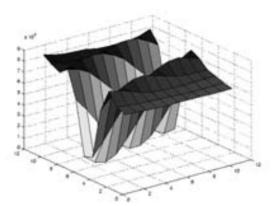
Edge: low along edge

Sky: low everywhere

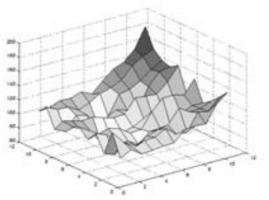












# Self-difference

Naive computation:

$$\Sigma_{d}\Sigma_{x,y}$$
 (I(x,y) - I(x+d\_x,y+d\_y))<sup>2</sup>



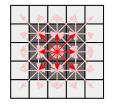
$$(I(x,y) - I(x+\mathbf{d}_x,y+\mathbf{d}_y))^2$$

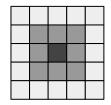
In practice we pool the previous indicator function over a small region (u,v) and we use a window w(u,v) to weight the contribution of each displacement to the global sum.

$$S(x, y) = \sum_{u} \sum_{v} w(u, v) \left( I(x + u + d_x, y + v + d_y) - I(x + u, y + v) \right)^2$$



 $(I(x,y) - I(x+\mathbf{d}_x,y+\mathbf{d}_y))^2$ 





V

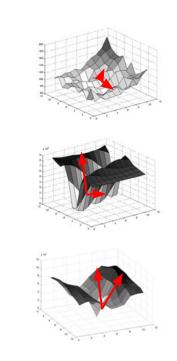
w(u, v)

$$\Sigma_{d}\Sigma_{x,y} (I(x,y) - I(x+d_x,y+d_y))^2$$

Lots of summing => Need an approximation

Look at nearby gradients Ix and Iy

- If gradients are mostly zero, not a lot going on
   ⇒ Low self-difference
- If gradients are mostly in one direction, edge
   ⇒ Still low self-difference
- If gradients are in twoish directions, corner!
   ⇒ High self-difference, good patch!



Trick to precompute the derivatives

$$I(x+d_x, y+d_y)$$

can be approximated by a Taylor expansion  $I(x + d_x, y + d_y) \approx I(x, y) + d_x \frac{\partial I(x, y)}{\partial x} + d_y \frac{\partial I(x, y)}{\partial y} + \cdots$ 

This allows us to "simplify" the original equation,

$$S(x, y) \approx \sum_{u} \sum_{v} w(u, v) \left( d_x \frac{\partial I(x + u, y + v)}{\partial x} + d_y \frac{\partial I(x + u, y + v)}{\partial y} \right)^2$$

and more important making it faster to compute,

thanks to simpler derivatives which can be computed for the whole image.

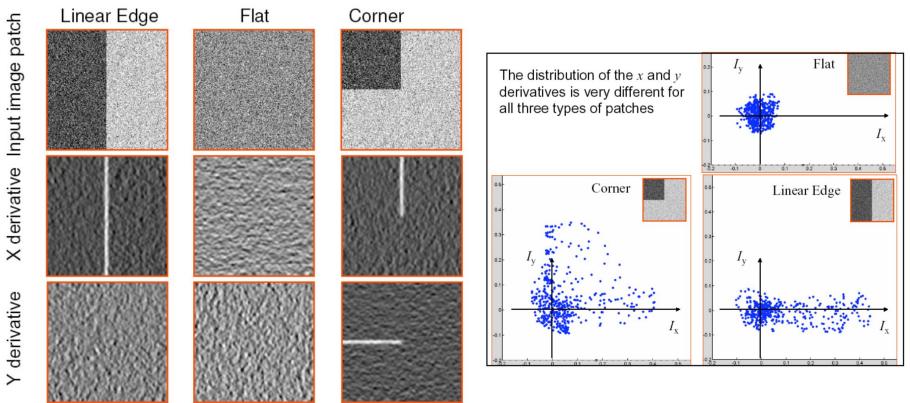
If we develop the equation and write is as usual matrix form, we get:

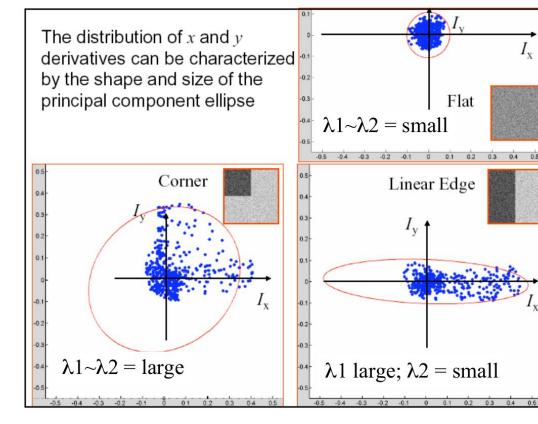
$$S(x, y) \approx \begin{pmatrix} d_x & d_y \end{pmatrix} A(x, y) \begin{pmatrix} d_x \\ d_y \end{pmatrix}$$

where A(x,y) is the structure tensor:

$$A = \sum_{u} \sum_{v} w(u, v) \begin{bmatrix} \frac{\partial I^2(x+u,y+v)}{\partial x} & \frac{\partial I(x+u,y+v)}{\partial x} \frac{\partial I(x+u,y+v)}{\partial y} \\ \frac{\partial I(x+u,y+v)}{\partial x} \frac{\partial I(x+u,y+v)}{\partial y} & \frac{\partial I^2(x+u,y+v)}{\partial y} \end{bmatrix}$$
$$= \begin{bmatrix} \langle I_x^2 \rangle & \langle I_x I_y \rangle \\ \langle I_x I_y \rangle & \langle I_y^2 \rangle \end{bmatrix}$$

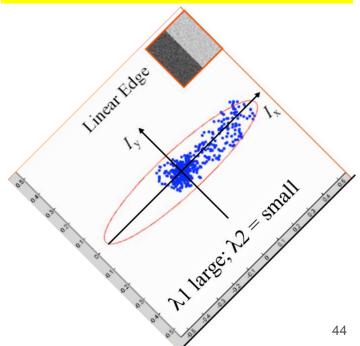
This trick is useful because Ix and Iy can be precomputed very simply.





<u>The need for eigenvalues:</u> If the edge is rotated, so are the values of  $I_x$  and  $I_y$ .

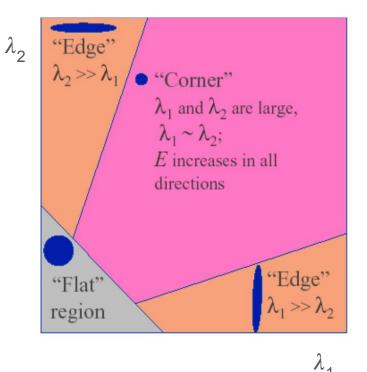
Eigenvalues give us the ellipsis axis len.



A corner is characterized by a large variation of S in all directions of the vector (x y).

Analyse the eigenvalues of A to check whether we have two large variations.

- If  $\lambda_1 \approx 0$  and  $\lambda_2 \approx 0$  then this pixel (x,y) has no features of interest.
- If  $\lambda_1 \approx 0$  and  $\lambda_2$  has some large positive value, then an edge is found.
- If  $\lambda_1$  and  $\lambda_2$  have large positive values, then a corner is found.



To avoid the computation of the eigenvalues, which used to be expensive, Harris and Stephens instead suggest the following function Mc, where  $\kappa$  is a tunable sensitivity parameter:

$$M_c = \lambda_1 \lambda_2 - \kappa (\lambda_1 + \lambda_2)^2 = \det(A) - \kappa \operatorname{trace}^2(A)$$

We will use Noble's trick to remove  $\kappa$ :

approximation

$$M_c' = 2 \frac{\det(A)}{\operatorname{trace}(A) + \epsilon}$$

 $\epsilon$  being a small positive constant.

*A* being a 2x2 matrix, we have the following relations:

- $det(A) = A_{1,1}A_{2,2} A_{2,1}A_{1,2}$
- trace(A)= $A_{1,1}+A_{2,2}$

Using previous definitions, we obtain:

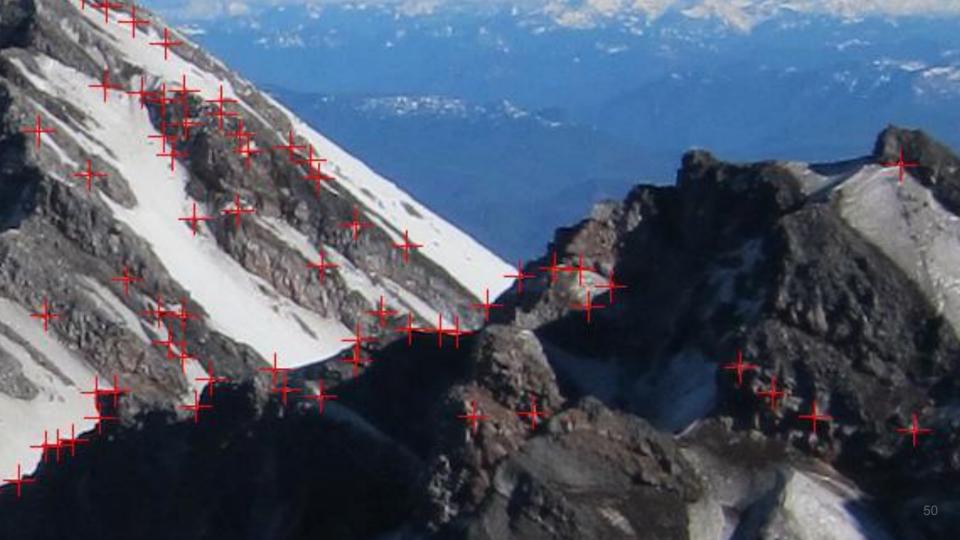
- det(A)= $\langle I^2 x \rangle \langle I^2 y \rangle \langle I x I y \rangle^2$
- trace(A)= $\langle I^2 x \rangle + \langle I^2 y \rangle$

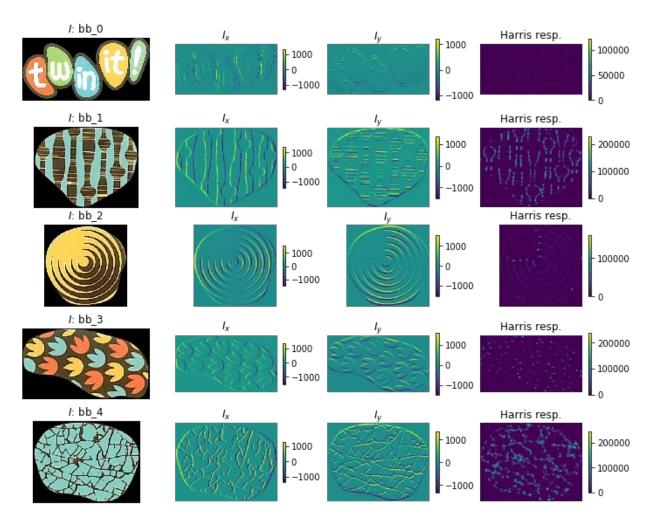
In summary, given an image, we can compute the Harris corner response image by simply computing:

- Ix: I's smoothed (interpolated) partial derivative with respect to x;
- Iy: I's smoothed (interpolated) partial derivative with respect to y;
- $\langle I^2 x \rangle$ : the windowed sum of  $I^2 x$ ;
- $\langle I^2 y \rangle$ : the windowed sum of  $I^2 y$ ;
- $\langle IxIy \rangle$ : the windowed sum of IxIy;
- det(A);
- trace(A);
- $M''_{c} = \det(A) / (\operatorname{trace}(A) + \epsilon).$

Then, we just perform **non-maximal suppression** to keep local maximas.



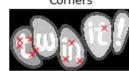






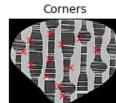
Harris resp.







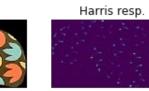
Harris resp.

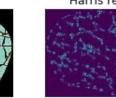


Corners



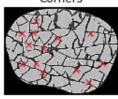
Corners

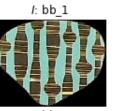






Corners



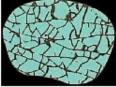






1: bb\_3









# Harris & Stephens Conclusion

### Good features to track aka Shi-Tomasi aka Kanade-Tomasi

Remember the Harris-Stephens trick to avoid computing the eigenvalues?

$$M_{c} = \lambda_{1} \lambda_{2} - \kappa (\lambda_{1} + \lambda_{2})^{2} = \det(A) - \kappa \operatorname{trace}^{2}(A)$$
approximation

Well, nowadays, linear algebra is cheap, so compute the real eigenvalues.

Then filter using  $min(\lambda_1, \lambda_2) > \lambda$ ,  $\lambda$  being a predefined threshold.

You get the Shi-Tomasi variant.

Jianbo Shi and Carlo Tomasi. Good features to track. In Computer Vision and Pattern Recognition, 1994. Proceedings CVPR'94., 1994 IEEE Computer Society Conference on, pages 593–600. IEEE, 1994.

### Build your own edge/corner detector

Hessian matrix with block-wise summing

You just need eigenvalues  $\lambda_1$  and  $\lambda_2$  of the structure tensor

$$A = \sum_{u} \sum_{v} w(u, v) \begin{bmatrix} \frac{\partial I^2(x+u,y+v)}{\partial x} & \frac{\partial I(x+u,y+v)}{\partial x} \frac{\partial I(x+u,y+v)}{\partial y} \\ \frac{\partial I(x+u,y+v)}{\partial x} \frac{\partial I(x+u,y+v)}{\partial y} & \frac{\partial I^2(x+u,y+v)}{\partial y} \end{bmatrix}$$
$$= \begin{bmatrix} \langle I_x^2 \rangle & \langle I_x I_y \rangle \\ \langle I_x I_y \rangle & \langle I_y^2 \rangle \end{bmatrix}$$

dst = cv2.cornerEigenValsAndVecs(src, neighborhood\_size, sobel\_aperture)
dst = cv2.cornerMinEigenVal(src, neighborhood\_size, sobel\_aperture)

# Harris summary

#### Pros

Translation invariant

- $\Rightarrow$  Large gradients in both directions
  - = stable point

#### Cons

Not so fast

 $\Rightarrow$  Avoid to compute all those derivatives

Not scale invariant

 $\Rightarrow$  Detect corners at different *scales* 

Not rotation invariant ⇒ Normalization orientation

# Corner detectors, binary tests FAST

# Features from accelerated segment test (FAST)

Keypoint detector used by ORB (described in next lecture)

#### Segment test:

compare pixel P intensity  $I_p$ with surrounding pixels (circle of 16 pixels)

If *n* contiguous pixels are either

- all darker than  $I_p t$
- all brighter than  $I_p + t$

then **P** is a detected as a corner

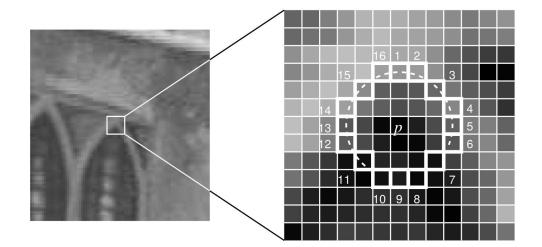


Figure 1. 12 point segment test corner detection in an image patch. The highlighted squares are the pixels used in the corner detection. The pixel at p is the centre of a candidate corner. The arc is indicated by the dashed line passes through 12 contiguous pixels which are brighter than p by more than the threshold.

# Tricks

- 1. **Cascading:** If n = 12 (<sup>3</sup>/<sub>4</sub> of the circle), then many non-corners can be discarded by testing pixels at the 4 compass directions. The full test is only applied to the candidates which passed the first test.
- 2. **Machine learning:** Learn on a dataset which pixels should be tested first to discard a non-corner as quickly as possible.

Learn a decision tree, then compile the decisions as nested if-then rules.

3. How to perform **non-maximal suppression**?

Need to assign a score *V* to each corner.

 $\Rightarrow$  The sum of the absolute difference between the pixels in the contiguous arc and the centre pixel

$$V = \max\left(\sum_{x \in S_{\text{bright}}} |I_{p \to x} - I_p| - t , \sum_{x \in S_{\text{dark}}} |I_p - I_{p \to x}| - t\right)$$

# FAST summary

#### Pros

Very fast

Authors tests:

- 20 times faster than Harris
- 40 times faster than DoG (next slide)

Very robust to transformations (perspective in particular)

#### Cons

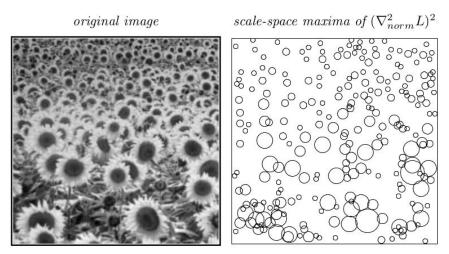
Very sensitive to blur

# Corner detectors at different scales LoG, DoG, DoH

# Laplacian of Gaussian (LoG)

The theoretical, slow way.

If you need to remember only 1 thing: it is a **band-pass filter** – it **detects objects of a certain size**.



T. Lindeberg, "Feature Detection with Automatic Scale Selection," Int. J. of Computer Vision, vol. 30, no. 2, p. 53, 1998.

### Laplacian (plain, not Gaussian here) = second derivative

Second derivative of an image? Like Sobel... with 1 more derivation...

Taylor, again:

$$f(x+h) = f(x) + hf'(x) + \frac{1}{2}h^2 f''(x) + \frac{1}{3!}h^3 f'''(x) + O(h^4)$$
  
+ 
$$\left[f(x-h) = f(x) - hf'(x) + \frac{1}{2}h^2 f''(x) - \frac{1}{3!}h^3 f'''(x) + O(h^4)\right]$$

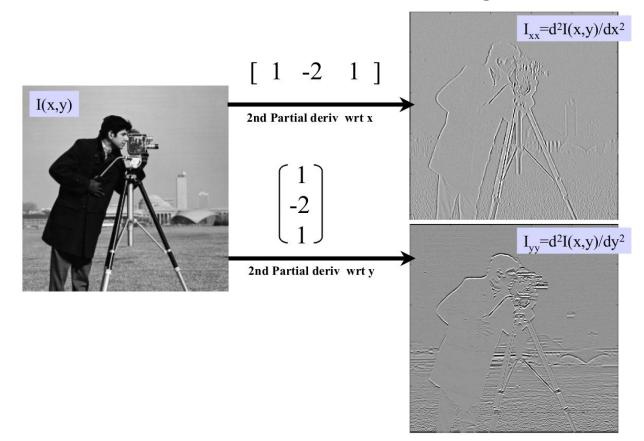
$$f(x+h) + f(x-h) = 2f(x) + h^2 f''(x) + O(h^4)$$

$$f(x-h) - 2f(x) + f(x+h)$$

$$h^2 = f''(x) + O(h^2)$$

New filter:  $I_{xx} = 1 -2 1 * I$ 

### Second partial derivatives of an image



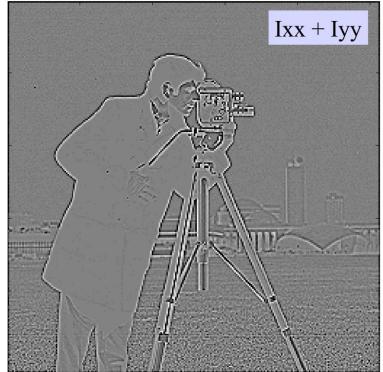
# Laplacian filter $\nabla^2 I(x,y)$

Edge detector, like Sobel but with 2nd derivatives

$$I_{xx} + I_{yy} = \left( \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right) * I$$
$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} * I$$

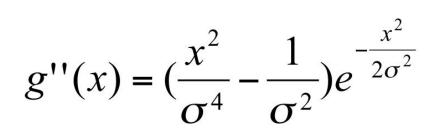


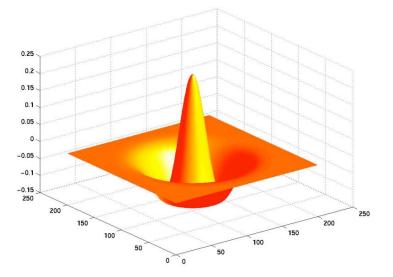
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} * I$$



# Laplacian of Gaussian

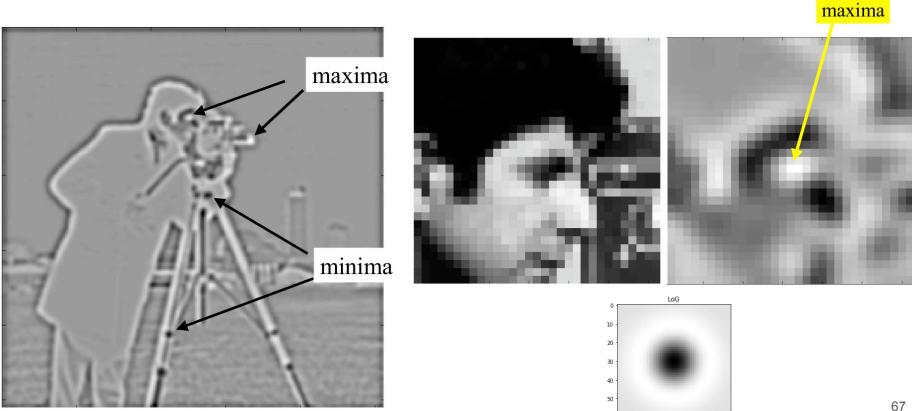
Second derivative of a Gaussian: "Mexican hat"





2D formula = exercise.

### LoG = detector of circular shapes



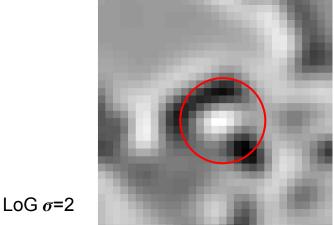
 

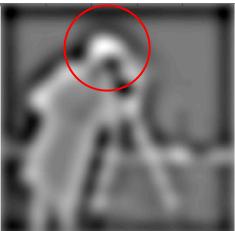
# LoG = detector of circular shapes

LoG filter extrema locates "blobs"

- maxima = dark blobs on light background
- minima = light blobs on dark background

Scale of blob (size ; radius in pixels) is determined by the sigma parameter of the LoG filter.



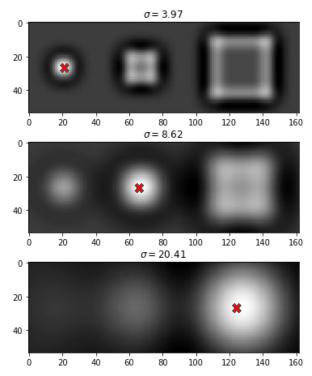


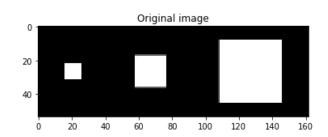
68

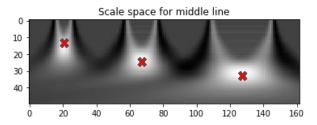
 $LoG \sigma = 10$ 

# Detecting corners / blobs

Build a scale space representation: stack of images (3D) with increasing sigma





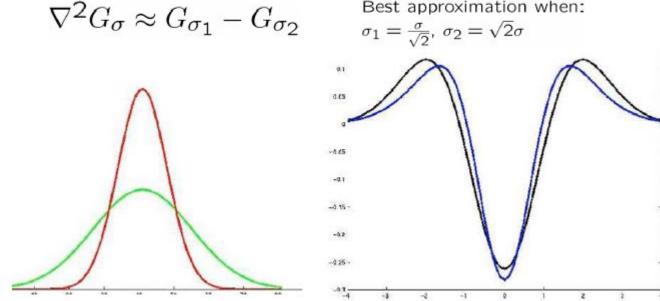


Then find local extremas in the scale space volume.

# Difference of Gaussian (DoG)

Fast approximation of LoG. Used by SIFT (next lecture).

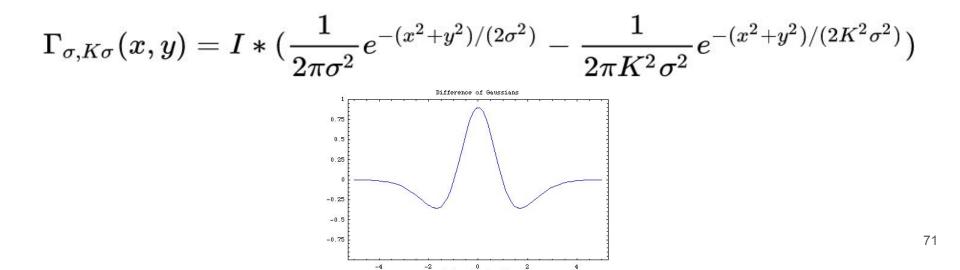
LoG can be approximate by a Difference of two Gaussians (DoG) at different scales. Best approximation when:



### DoG filter

It is a band-pass filter.

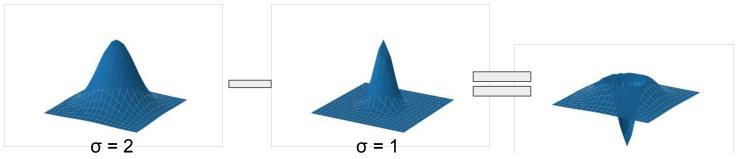
$$\Gamma_{\sigma,K\sigma}(x,y) = I * rac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/(2\sigma^2)} - I * rac{1}{2\pi K^2\sigma^2} e^{-(x^2+y^2)/(2K^2\sigma^2)}$$



# DoG filter

Intuition

- Gaussian (g) is a low pass filter
- Strongly reduce components with frequency f <  $\sigma$
- (g\*l) low frequency components
- I (g\*I) high frequency components
- $g(\sigma_1)^*I g(\sigma_2)^*I \leftarrow$  Components in between these frequencies
- $g(\sigma_1)^* | g(\sigma_2)^* | = [g(\sigma_1) g(\sigma_2)]^* |$



#### DoG computation in practice

Take a image.







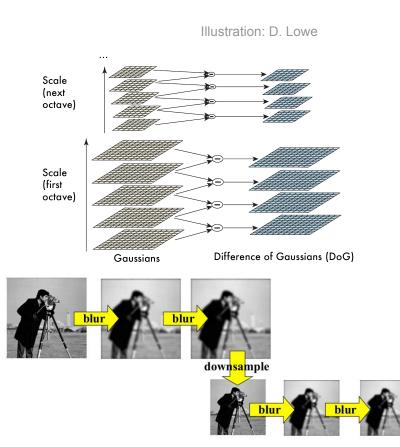


Take the difference.

# DoG scale generation trick

DoG computation: use "octaves"

- "Octave" because frequency doubles/halves between octaves
- If sigma = sqrt(2),
   then 3 levels per octave
- Downsample images for next octave (like double sized kernel)
- Compute the DoG between images



Crowley et.al., "Fast Computation of Characteristic Scale using a Half-Octave Pyramid." Proc International Workshop on Cognitive Vision (CogVis), Zurich, <sup>74</sup> Switzerland, 2002.

### **DoG: Corner selection**

Throw out weak responses and edges

Estimate gradients

- Similar to Harris, look at nearby responses
- Not whole image, only a few points! Faster!
- Throw out weak responses

Find cornery things

- Same deal, structure matrix, use det and trace information (SIFT variant)

D. G. Lowe, "Distinctive image features from scale-invariant keypoints," *International journal of computer vision*, vol. 60, no. 2, pp. 91–110, 2004., see p. 12

$$\frac{\operatorname{Tr}(\mathbf{H})^2}{\operatorname{Det}(\mathbf{H})} < \frac{(r+1)^2}{r} \quad \mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}_{_{75}}$$

# Determinant of Hessian (DoH)

Faster approximation. Used by SURF. Better resistance to perspective

Computes the scale-normalized determinant of the Hessian (strength of the curvature at a given point)

$$\det H_{norm}L = \sigma^2 (L_{xx}L_{yy} - L_{xy}^2)$$

⇒ Precompute *Lxx*, *Lyy*, *Lxy* 

⇒ Blur them with the right sigma while computing **det** *H L*: 3 additions

⇒ normalize: different scales – same value range



 $\nabla^2 L$ 



 $\det \mathcal{H}L$ 

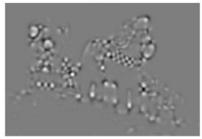
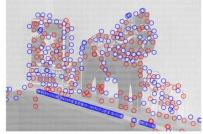
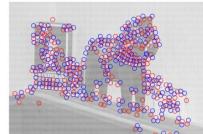


Illustration: T. Lindeberg

local extrema of  $\nabla^2 L$ 

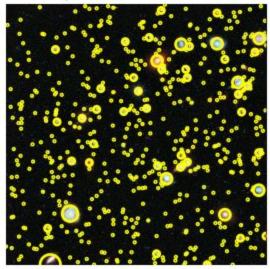


local extrema of  $\det \mathcal{H}L$ 

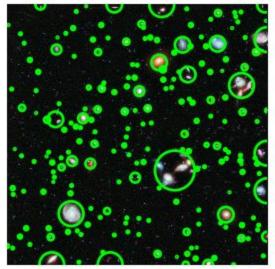


#### LoG vs DoG vs DoH

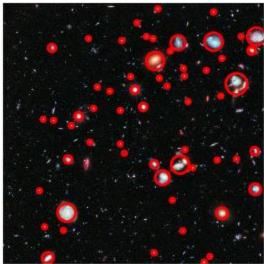
Laplacian of Gaussian



Difference of Gaussian



Determinant of Hessian



# LoG, DoG, DoH summary

#### Pros

Very robust to transformations

- Perspective
- Blur

Adjustable size (scale)

Cons

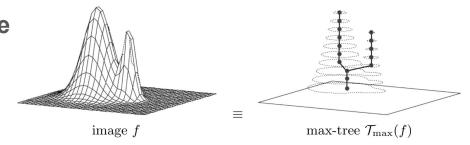
Slow

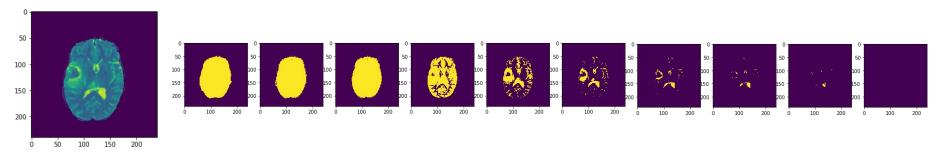
# Blob detectors MSER

## Maximally Stable Extremal Regions (MSER)

Detects regions which are stable over thresholds.

 Create min- & max-tree of the image tree of included components when thresholding the image at each possible level

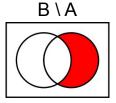


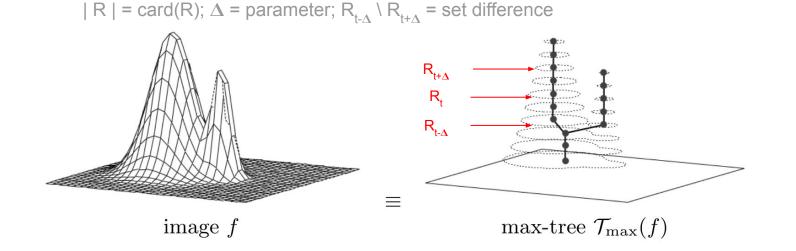


J. Matas, O. Chum, M. Urban, and T. Pajdla, "Robust wide-baseline stereo from maximally stable extremal regions," Image and vision computing, vol. 22, no. 10, pp. 761–767, 2004.

## Maximally Stable Extremal Regions (MSER)

2. Select most stable regions between t- $\Delta$  and t+ $\Delta$ R<sub>t\*</sub> is maximally stable iif q(t) = | R<sub>t- $\Delta$ </sub> \ R<sub>t+ $\Delta$ </sub> | / | R<sub>t</sub> | as local minimum at t\*





## MSER summary

#### Pros

Very robust to transformations

- Affine transformations
- Intensity changes

Quit fast

#### Cons

Does not support blur

# Local feature detectors Conclusion

#### Local feature detectors: Conclusion

Harris Stephens: Can be very stable when combined with DoG

Shi-Tomasi: Assumes affine transformation (avoid it with perspective)

DoG: slow but very robust (perspective, blur, illumination)

DoH: faster than DoG, misses small elements, better with perspective.

FAST: very fast, robust to perspective change (like DoG), but blur quickly kills it

MSER: fast, very stable, good choice when no blur

#### Classification

Feature detector	<u>Edge</u>	<u>Corner</u>	<u>Blob</u>
Canny	Х		
Sobel	Х		
Harris & Stephens / Plessey / Shi–Tomasi	Х	Х	
Shi & Tomasi		Х	
FAST		Х	
Laplacian of Gaussian		Х	Х
Difference of Gaussians		Х	Х
Determinant of Hessian		Х	Х
MSER			Х

https://en.wikipedia.org/wiki/Feature\_detection\_(computer\_vision)