The Quest for an Efficient LTL Model-Checking

E. Renault

Friday, May 18th

What is Model-Checking? (Trebuchet Example)

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Finally Pivot Bar released?

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Model-Checking

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Model-Checking

Verified

Violated

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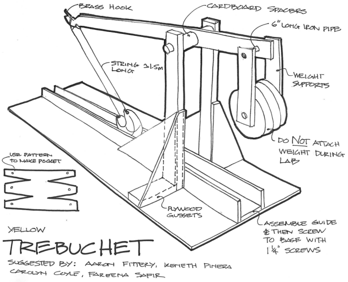

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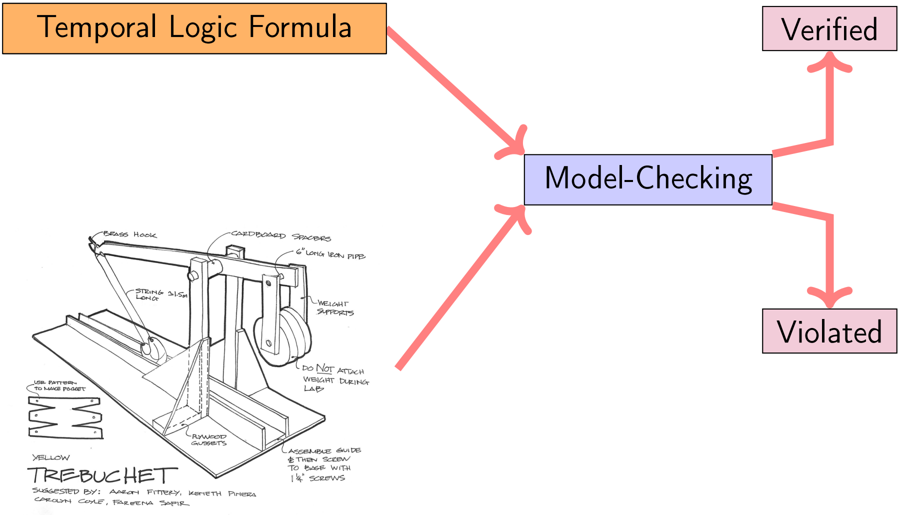
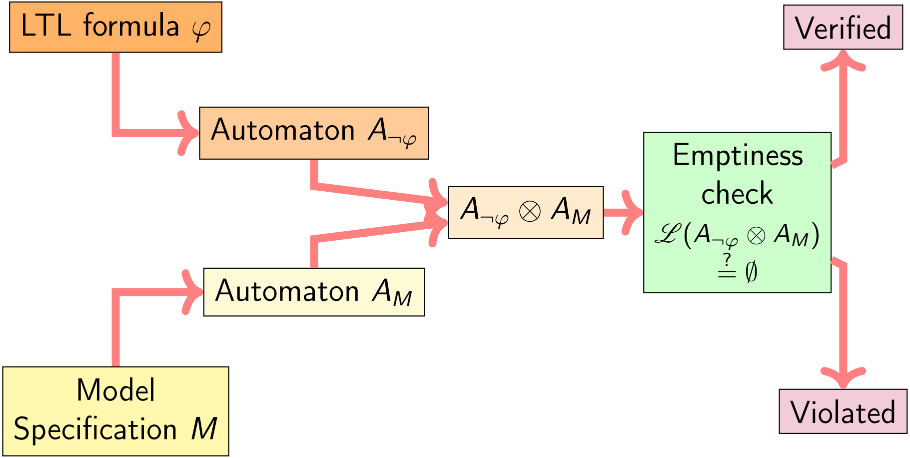


Automata-Theoretic Approach to Model Checking



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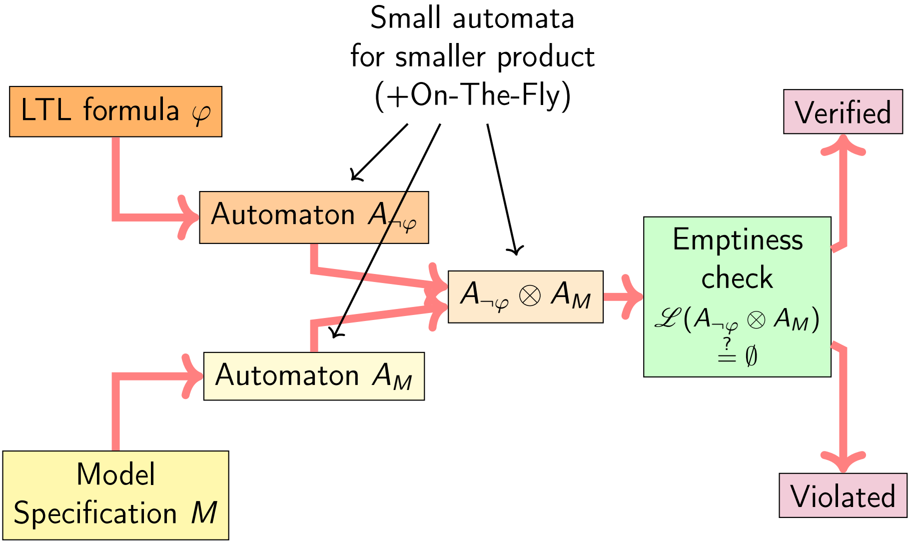

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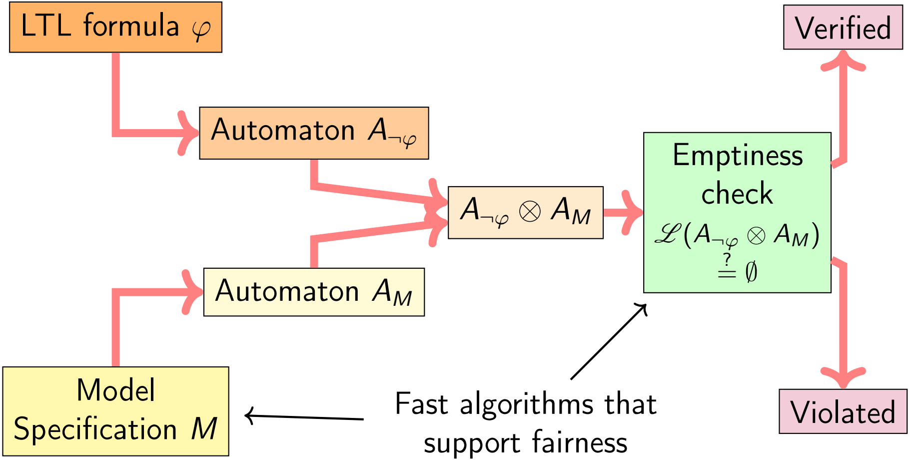

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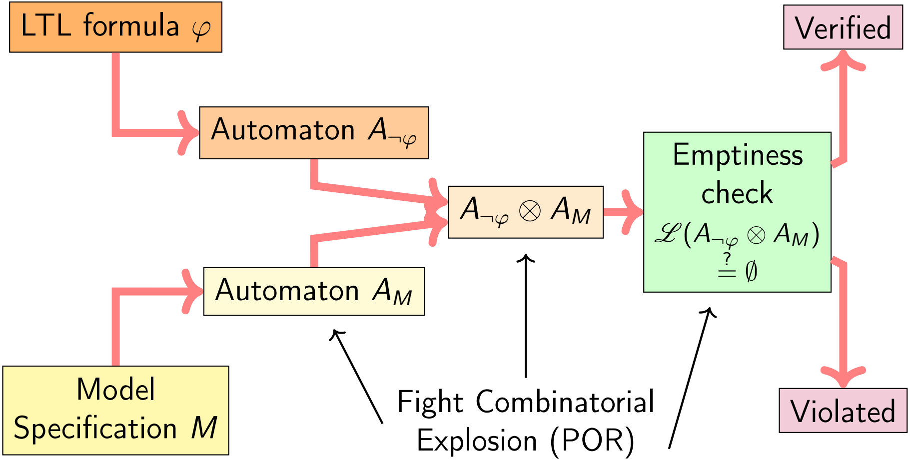


Explicit Techniques Valley
strong
terminal
weak
NDFS
SCC
Symbolic techniques Mountains
ω-automata villages
The forest of the Emptiness
Dijkstra Temple
SCC HILLS
Bitstate haching
State space caching
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ω-Automata Villages


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Many automata . . .

- Büchi, Co-Büchi, Streett, Rabin, Parity, Muller, other?
- Generalized or not?
- Transition-based or state-based?
- Support fairness (weak or strong)

The HOA format support all these variations. HOA is supported by many tools: Spot ltl3ba, Rabinizer3, ltl3dra

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Transition-based Generalized Büchi Automata (TGBA) seems to be a good compromise:

- Support for weak fairness
- Emptiness checks may be linear regardless the acceptance condition

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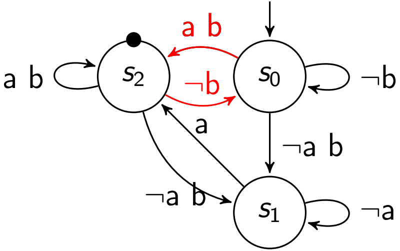
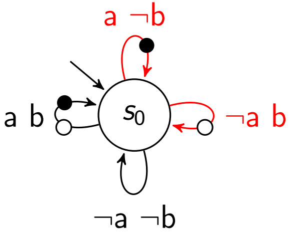
The HOA format support all these variations. HOA is supported by many tools: Spot ltl3ba, Rabinizer3, ltl3dra

Fight Combinatorial Explosion

Büchi Automata (BA) $\mathcal{F}$={●}	Transition-based Generalized Büchi Automata (TGBA) $\mathcal{F}$={●, ○}





Infinite runs are accepting if they visit each acceptance set infinitely often. If there is such a run: $\mathscr{L}(A) \neq \emptyset$.

Two equivalent and minimal automata for the LTL formula GF a ∧ GF b

Support Fairness

Weak fairness can be expressed using the LTL property:

$$\bigwedge_{i \in Processes} \text{GF progress}_i$$

	Min. det. BA		Min. det. TGBA	
Nb. Processes	states	transitions	states	transitions
1	2	4	1	2
2	3	12	1	4
4	5	80	1	16
8	9	2 304	1	256
n	$(n+1)$	$(n+1).2^n$	1	2^n

TGBA are never worst than BA!

The Forest of the Emptiness and the SCC Hills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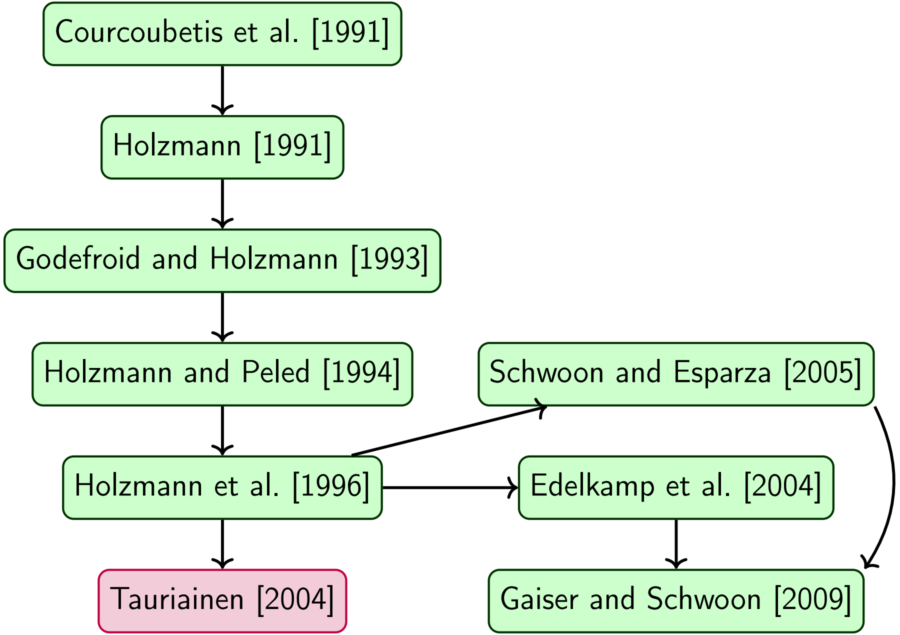
Sequential Emptiness Checks

- **NDFS-based**: look for accepting runs of the automaton using a second interleaved DFS


Courcoubetis et al. [1991]
Holzmann [1991]
Godefroid and Holzmann [1993]
Holzmann and Peled [1994]
Schwoon and Esparza [2005]
Holzmann et al. [1996]
Edelkamp et al. [2004]
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Gaiser and Schwoon [2009]

Sequential Emptiness Checks

- **NDFS-based**: look for accepting runs of the automaton using a second interleaved DFS

	NDFS-based	
Memory requirements	2 extra bits per state	
Closing edge detect.	easy only on DFS stack	
On-the-fly	✓	
Bit state hashing	✓	
State space caching	✓	
Generalization	Proportionnal to $\mid \mathcal{F} \mid$	

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- **SCC-based**: compute SCC of the automaton and look for accepting SCC using only one DFS

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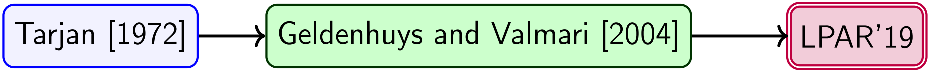
Tarjan [1972]
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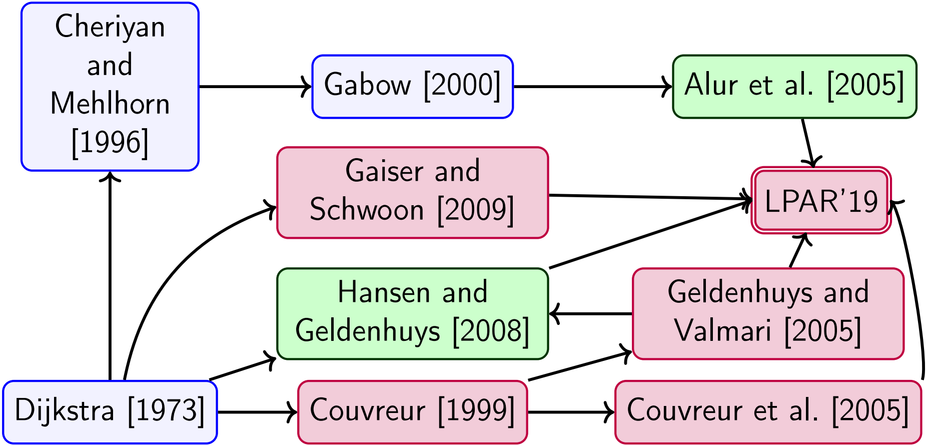
LPAR'19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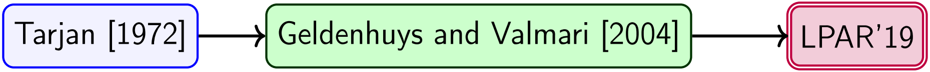
	NDFS-based	SCC-based
Memory requirements	2 extra bits per state	1 or 2 int per state
Closing edge detect.	easy only on DFS stack	easy
On-the-fly	✓	✓
Bit state hashing	✓	✓
State space caching	✓	✓
Generalization	Proportionnal to $\mid \mathcal{F} \mid$	Independant to $\mid \mathcal{F} \mid$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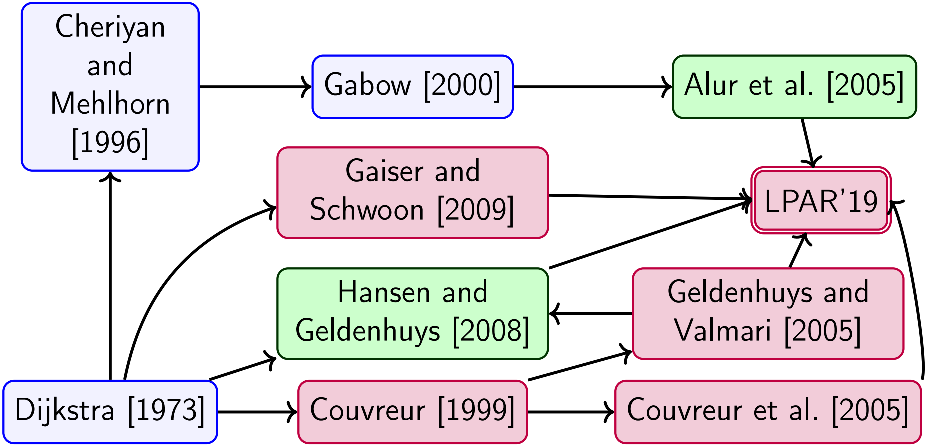
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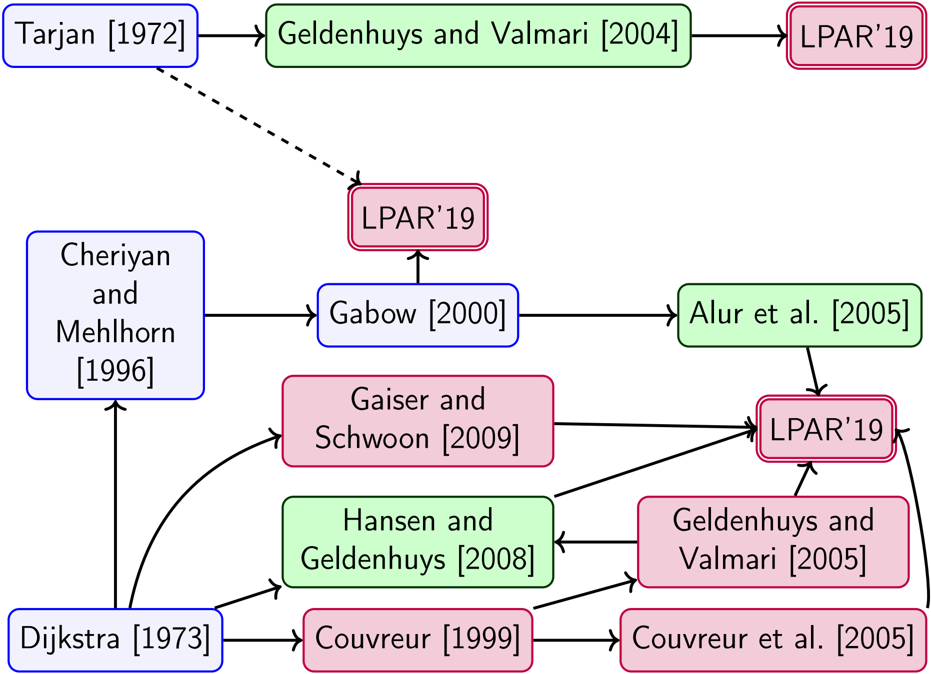
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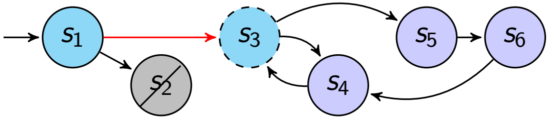
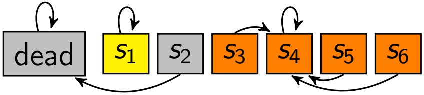
Using Union-Find for Emptiness Check

Main Idea

- Store state's SCC-membership in a Union-Find
- Marking an SCC of size S as *Dead* in $O(Ack^{-1}(S))$ (quasi-constant) rather that in $O(S)$
- Independant from the underlying algorithm (Tarjan/Dijkstra)





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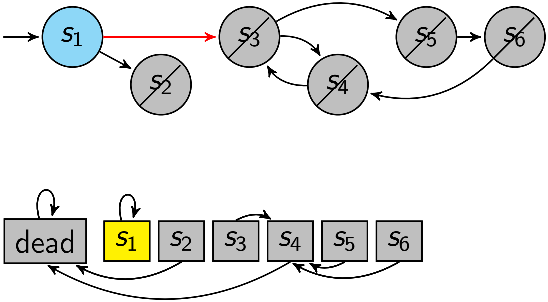

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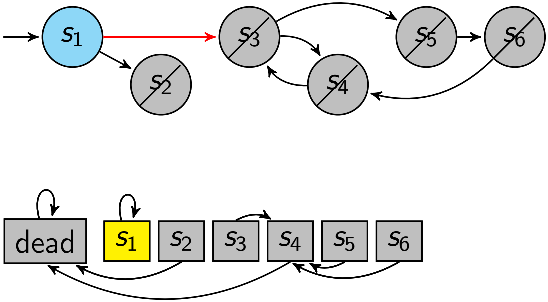
- Easy to parallelize (later on this talk!)



The outpost of the parallelism

The Decomposition Tower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Strength of $A_{\neg\varphi}$ & Emptiness Check of $A_{\neg\varphi} \otimes A_{Sys}$ [Bloem al., 1999]

Terminal Automaton	Weak Automaton	Strong Automaton
Accepting SCC are complete and contain only accepting cycles	*Accepting SCC contain only accepting cycles*	*Accepting SCC can mix accepting cycles and non accepting cycles*
Reachability Assumption on A_{Sys}: no deadlock.	Simple cycle search	NDFS-based or SCC-based

Strength of $A_{\neg\varphi}$ & Emptiness Check of $A_{\neg\varphi} \otimes A_{Sys}$ [Bloem al., 1999]

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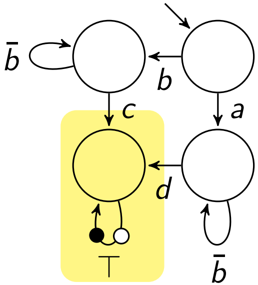	*Accepting SCC contain only accepting cycles*	*Accepting SCC can mix accepting cycles and non accepting cycles*
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[Bloem al., 1999]

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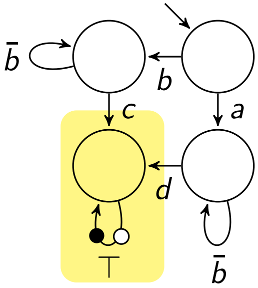	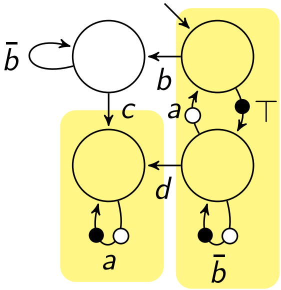	*Accepting SCC can mix accepting cycles and non accepting cycles*
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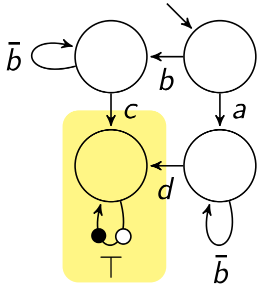
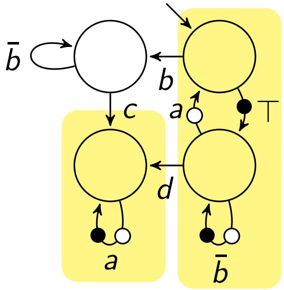
[Bloem al., 1999]

Terminal Automaton



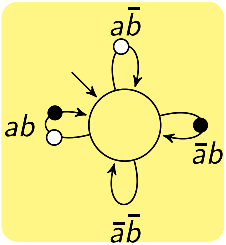
Reachability
Assumption on A_{Sys}: no deadlock.

Weak Automaton



Simple cycle search

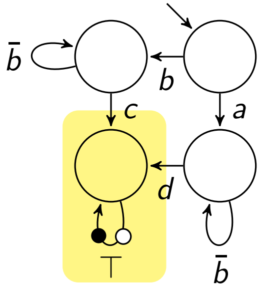
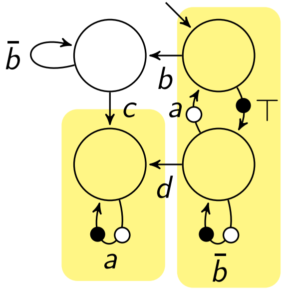
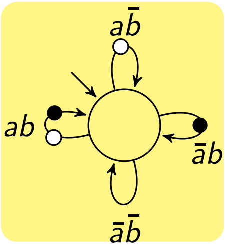
Strong Automaton



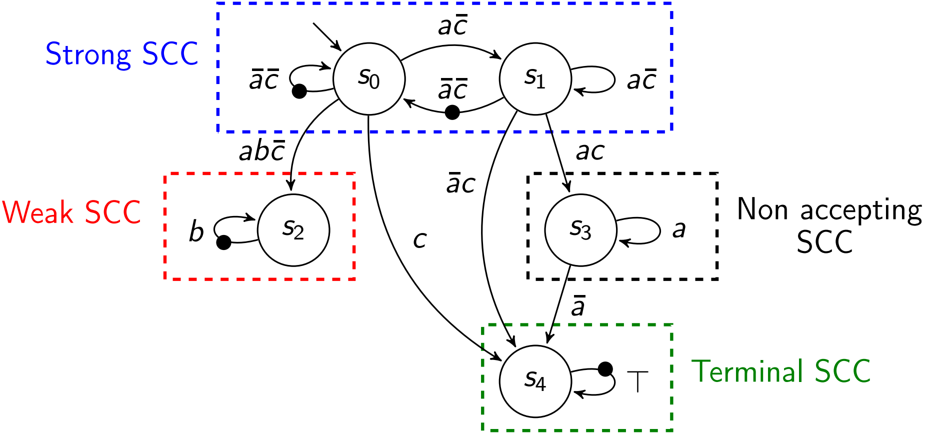
NDFS-based or SCC-based

Strength of $A_{\neg\varphi}$ & Emptiness Check of $A_{\neg\varphi} \otimes A_{Sys}$

[Bloem al., 1999]

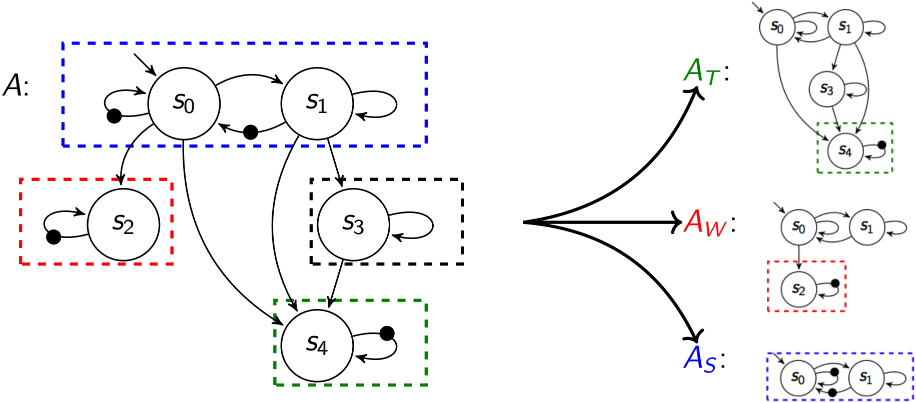
Terminal Automaton	$\subset$	Weak Automaton	$\subset$	Strong Automaton
				
Reachability Assumption on A_{Sys}: no deadlock.	$<$	Simple cycle search	$<$	NDFS-based or SCC-based

Strong Automaton with Multiple SCC Strengths
[Edelkamp et al., 2004]



$A_{\neg\varphi}$ for $\neg\varphi = (\mathsf{G}\, a \rightarrow \mathsf{G}\, b)\, \mathsf{W}\, c$

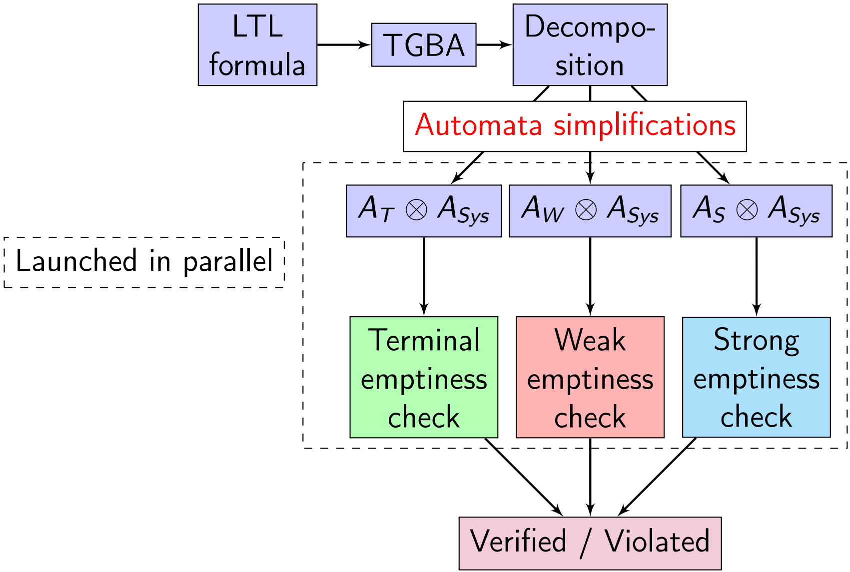
Decomposing the Property Automaton



$$\mathscr{L}(A) = \mathscr{L}(A_T) \cup \mathscr{L}(A_W) \cup \mathscr{L}(A_S).$$

- A_T: captures the terminal behaviors of A
- A_W: captures the weak behaviors of A
- A_S: captures the strong behaviors of A

Decomposition Canevas



Note: emptiness-check agnostic.

Results

On 10 models from BEEM and 3 268 random formula

	No simpl.			With simpl.		
	A_T	A_W	A_S	A_T	A_W	A_S
States Reduction (%)	20	27	54	47	40	60
Transitions Reduction (%)	25	35	67	50	42	67

After simplifications

- Reduction of 86% of states for $A_{sys} \otimes A_T$
- Reduction of 39% of states for $A_{sys} \otimes A_W$
- Reduction of 42% of states for $A_{sys} \otimes A_S$

Average Speedup

- 15% for empty products,
- 70% for non-empty products.

The outpost of the parallelism

The Dead forest of the Union-Find (UFSCC & CNDFS)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Problem Statement

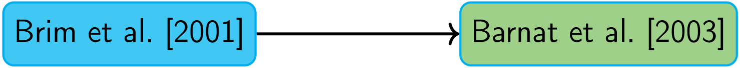
Reif [1985]

Depth-First Search is Inherently Sequential

Brim et al. [2001]

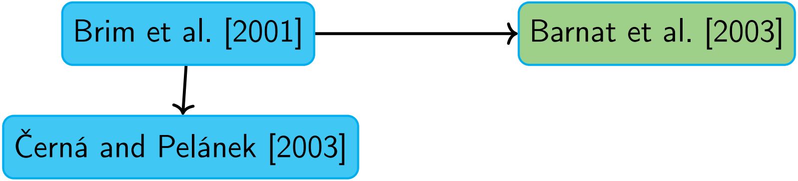
Brim et al. [2001]

- Detects *negative cycles*
- Transitions are tagged 0 except the one from an accepting state (tagged -1
- Maintains shortest distance from the initial state
- If negative distance, a counterexample is reported


Brim et al. [2001]
Barnat et al. [2003]

Brim et al. [2001] → Barnat et al. [2003]

- Track BFS detph of each state
- When a transition goes to an highest state: launch a sequential DFS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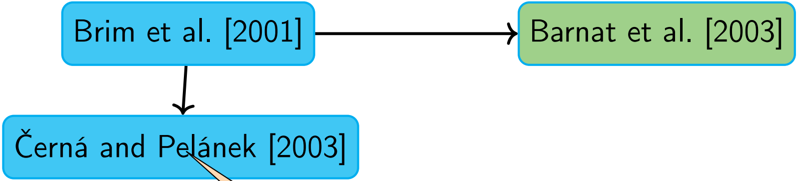
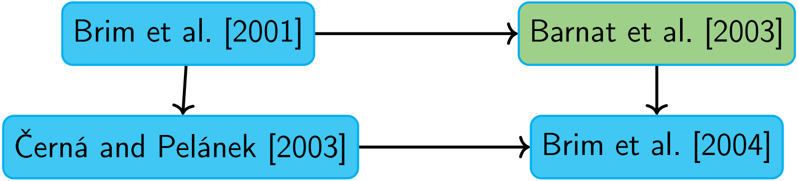


- Explicit OWCTY
- Compute SCCs with accepting states
- If such an SCC, a counterexample exists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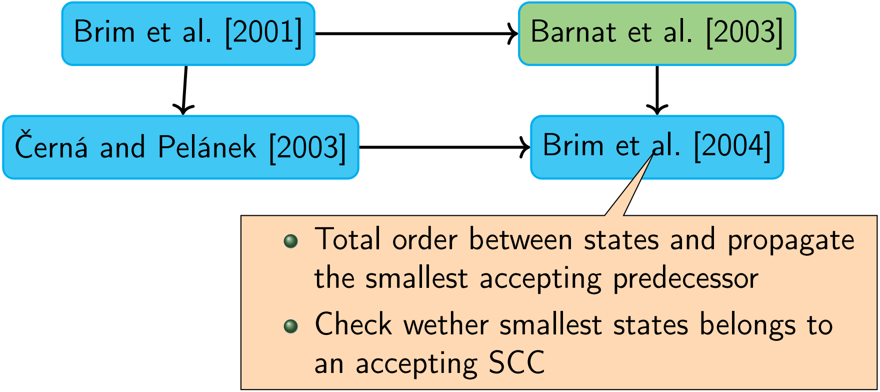
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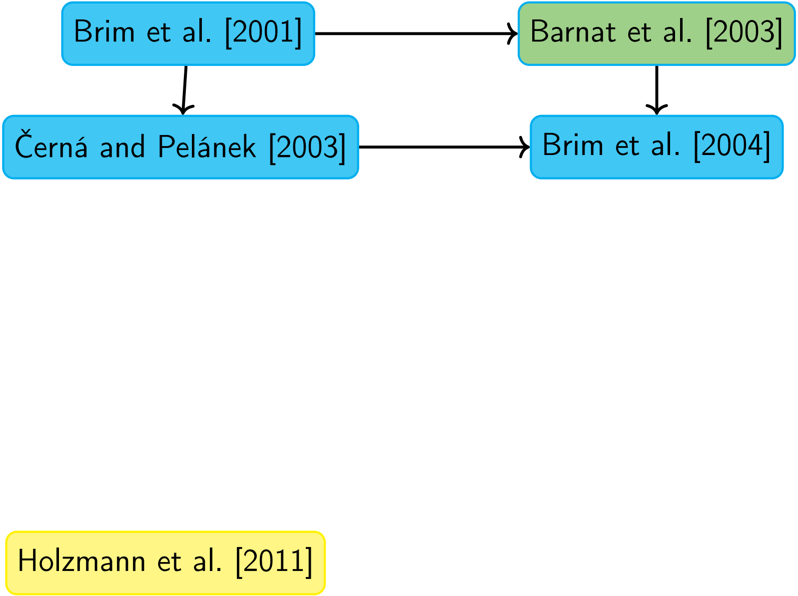
Total order between states and propagate the smallest accepting predecessor
Check wether smallest states belongs to an accepting SCC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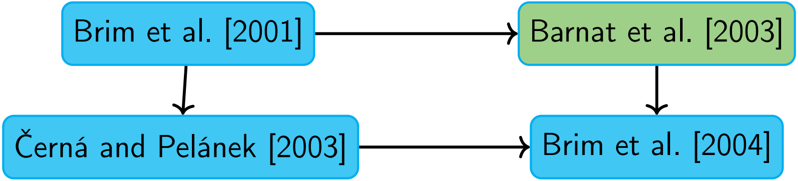
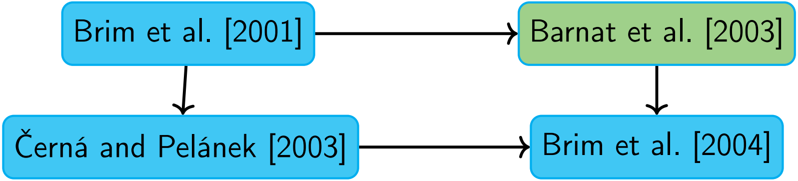
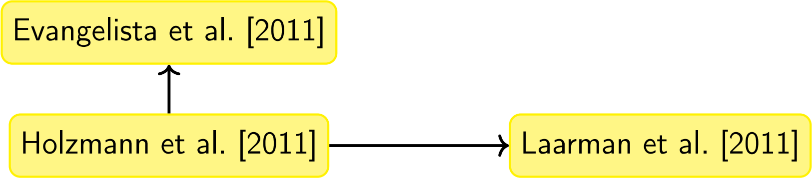
Černá and Pelánek [2003]
Brim et al. [2004]
Holzmann et al. [2011]

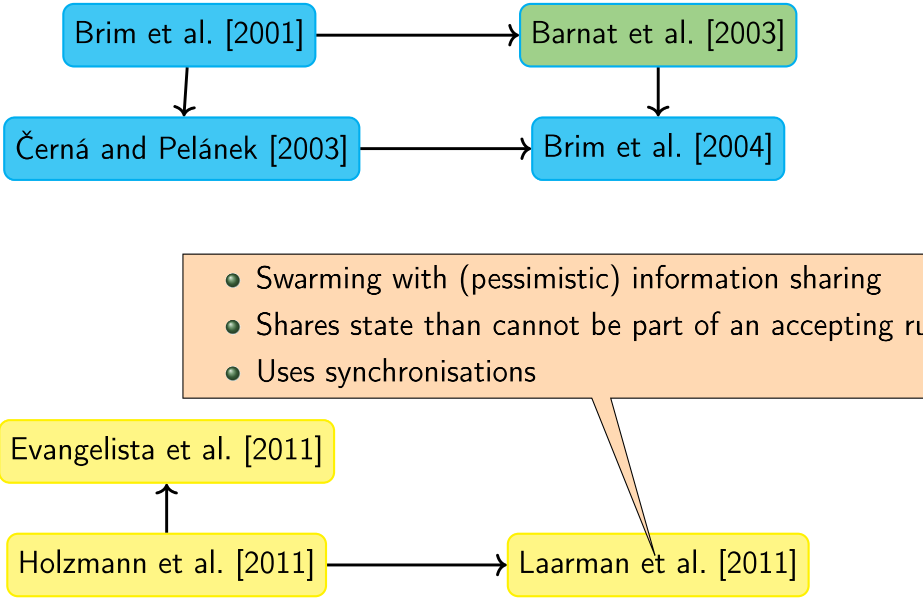


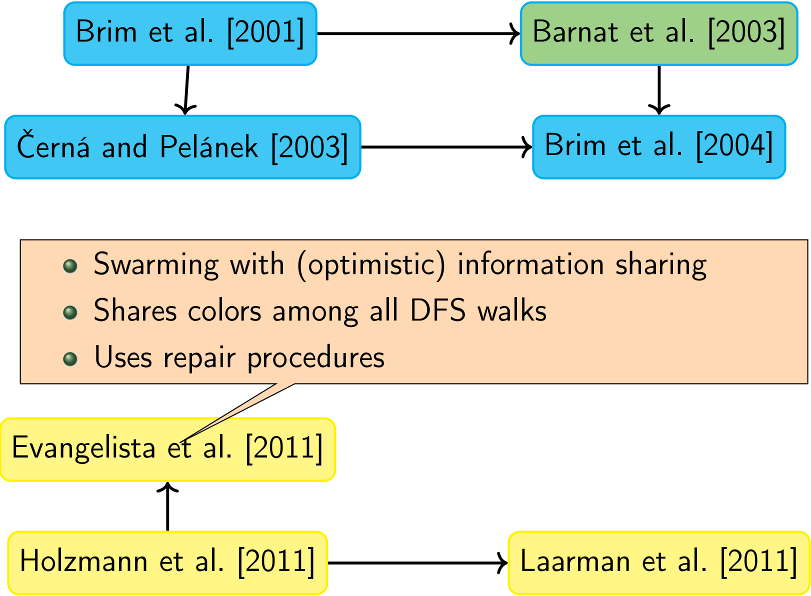
- Run multiple independant emptiness check in parallel
- Each thead has it own transition order

Holzmann et al. [2011]

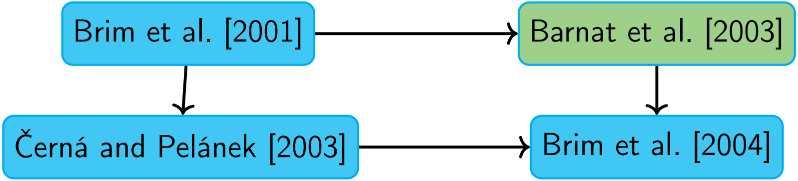

Brim et al. [2001]
Barnat et al. [2003]
Černá and Pelánek [2003]
Brim et al. [2004]


Evangelista et al. [2011]
Holzmann et al. [2011]
Laarman et al. [2011]


Brim et al. [2001]
Barnat et al. [2003]
Černá and Pelánek [2003]
Brim et al. [2004]
Swarming with (pessimistic) information sharing
Shares state than cannot be part of an accepting ru
Uses synchronisations
Evangelista et al. [2011]
Holzmann et al. [2011]
Laarman et al. [2011]



- Swarming with (optimistic) information sharing
- Shares colors among all DFS walks
- Uses repair procedures


Brim et al. [2001]
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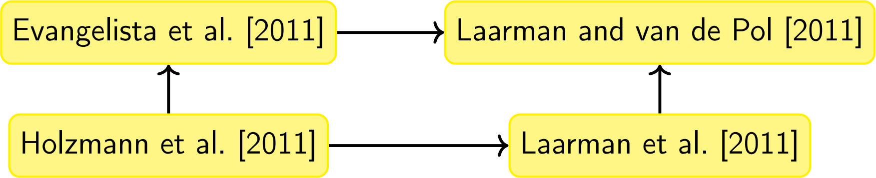
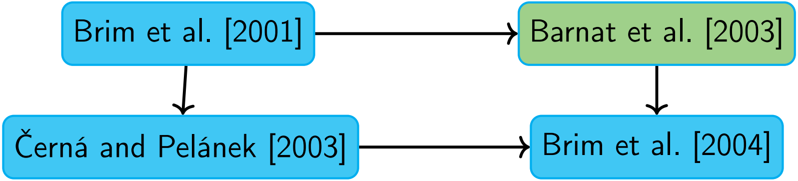
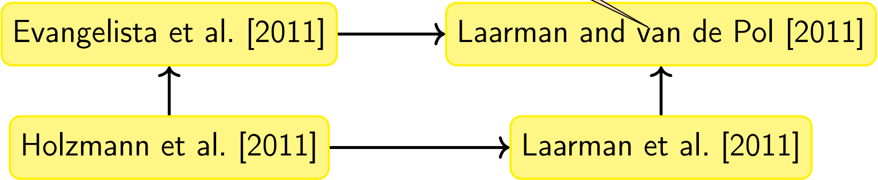
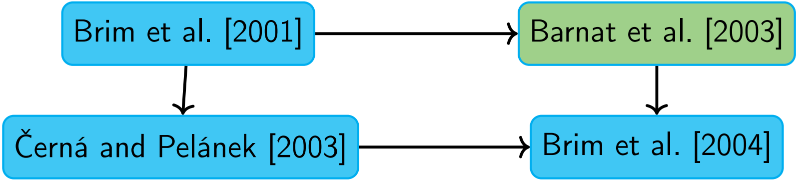
Brim et al. [2004]

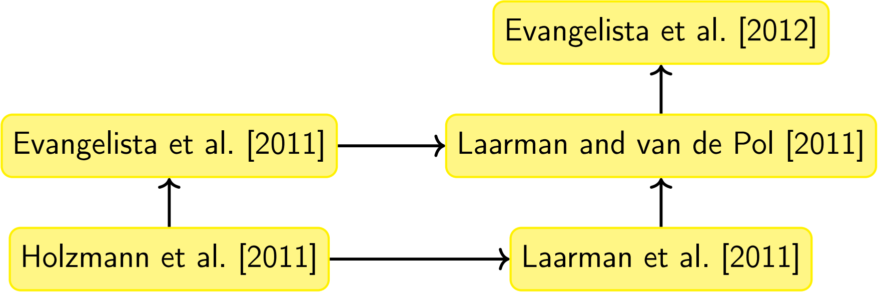

Evangelista et al. [2011]
Laarman and van de Pol [2011]
Holzmann et al. [2011]
Laarman et al. [2011]

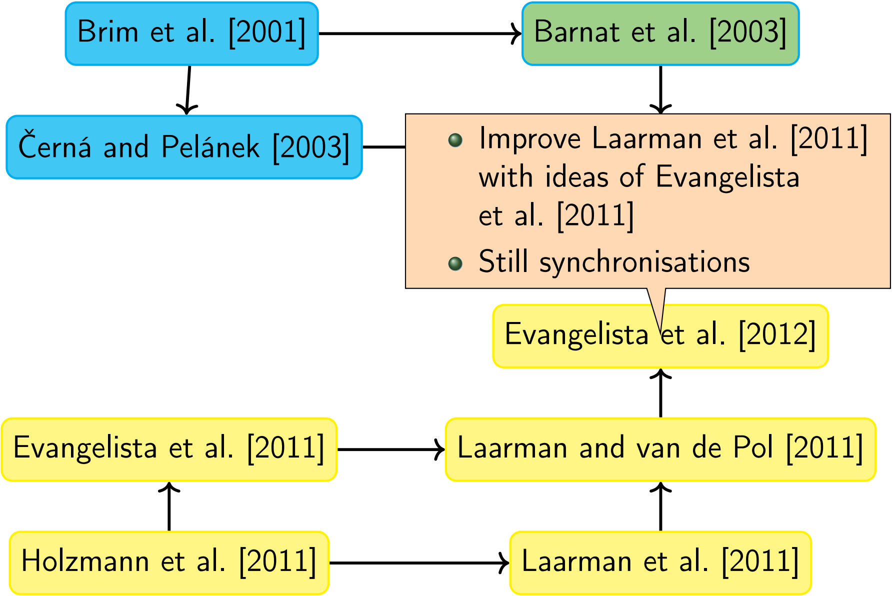


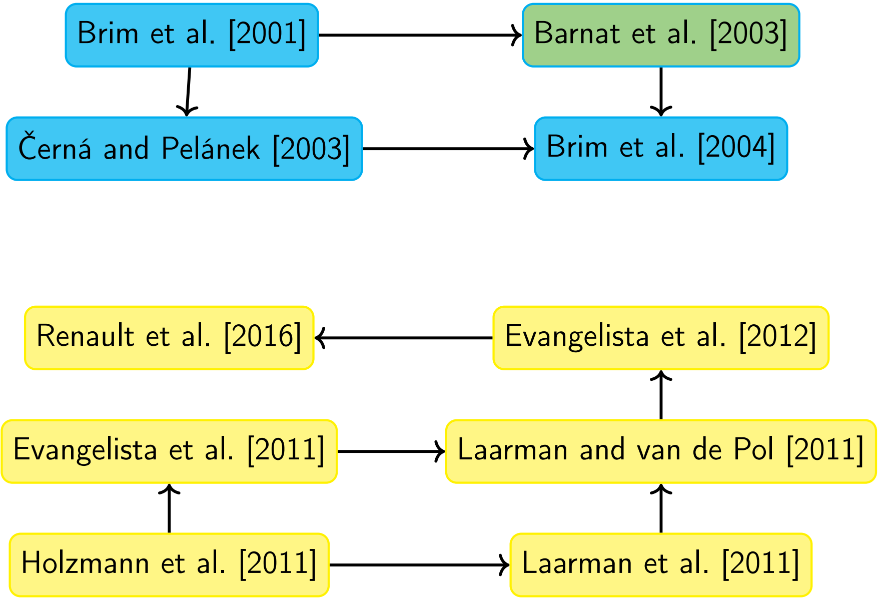
- Mix the 2 previous algorithms: Laarman et al. [2011] is used as a repair procedure



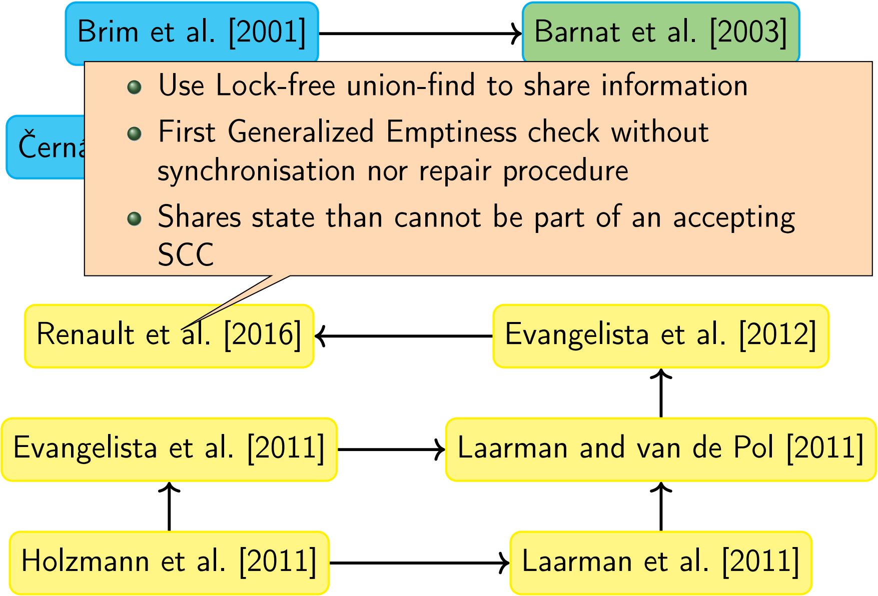

Brim et al. [2001]
Barnat et al. [2003]
Černá and Pelánek [2003]
Brim et al. [2004]

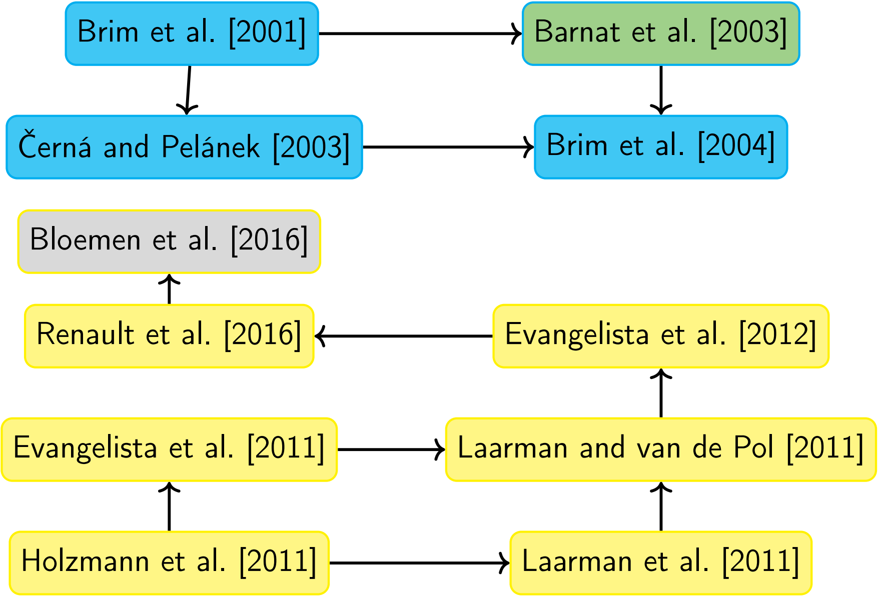

Evangelista et al. [2012]
Evangelista et al. [2011]
Laarman and van de Pol [2011]
Holzmann et al. [2011]
Laarman et al. [2011]


Brim et al. [2001]
Barnat et al. [2003]
Černá and Pelánek [2003]
Improve Laarman et al. [2011] with ideas of Evangelista et al. [2011]
Still synchronisations
Evangelista et al. [2012]
Evangelista et al. [2011]
Laarman and van de Pol [2011]
Holzmann et al. [2011]
Laarman et al. [2011]

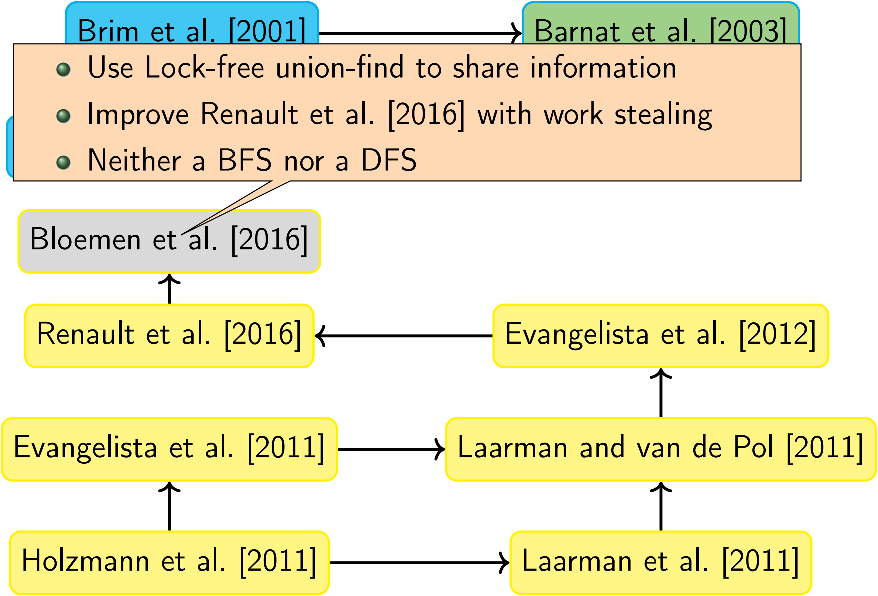

Brim et al. [2001]
Barnat et al. [2003]
Černá and Pelánek [2003]
Brim et al. [2004]
Renault et al. [2016]
Evangelista et al. [2012]
Evangelista et al. [2011]
Laarman and van de Pol [2011]
Holzmann et al. [2011]
Laarman et al. [2011]

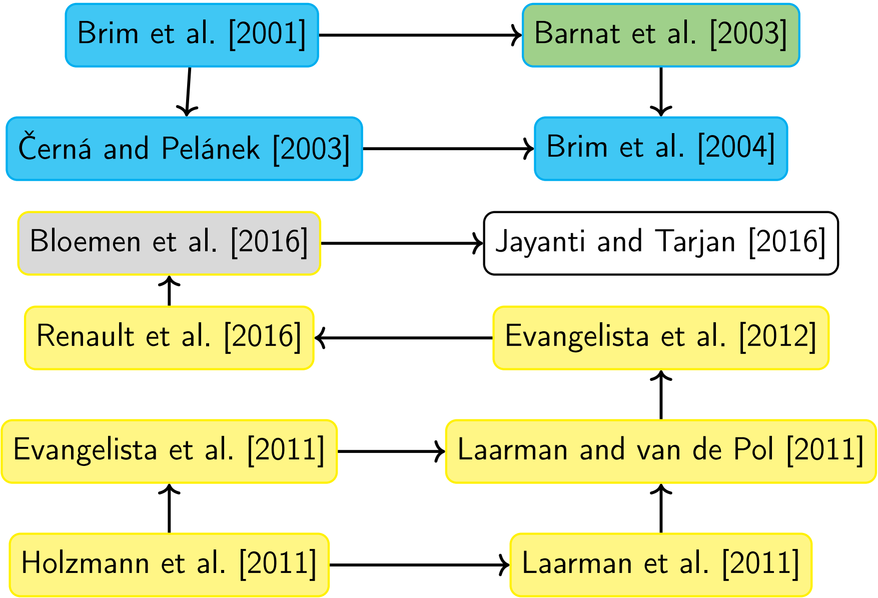
- Use Lock-free union-find to share information
- First Generalized Emptiness check without synchronisation nor repair procedure
- Shares state than cannot be part of an accepting SCC




Brim et al. [2001]
Barnat et al. [2003]
Černá and Pelánek [2003]
Brim et al. [2004]
Bloemen et al. [2016]
Renault et al. [2016]
Evangelista et al. [2012]
Evangelista et al. [2011]
Laarman and van de Pol [2011]
Holzmann et al. [2011]
Laarman et al. [2011]

- Use Lock-free union-find to share information
- Improve Renault et al. [2016] with work stealing
- Neither a BFS nor a DFS

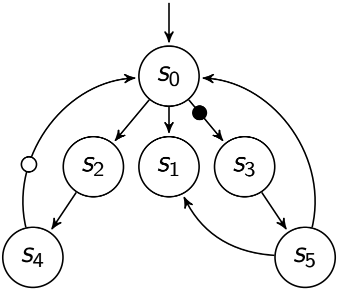


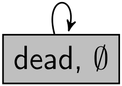

Brim et al. [2001]
Barnat et al. [2003]
Černá and Pelánek [2003]
Brim et al. [2004]
Bloemen et al. [2016]
Jayanti and Tarjan [2016]
Renault et al. [2016]
Evangelista et al. [2012]
Evangelista et al. [2011]
Laarman and van de Pol [2011]
Holzmann et al. [2011]
Laarman et al. [2011]

Example for Bloemen et al.

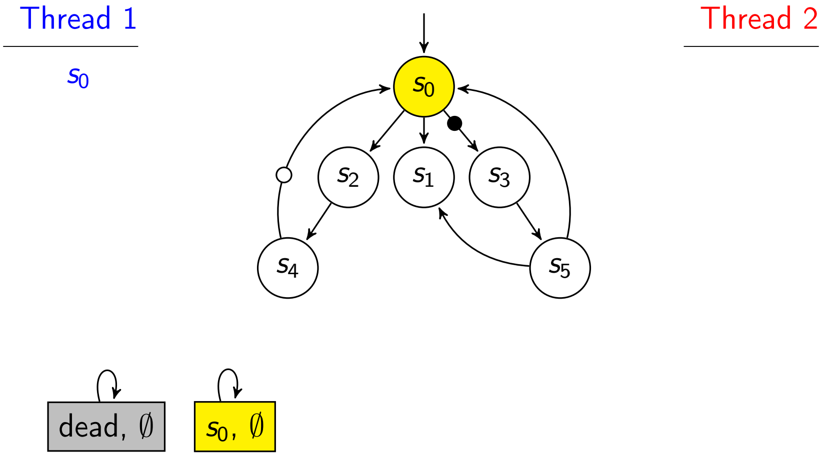
Thread 1

Thread 2





Example for Bloemen et al.



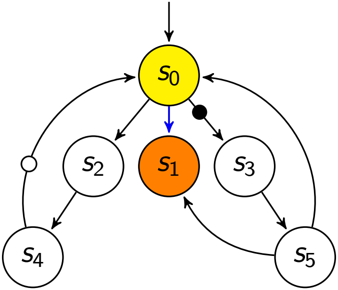
Example for Bloemen et al.

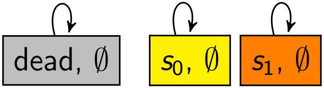
Thread 1

s_0

s_1

Thread 2



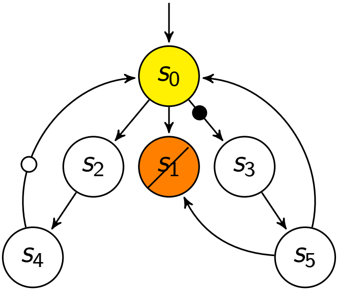


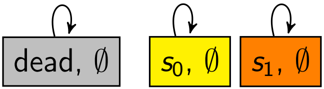
Example for Bloemen et al.

Thread 1

s_0
$s_1[s_1]$

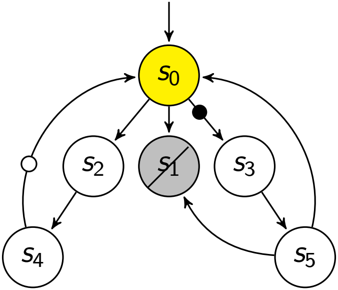
Thread 2

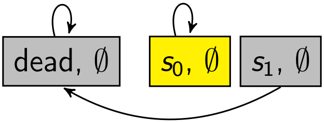




Example for Bloemen et al.

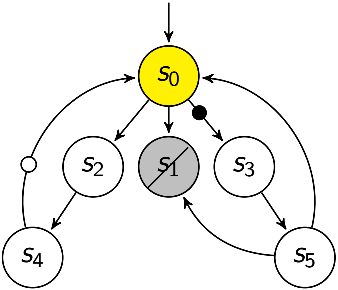
Thread 1	Thread 2
s_0	

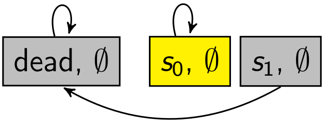




Example for Bloemen et al.

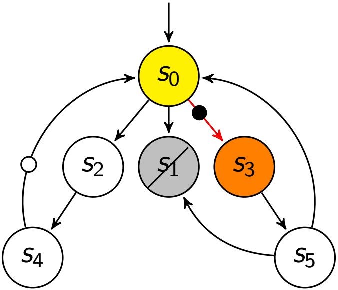
Thread 1	Thread 2
s_0	s_0



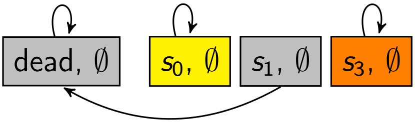


Example for Bloemen et al.

Thread 1
s_0

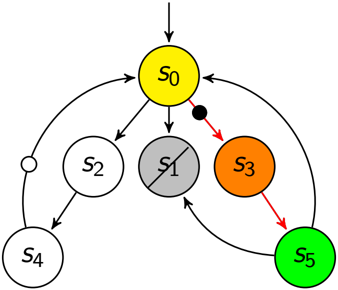


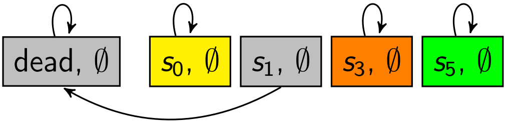
Thread 2
s_0
s_3



Example for Bloemen et al.

Thread 1
s_0





Thread 2
s_0
s_3
s_5

Example for Bloemen et al.

Thread 1

s_0

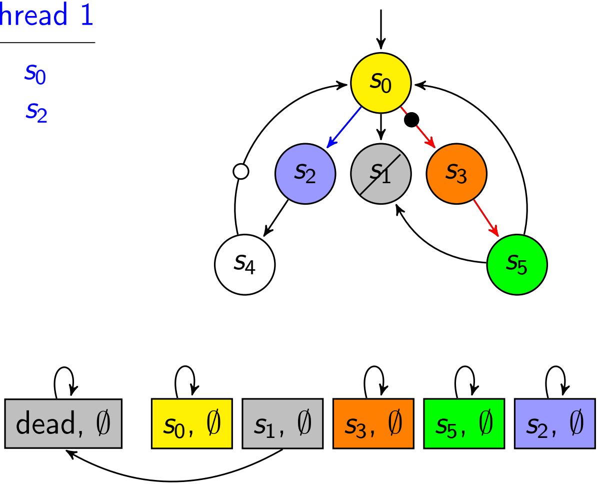
s_2

Thread 2

s_0

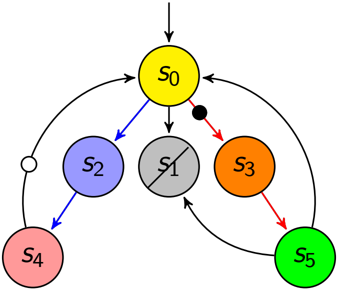
s_3

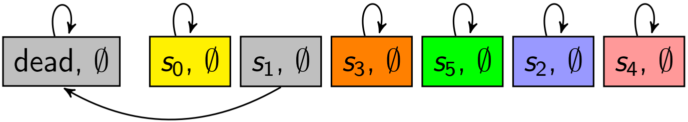
s_5



Example for Bloemen et al.

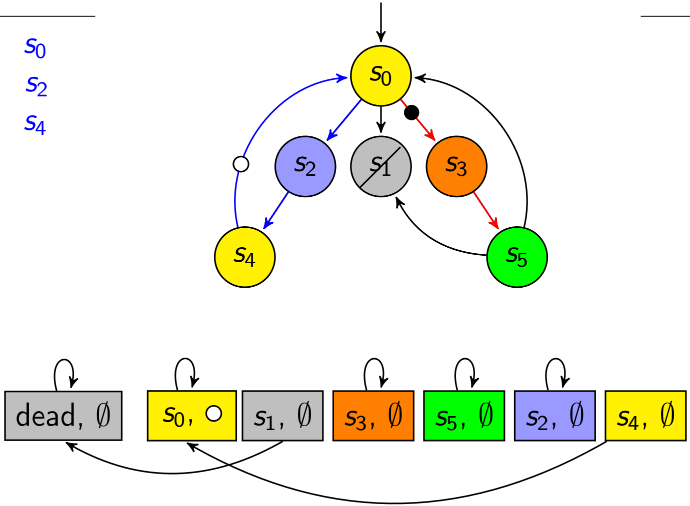
Thread 1	Thread 2
s_0	s_0
s_2	s_3
s_4	s_5



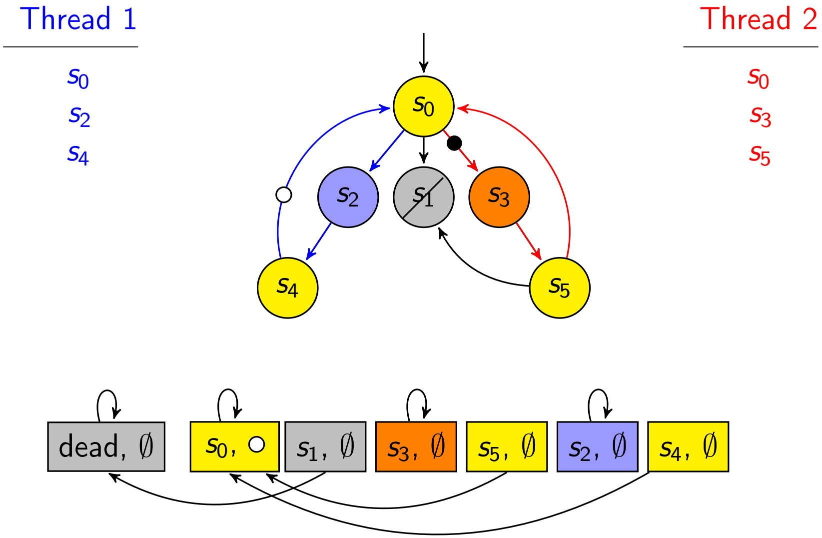


Example for Bloemen et al.

Thread 1	Thread 2
s_0	s_0
s_2	s_3
s_4	s_5

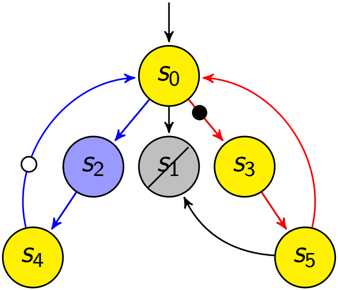


Example for Bloemen et al.

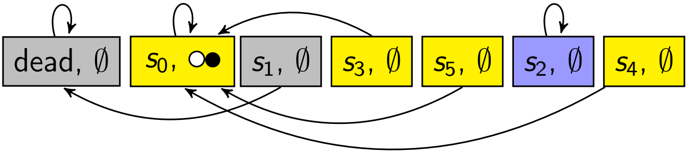


Example for Bloemen et al.

Thread 1
s_0
s_2
s_4



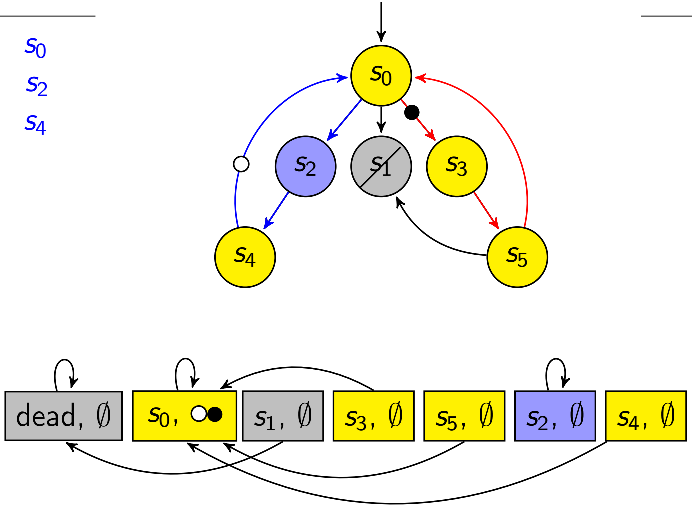
Thread 2
s_0
s_3
s_5



If $\mathcal{F} = \{\bullet, \circ\}$ then report counterexampe otherwise continue!

Example for Bloemen et al.

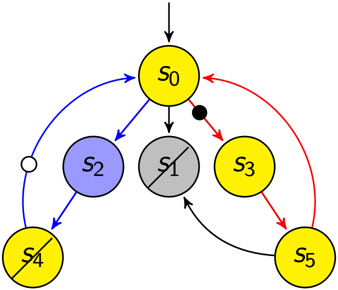
Thread 1
s_0
s_2
s_4



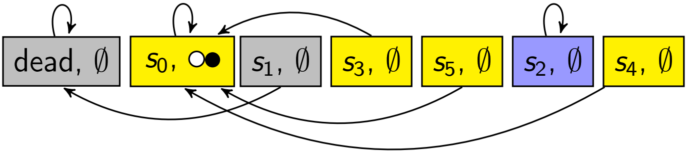
Thread 2
s_0
s_3
s_5

Example for Bloemen et al.

Thread 1
s_0
s_2
s_4



Thread 2
s_0
s_3
$s_5[s_4]$



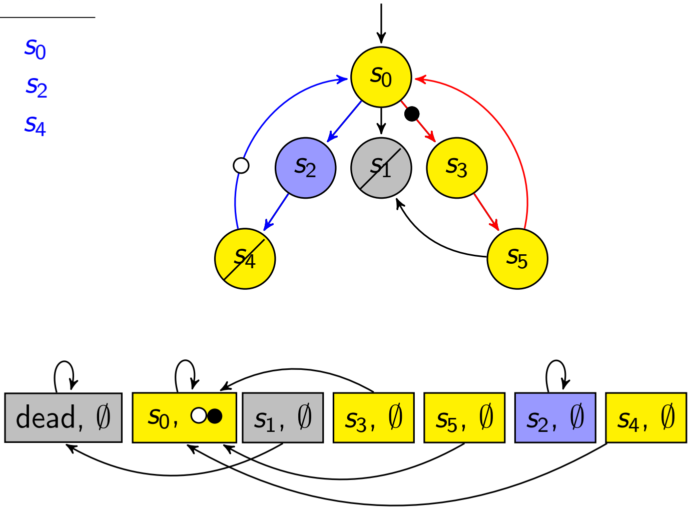
Example for Bloemen et al.

Thread 1

s_0
s_2
s_4

Thread 2

s_0
s_3
$s_5[s_0]$
s_2



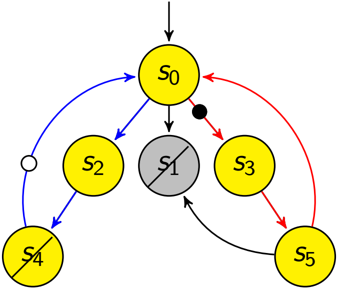
Example for Bloemen et al.

Thread 1

s_0

s_2

s_4



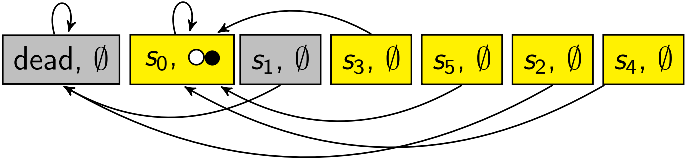
Thread 2

s_0

s_3

$s_5[s_0]$

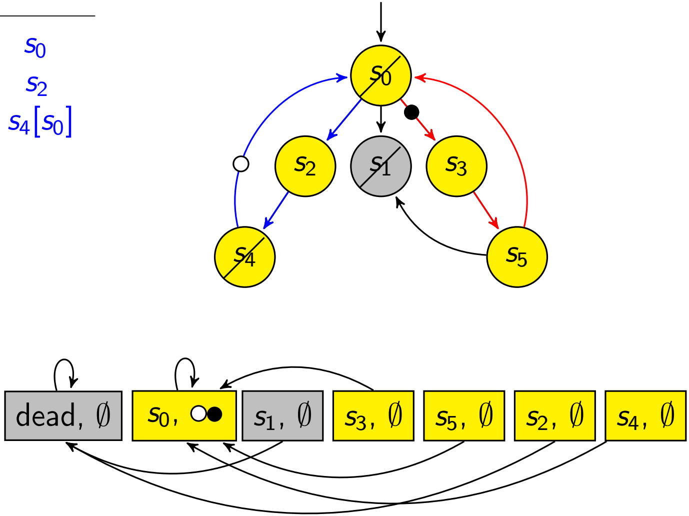
s_2



Example for Bloemen et al.

Thread 1

s_0
s_2
$s_4[s_0]$



Thread 2

s_0
s_3
$s_5[s_0]$
s_2

Example for Bloemen et al.

Thread 1

s_0

s_2

$s_4[s_3]$

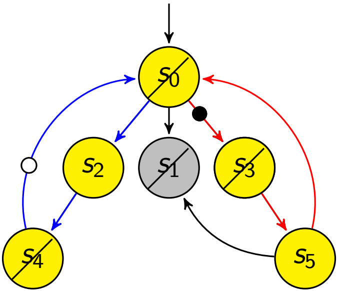
Thread 2

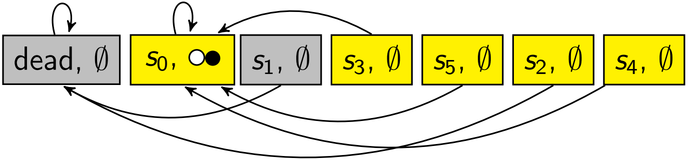
s_0

s_3

$s_5[s_0]$

s_2





Example for Bloemen et al.

Thread 1

s_0

s_2

$s_4[s_3]$

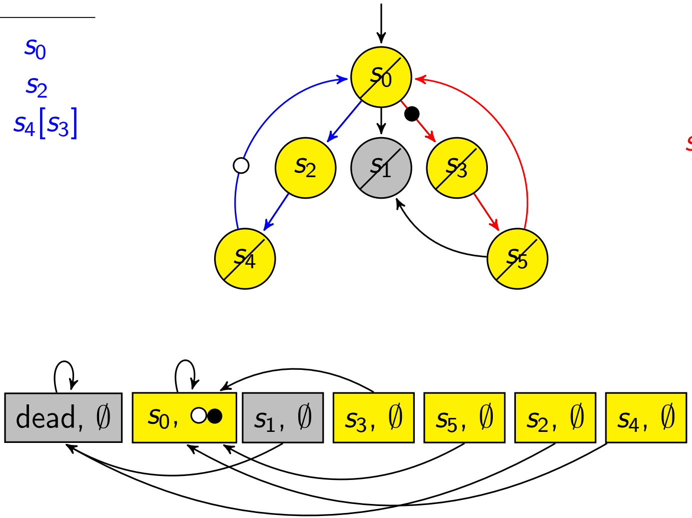
Thread 2

s_0

s_3

$s_5[s_0]$

$s_2[s_5]$



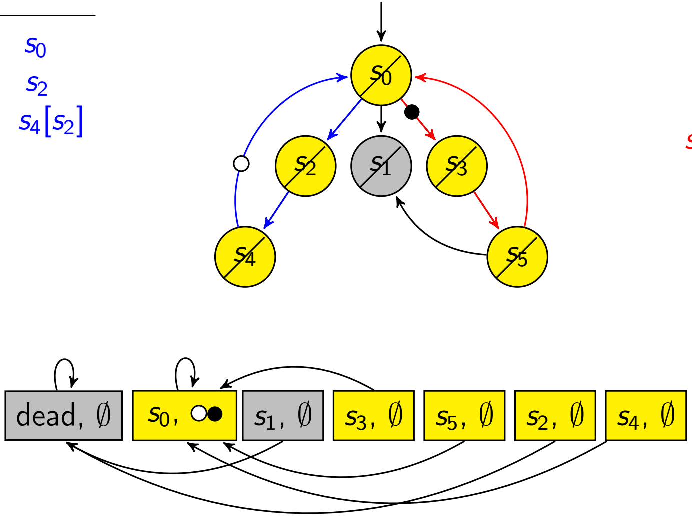
Example for Bloemen et al.

Thread 1

s_0

s_2

$s_4[s_2]$



Thread 2

s_0

s_3

$s_5[s_0]$

$s_2[s_5]$

Example for Bloemen et al.

Thread 1

s_0

s_2

$s_4[s_2]$

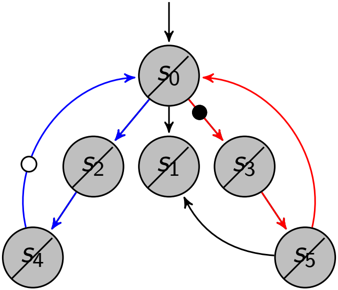
Thread 2

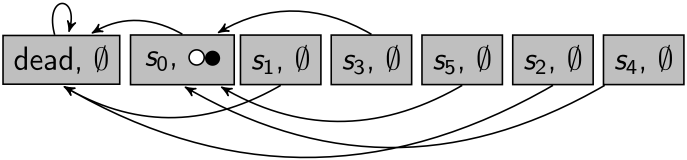
s_0

s_3

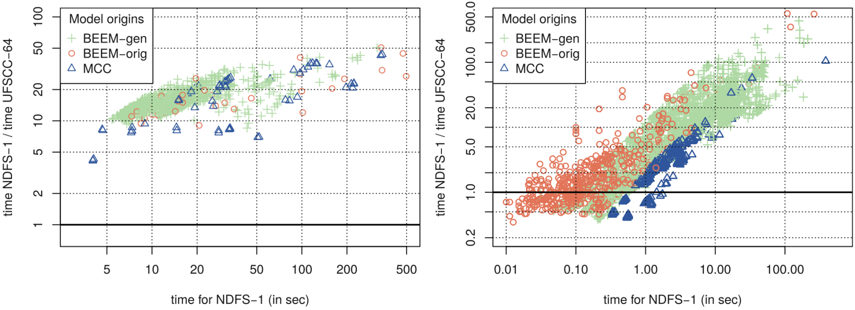
$s_5[s_0]$

$s_2[s_5]$



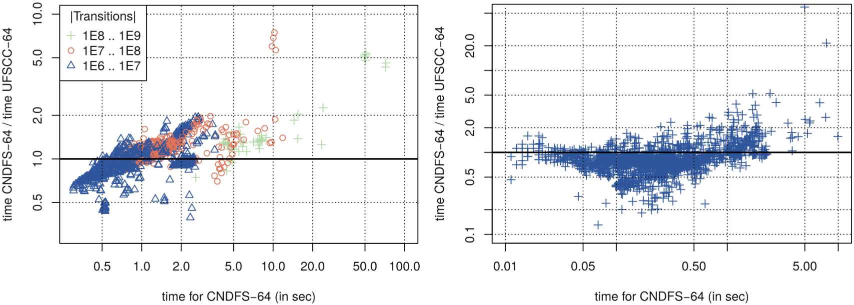


Results 1/2 (Bloemen and van de Pol [2016])



(a) Without counterexamples (b) With counterexamples

Results 1/2 (Bloemen and van de Pol [2016])



(a) Without counterexamples (b) With counterexamples

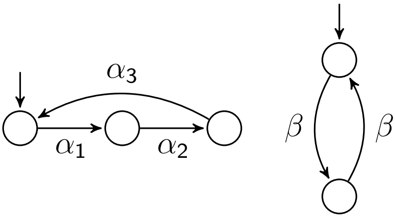
The lakes of the POR & the one hundred bridges of the proviso


Explicit Techniques Valley
strong
terminal
weak
NDFS
SCC
Symbolic techniques
Mountains
ω-automata
villages
The forest of the Emptiness
Dijkstra Temple
SCC hills
Bitstate haching
State space caching
Tarjan Temple
On the fly
The one hundred bridges of the proviso
The dead Forest of the
Union-Find
Decomposition Tower
Renault
UFSCC
Map
Stubborn
Lakes of the POR
Conflicting
bledge
The outpost of the parellism
Persistent
owcty
swarming
Linear Topology
SCC+POR
Timed Automata
endfs
lndfs
ω-automata parallel emptiness checks
CNDFS
mc-ndfs
Satisfiability
Distributed islands
1
2
3
4
5

State Space Explosion

- Two concurrent processes
- β independent of α_1, α_2, and α_3

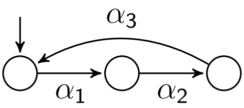
Process 1	Process 2



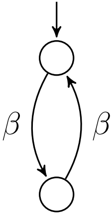
State Space Explosion

- Two concurrent processes
- β independent of α_1, α_2, and α_3

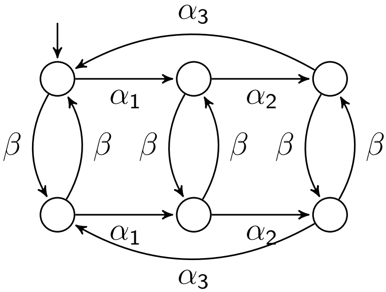
Process 1



Process 2



State Space

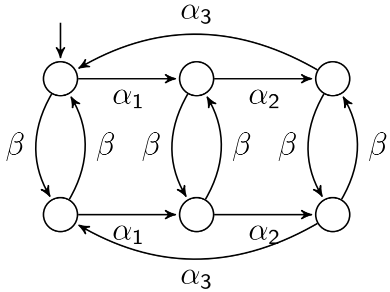


Process interleavings are one of the main sources of state-space explosion for explicit model checkers

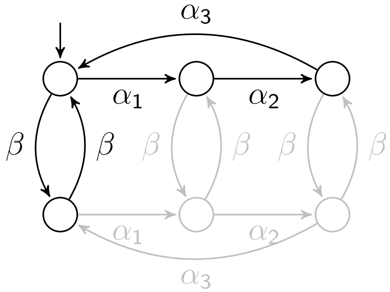
Partial Order Reductions (POR)

- Build a reduced state space
- For each state only consider a **reduced** subset of actions

State Space



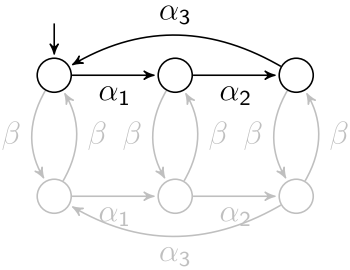
Possible Reduced State Space



POR work only iff the property to check belongs to LTL\X

The Ignoring Problem for Liveness Properties

- If the same actions are consistently ignored along a cycle, they may never be executed (below β is never executed)



The Ignoring Problem for Liveness Properties

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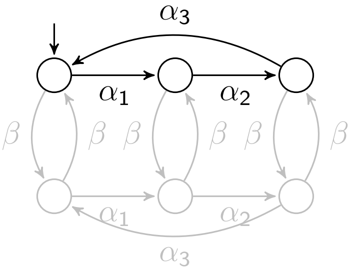


Requires an extra condition: the proviso

A proviso[a] ensures that every cycle in the reduced graph contains at least one **expanded state**, i.e, a state where all actions are considered.

[a]More simpler provisos can be applied for safety properties Evangelista and Pajault [2010]

Model Checking LTL\X with POR

Use classical DFS-based emptiness checks

During DFS:

- how to detect cycles without expanded states?
- which state to expand in a cycle?

Objectives:

- Choose states to expand states in order to have the smallest reduced state space

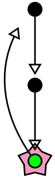
Variations on SPIN's proviso

SOURCE [Peled, 1994]

Expanded state  Not expanded state ● Already visited edge →

Variations on SPIN's proviso

SOURCE [Peled, 1994]

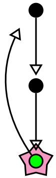


Systematically expands the source of a backedge

Expanded state  Not expanded state ● Already visited edge →

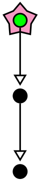
Variations on SPIN's proviso

SOURCE [Peled, 1994]



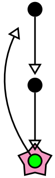
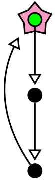
Systematically expands the source of a backedge

CONDSOURCE



Expanded state · Not expanded state ● · Already visited edge →

Variations on SPIN's proviso

SOURCE [Peled, 1994]	CONDSOURCE
	
Systematically expands the source of a backedge	Expands the source of backedge iff destination is not expanded

Expanded state ✩ Not expanded state ● Already visited edge →

Evaluation

- 38 models from the BEEM benchmark
- *reduced* implements the stubborn-set method from Valmari
- Each model is run 100 times with different transition order

	states (10^6)		transitions (10^6)		st/ms
Full	784.45	100.00%	2,677.73	100.00%	17.90
SOURCE [Peled, 1994]	303.21	38.65%	679.16	25.36%	12.33
CONDSOURCE	252.83	32.23%	518.80	19.37%	11.85
None	57.58	7.34%	97.65	3.65%	22.65

Deconstructing Evangelista and Pajault [2010] proviso

- Based on CONDSOURCE

Deconstructing Evangelista and Pajault [2010] proviso

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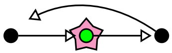
- Try to reduce useless expansions: 

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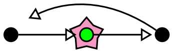
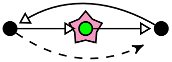
- Try to reduce useless expansions: 
- Must consider all closing-edges: 

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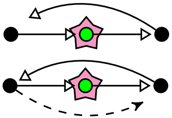

- Colors: safe, dangerous, on-dfs & not expanded

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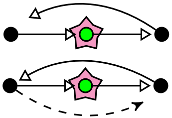

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WEIGHTED	SCAN	KNOWN

Deconstructing Evangelista and Pajault [2010] proviso

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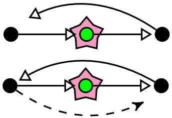

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Weighted	Scan	Known

Keep track of exp-
-anded states on DFS

Deconstructing Evangelista and Pajault [2010] proviso

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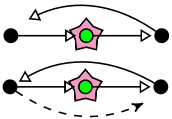

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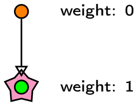
Weighted	Scan	Known



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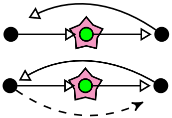

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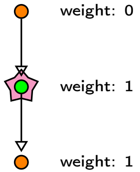

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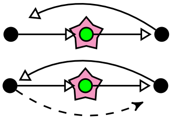

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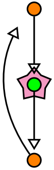
WEIGHTED	SCAN	KNOWN



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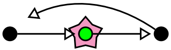
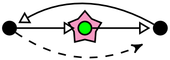
- Try to reduce useless expansions: 
- Must consider all closing-edges: 
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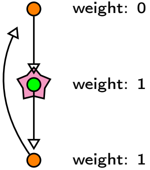
Weighted	Scan	Known
		
Keep track of exp- -anded states on DFS	Early tag "safe" states	

Deconstructing Evangelista and Pajault [2010] proviso

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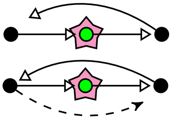

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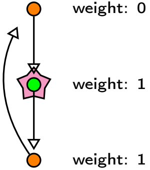
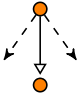
WEIGHTED	SCAN	KNOWN





Keep track of exp- -anded states on DFS

Early tag "safe" states

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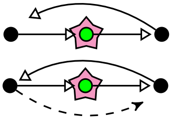

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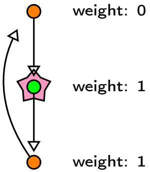

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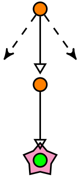

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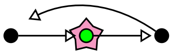

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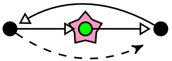

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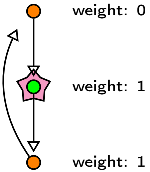
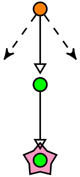
Weighted	Scan	Known





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Early tag "safe" states

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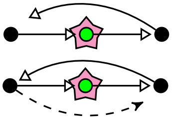

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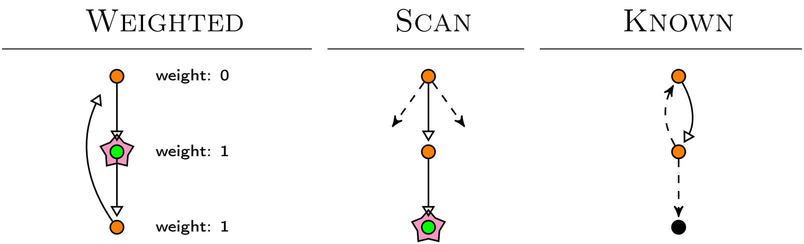

Weighted	Scan	Known
Keep track of exp- -anded states on DFS	Early tag "safe" states	Prioritizing known successors

Evaluation of each optimization

	states (10^6)		transitions (10^6)		st/ms
Full	784.45	100.00%	2,677.73	100.00%	17.90
Source [Peled, 1994]	303.21	38.65%	679.16	25.36%	12.33
WeightedSource	263.43	33.58%	537.56	20.08%	11.68
WeightedSourceKnown[1]	262.63	33.48%	534.35	19.96%	11.77
CondSource	252.83	32.23%	518.80	19.37%	11.85
CondSourceKnown	251.05	32.00%	510.91	19.08%	11.89
WeightedSourceScan	250.49	31.93%	505.98	18.90%	11.67
WeightedSourceKnownScan[1]	248.11	31.63%	498.68	18.62%	11.70
None	57.58	7.34%	97.65	3.65%	22.65

- SOURCE have the best throughput
- Most of the improvement comes from COND
- Evangelista's provisos outperforms SOURCE

[1] [Evangelista and Pajault, 2010]

Provisos Based on Destination Expansion

- Proposed by Nalumasu and Gopalakrishnan [2002] in a narrower context

SOURCE

DEST

Systematically expands the source of a backegde

Provisos Based on Destination Expansion

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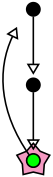

Systematically expands the source of a backegde

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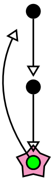
SOURCE	DEST



Systematically expands the source of a backegde

Systematically expands the destination of a backedge

Provisos Based on Destination Expansion

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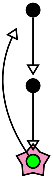
Source	Dest
	
Systematically expands the source of a backegde	Systematically expands the destination of a backedge

Provisos Based on Destination Expansion

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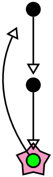	
Systematically expands the source of a backegde	Systematically expands the destination of a backedge

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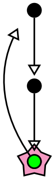
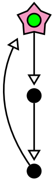
	
Systematically expands the source of a backegde	Systematically expands the destination of a backedge

Optimizations for these new provisos

- Compatible with: COND, WEIGHTED, KNOWN

Mark for expansion ■ Already visited edge → Not yet visited edge ⇢

Optimizations for these new provisos

- Compatible with: Cond, Weighted, Known

Colored	Unknown	Deepest

Mark for expansion ■ Already visited edge → Not yet visited edge ⇢

Optimizations for these new provisos

- Compatible with: Cond, Weighted, Known

Colored	Unknown	Deepest

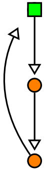
Reuse colors
Mark for expansion
Expand iff necessary

Mark for expansion ■ Already visited edge → Not yet visited edge ⇢

Optimizations for these new provisos

- Compatible with: COND, WEIGHTED, KNOWN

COLORED	UNKNOWN	DEEPEST

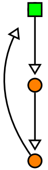
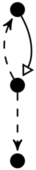


Reuse colors
Mark for expansion
Expand iff necessary

Mark for expansion ■ Already visited edge → Not yet visited edge ⇢

Optimizations for these new provisos

- Compatible with: COND, WEIGHTED, KNOWN

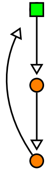
COLORED	UNKNOWN	DEEPEST
		
Reuse colors Mark for expansion Expand iff necessary	Prioritizing unknown successsors	

Mark for expansion ■ Already visited edge → Not yet visited edge ⇢

Optimizations for these new provisos

- Compatible with: Cond, Weighted, Known

Colored



Reuse colors
Mark for expansion
Expand iff necessary

Unknown

Prioritizing
unknown
successsors

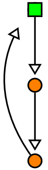
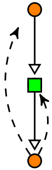
Deepest

Only mark the deepest
dest. for expansion

Mark for expansion ■ Already visited edge → Not yet visited edge - ->

Optimizations for these new provisos

- Compatible with: COND, WEIGHTED, KNOWN

COLORED	UNKNOWN	DEEPEST
		
Reuse colors Mark for expansion Expand iff necessary	Prioritizing unknown successsors	Only mark the deepest dest. for expansion

Mark for expansion ■ Already visited edge → Not yet visited edge ⇢

Evaluation

	states (10^6)		transitions (10^6)		st/ms
DeepestDestUnknown	276.51	35.25%	570.52	21.31%	11.81
DeepestDest	275.31	35.10%	566.63	21.16%	11.87
WeightedDestUnknown	273.94	34.92%	563.61	21.05%	11.83
Dest	272.79	34.77%	508.17	18.98%	14.48
WeightedDest	272.68	34.76%	559.73	20.90%	11.80
WeightedSourceKnownScan	248.11	31.63%	498.68	18.62%	11.70
CondDest	213.98	27.28%	413.15	15.43%	12.57
CondDestUnknown	213.92	27.27%	412.75	15.41%	12.52
ColoredDest	213.92	27.27%	412.93	15.42%	12.54
ColoredDestUnknown	213.83	27.26%	412.27	15.40%	12.46

- CondDest outperforms state-of-the-art provisos
- Weighted and Deepest variants are disappointing

Improving Provisos With SCCs information

- When destination is red, an expansion is required:
  - Until now, the source was expanded

Improving Provisos With SCCs information

- When destination is red, an expansion is required:
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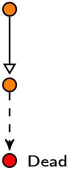
DEAD	HIGHLINKS

Improving Provisos With SCCs information

- When destination is red, an expansion is required:
  - Until now, the source was expanded

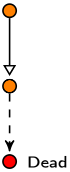
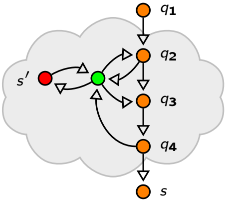
Dead	Highlinks



Avoid expansions when dest. is dead, i.e. in a fully visited SCC

Improving Provisos With SCCs information

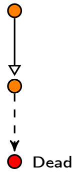
- When destination is red, an expansion is required:
  - Until now, the source was expanded

Dead	Highlinks
	
Avoid expansions when dest. is dead, i.e. in a fully visited SCC	Adaptation of Deepest when dest. is not on the DFS and not dead

Improving Provisos With SCCs information

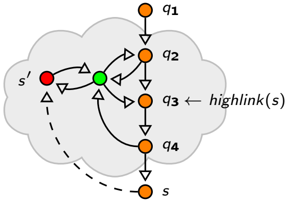
- When destination is red, an expansion is required:
  - Until now, the source was expanded

Dead



Avoid expansions when dest. is dead, i.e. in a fully visited SCC

Highlinks



Adaptation of Deepest when dest. is not on the DFS and not dead

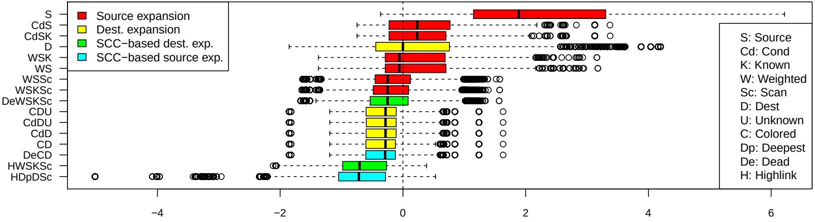
DEAD and HIGHLINKS are compatibles with both source and destination expansion-based provisos.

Evaluation 1/2

	states (10^6)		transitions (10^6)	
DeepestDest	275.31	35.10%	566.63	21.16%
DeadDeepestDest	269.10	34.30%	543.64	20.30%
WeightedDest	272.68	34.76%	559.73	20.90%
DeadWeightedDest	270.62	34.50%	554.91	20.72%
DeadWeightedSourceKnownScan	247.68	31.57%	497.79	18.59%
ColoredDest	213.92	27.27%	412.93	15.42%
DeadColoredDest	213.87	27.26%	412.80	15.42%
HighlinkWeightedDest	207.41	26.44%	393.22	14.68%
HighlinkWeightedDestScan	206.23	26.29%	391.05	14.60%
HighlinkWeightedSourceKnown	203.20	25.90%	386.84	14.45%
HighlinkWeightedSourceKnownScan	203.08	25.89%	386.60	14.44%
HighlinkDeepestDest	192.84	24.58%	349.89	13.07%
HighlinkDeepestDestScan	191.78	24.45%	347.95	12.99%

Evaluation 2/2

- Standard score for selected provisos
  - take the set of 1600 runs generated
  - compute a mean number μ_M for each model M
  - compute a standard deviation σ_M for each model M
  - standard score for a run r is then $\frac{states(r) - \mu_M}{\sigma_M}$
- Boxplot standard score



Results

- Overview of state-of-the-art provisos for checking liveness properties
- New heuristics: Colored, Deepest, Dead, Highlink
- Combination with existing heuristics
- Intensive evaluation
- Independant of the reduction technique: ample set, sttuborn set, etc. (see [Laarman et al., 2014] for survey)

Our recommended provisos:

- CondDest in NDFS-based emptiness-checks
- HighlinkWeightedSourceKnown in SCC-based emptiness checks (no scan required)

Explore new Lands . . .

Perspectives

- Parallel Algorithms
  - **Exploit Topology**:
    - *If the automaton to check is linear, parallel algorithms can't help to speed up computation*
  - **Mix UFSCC with POR**:
    - *CNDFS has been successfuly mixed with POR and can benefits from all previous techniques.*
  - **Improve classical ω-automata algorithms**
- Distributed Algorithms
  - **Improve existing algorithms**
  - **Build message-passing algorithms rather than shared memory-one**


Explicit Techniques Valley
strong
terminal
weak
NDFS
SCC
Symbolic techniques Mountains
ω-automata villages
The forest of the Emptiness
Dijkstra Temple
SCC hills
Tarjan Temple
Bitstate haching
State space caching
On the fly
The one hundred bridges of the proviso
Decomposition Tower
The dead Forest of the Union-Find
Renault
UFSCC
Map
Stubborn
Lakes of the POR
Conflicting
Persistent
Timed Automata
The outpost of the parellism
bledge
nege
owcty
swarming
Linear Topology
SCC+POR
endfs
lndfs
ω-automata parallel emptiness checks
CNDFS
mc-ndfs
Satisfiability
Distributed islands

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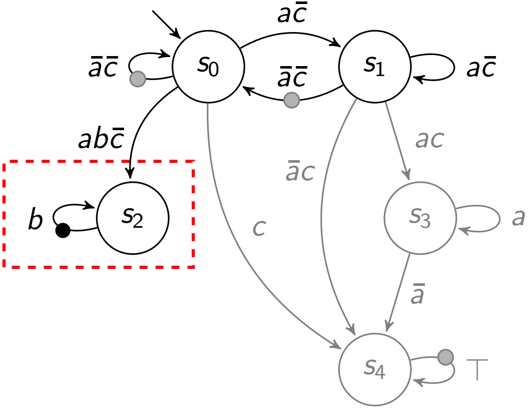
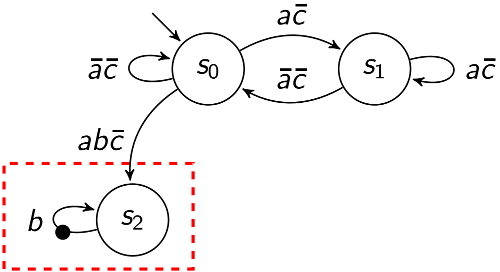
Construction of A_W



All acceptance sets are removed and
a single acceptance set labels all transitions of *weak* SCC.

Construction of A_W



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