Evidence Theory [episode 2]
Implementation Issues and Applications

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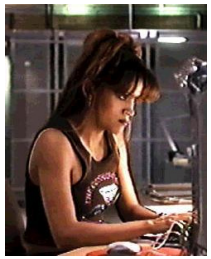
Fed up with TV series?

Someone is a traitor...



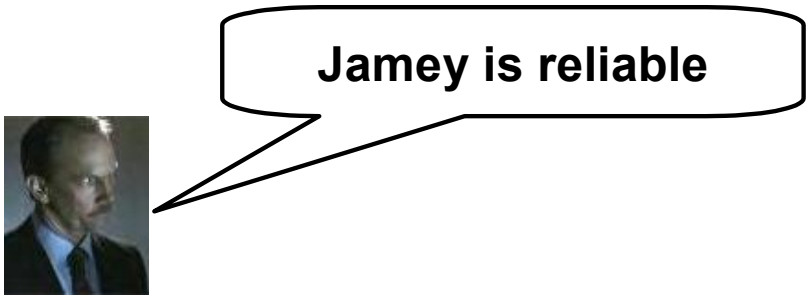
Jamey Farrell

Tony Almeida

Nina Myers

- 1 variable: traitor = { Jamey, Tony, Nina }
- belief mass on focal element X = belief that X contains the traitor
- potentials = episodes

Episode 1



Richard Walsh

▷ $m(\{Tony, Nina\}) = 0.7$

Richard is not omnipotent.

▷ $m(\{Jamey, Tony, Nina\}) = 0.3$

- Reminder:

$$\sum_{A \subseteq \Omega} m(A) = 1$$

$$ep_1 = \{\{Tony, Nina\}, \{Jamey, Tony, Nina\}\}$$

Episode 2

Jack Bauer

Statistically, people with a family are more sensitive to pressure or money temptation

- $m(\{Nina\}) = 0.4$
- $m(\{Jamey, Tony\}) = 0.6$

$$ep_2 = \{\{Nina\}, \{Jamey, Tony\}\}$$

➔ Statistical support

- Combination of ep_1 and ep_2

$$ep_1 = \{\{T, N\}[0.7], \{J, T, N\}[0.3]\}$$
$$ep_2 = \{\{N\}[0.4], \{J, T\}[0.6]\}$$

  - **Reminder**: Combination
    - focal elements intersections
    - $m_{D=A\cap B=A\cap C} = m_A \times m_B + m_A \times m_C$

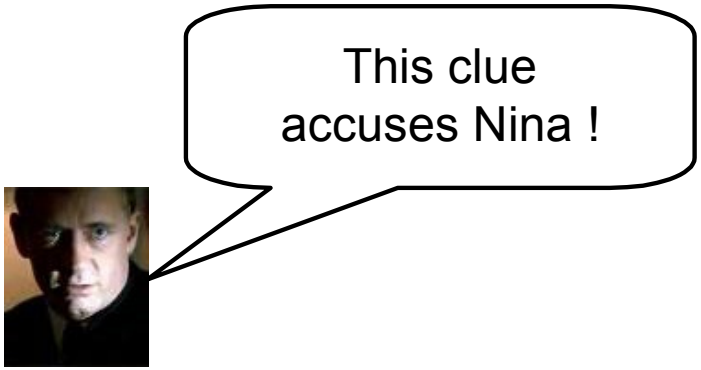
  - Generated focals elements:

$$ep_{1\oplus 2} = \{\{N\}, \{T\}, \{J, T\}\}$$

  - Belief Masses:

$$m(\{Nina\}) = 0.7 \times 0.4 + 0.3 \times 0.4 = 0.4$$
$$m(\{Tony\}) = 0.7 \times 0.6 = 0.42$$
$$m(\{Jamey, Tony\}) = 0.3 \times 0.6 = 0.18$$

Episode 3



Georges
Masson

- $m(\{Nina\}) = 0.8$
- $m(\{Jamey, Tony\}) = 0.2$

$$ep_3 = \{\{Nina\}, \{Jamey, Tony\}\}$$

- Combination of ep_3 and $ep_{1\oplus 2}$

$$ep_3 = \{\{N\}[0.8], \{J,T\}[0.2]\}$$
$$ep_{1\oplus 2} = \{\{N\}[0.4], \{T\}[0.42], \{J,T\}[0.18]\}$$

  - ▷ Generated focals elements:

$$ep_{1\oplus 2\oplus 3} = \{\{N\}, \{T\}, \{J,T\}\}$$

  - ▷ Conflict C:

$$\{N\} \cap \{T\} = \varnothing \text{ and } \{N\} \cap \{J,T\} = \varnothing$$
$$C = 0.8 \times 0.42 + 0.8 \times 0.18 + 0.2 \times 0.4 = 0.56$$

  - ▷ Belief Masses:

$$m(\{Nina\}) = \frac{0.8 \times 0.4}{1-C} \simeq 0.73$$
$$m(\{Tony\}) = \frac{0.2 \times 0.42}{1-C} \simeq 0.19$$
$$m(\{Jamey, Tony\}) = \frac{0.2 \times 0.18}{1-C} \simeq 0.08$$

Belief State and Decision after 3 episodes

- **Reminder:**

$$bel_A = \sum_{\emptyset \neq X \subseteq A} m(X)$$

$$pl_A = 1 - bel_{\overline{A}} = \sum_{X \cap A \neq \emptyset} m(X)$$

$$ep_{1\oplus 2\oplus 3} = \{\{N\}[0.73], \{T\}[0.19], \{J,T\}[0.08]\}$$

- **{Jamey}**

$bel_{\{Jamey\}} = 0$ $\quad$ $pls_{\{Jamey\}} = 0.08$

- **{Tony}**

$bel_{\{Tony\}} = 0.19$ $\quad$ $pls_{\{Tony\}} = 0.19 + 0.08 = 0.27$

- **{Nina}**

$bel_{\{Nina\}} = 0.73$ $\quad$ $pls_{\{Nina\}} = 0.73$

➔ Take care about interpretations !

- Decision ➔ probability construction (Smets et al., 1992)

$$BetP(x,m) = \sum_{x \in A \subseteq \Omega} \frac{m(A)}{|A|}$$

$$ep_{1 \oplus 2 \oplus 3} = \{\{N\}[0.73], \{T\}[0.19], \{J,T\}[0.08]\}$$

▷ **{Jamey}**

$$BetP(J) = \frac{0.08}{2} = 0.04$$

▷ **{Tony}**

$$BetP(T) = 0.19 + \frac{0.08}{2} = 0.24$$

▷ **{Nina}**

$$BetP(N) = 0.73$$

Is this theory right?
To be continued...

illustrations from “24” TV series: http://www.fox.com/24/

Implementation

- Mass Functions representation
- Focal Elements representation
- Potentials representation
- Combination & fusion issues

Mass Functions representation

- Mass function = distribution of belief on Ω

▷ **In theory**:
mass functions assign mass **values** to sets.

▷ **In reality**:
masses depend on **context** (events).

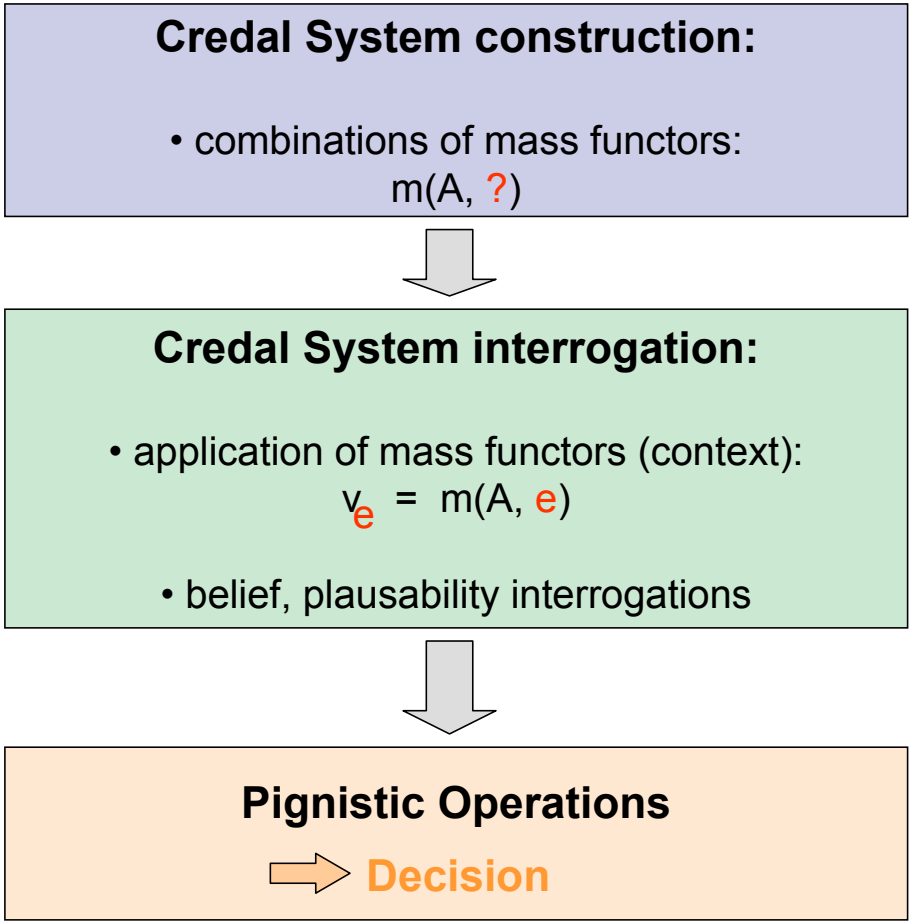
Example, in the field of **Medical Diagnosis**:

Combining “general rules” (potentials), and applying the system obtained on multiple cases, according to patient’s characteristics.

➔ Need for **functors**

$$\begin{array}{rccc} m: & 2^{\Omega} \times E & \rightarrow & [0,1]_{\Re} \\ & (A,e) & \rightarrow & v \end{array}$$

➔ Model principles:



- **Consequences of this model**
  - ▷ Different levels of evaluation
  - ▷ Approximations impossible

Focal Elements representation

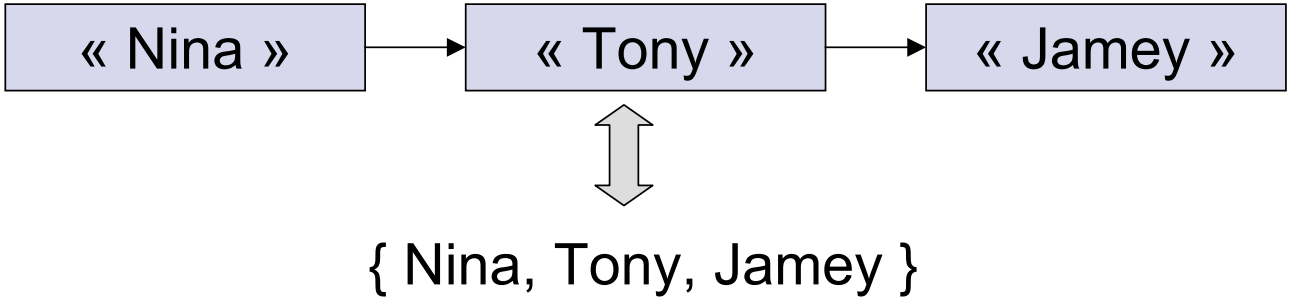
- Focal Element = set of variable configurations ➔ **hypothesis**

▷ Operations: **set intersections, projections, extensions**

▷ Size issues

▷ Efficiency issues

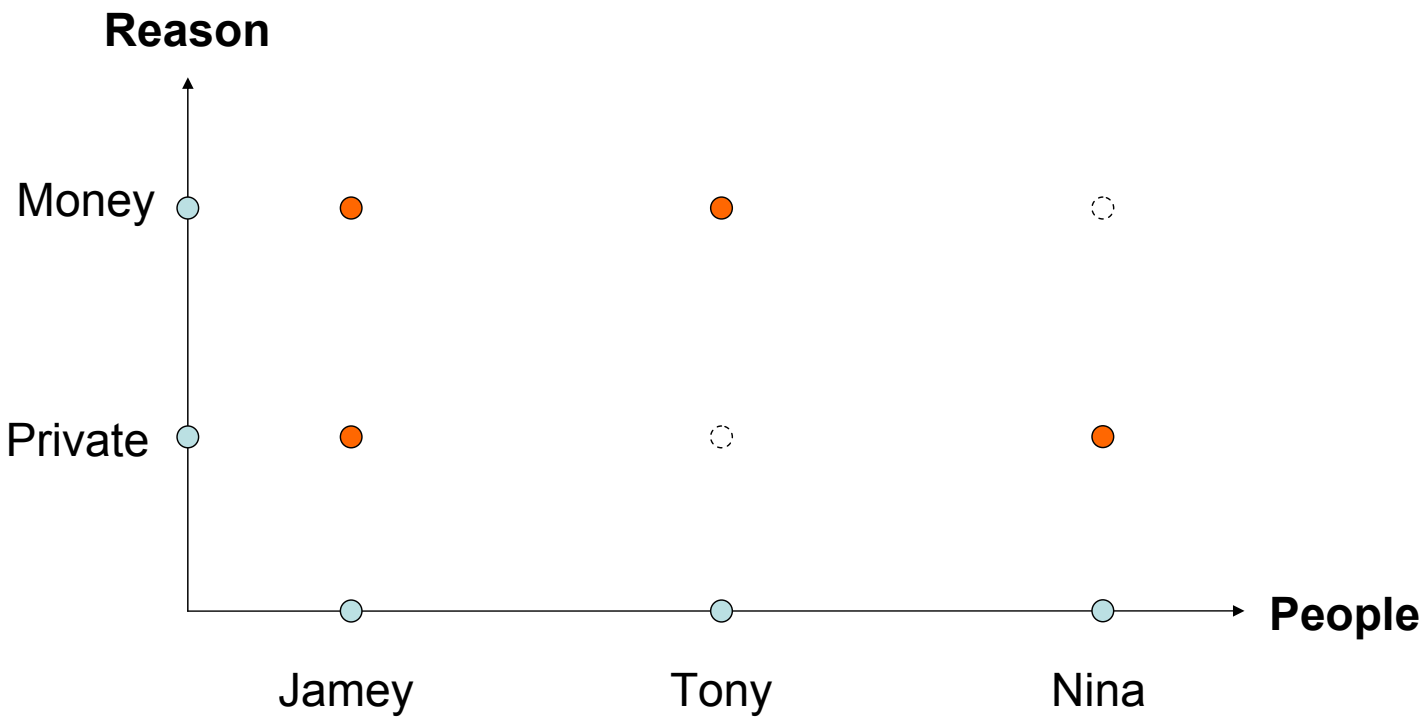
- **Lists?**



  - ▷ speed, size, ordering problems

- **Ordered sets?**
  - ▷ solve only ordering problems

- **Bitsets?**



{ (Jamey, Money), (Jamey, Private),
(Tony, Money),
(Nina, Private) }

⟺ **< 11 10 01 >**

Bitsets properties

- ▷ efficient operations on sets
- ▷ set bit number = configurations number
- ▷ constant size:

$$S = \prod_{i=1}^{nbvars} \#X_i bits$$

ex: for 23 binary variables, $size(FS) = 2^{23}$ bits $= 1$ Mo

- ▷ $Index \Leftrightarrow Configuration$ correspondancy formulas
- ▷ Independence towards original representation !

- **Other representations**
  - ▷ Normal forms

Disjunctive forms

$$r(A) = \bigcup_{i=1}^{h1} \bigcap_{j=1}^{n} S_j^i$$

$$S_j^i \in \Theta_{x_j}$$

Conjunctive forms

$$r(A) = \bigcap_{i=1}^{h1} \bigcup_{j=1}^{n} S_j^i$$

$$S_j^i \in \Theta_{x_j}$$

Potentials representation

- Potential = collection of pairs (F_i, m_i)
  with $F_i \in FS$ and m_i mass function ➔ **1 source of information**
  - ▷ Operations on potentials: **combinations, fusions**
  - ▷ Need for efficient research and combination ➔ **regroupments**
  - ▷ **Lists?** research: *O(n)* regroupment: *O(n^2)*
  - ▷ **Balanced Trees?** research: *O($log(n)$)* regroupment: *O($n \times log(n)$)*
  - ▷ **Hash Tables?** research: *O(1)* regroupment: *O(n^2/s)*

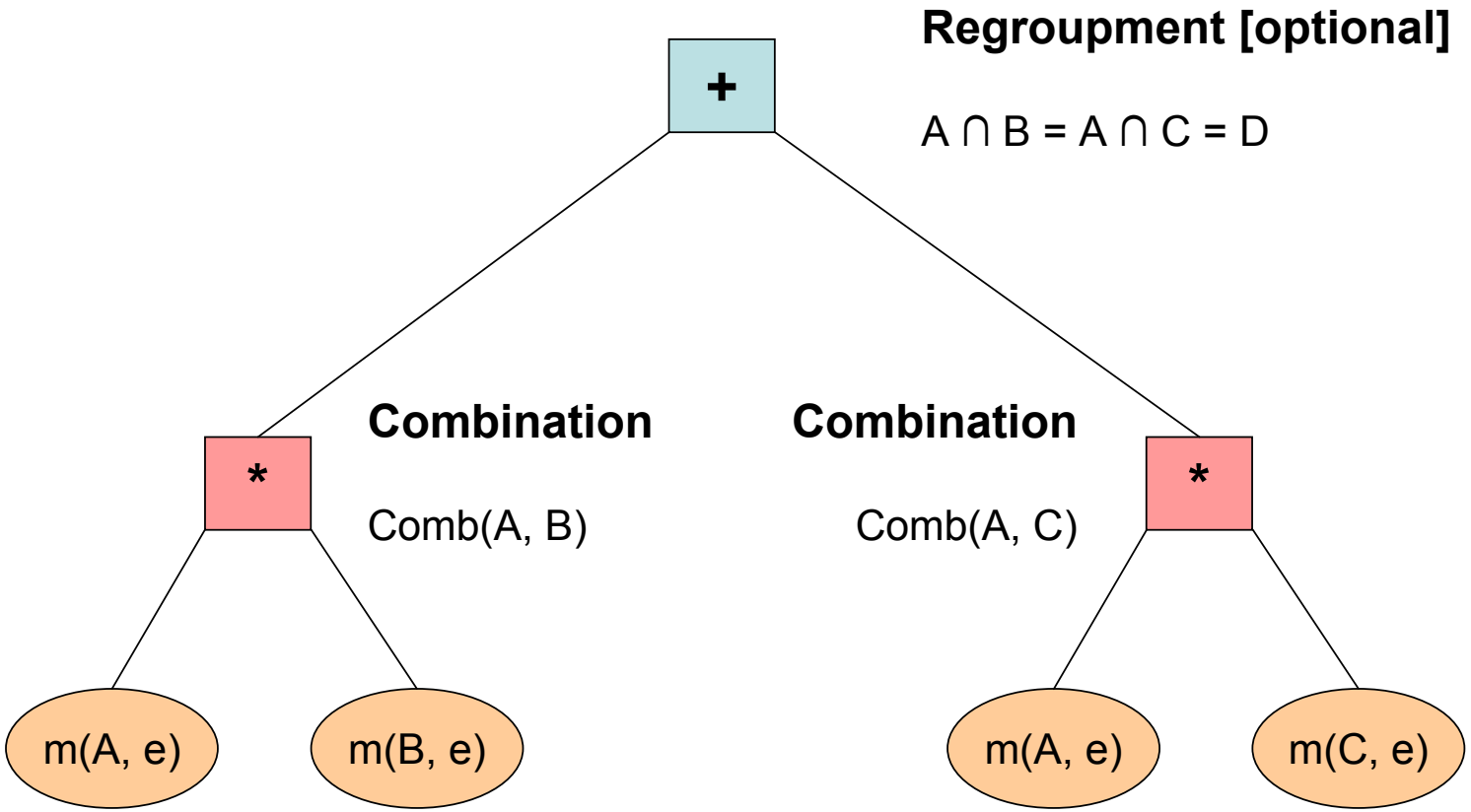
Example of a combination of 2 potentials of 1000 focal elements with 1000 configurations:

balanced trees (AVL)	6 seconds
lists	14 hours 22 minutes

Combination & fusion issues

- New potentials creation ➔ focal elements creation (by intersection)
- New mass functions creation ➔ functors creation
- Mass functions combinations and regroupment

Mass Function of D = A ∩ B = A ∩ C



▷ **indirections** due to functors !

▷ example, with 1 potential P, 10 focal elements of 10 configurations, on 1 variable with 10 realisations, after 5 combinations:

construction time	1 second
memory load	1 Mo
original focal elements	10
final focal elements	163
interrogation time (1 Focal element)	2 minutes
mass regroupments (plus indirections)	1.6059×10^9
mass combinations (times indirections)	3.2642×10^9

- optimization?

Evidenz: an evidence theory engine

- Existing engines: research prototypes, written in Lisp...
  (Haenni and Lehmann, 2001) (Saffiotti and Umkehrer, 1991)

  - Evidenz characteristics
  - Performances
  - Evidenz engine future improvements

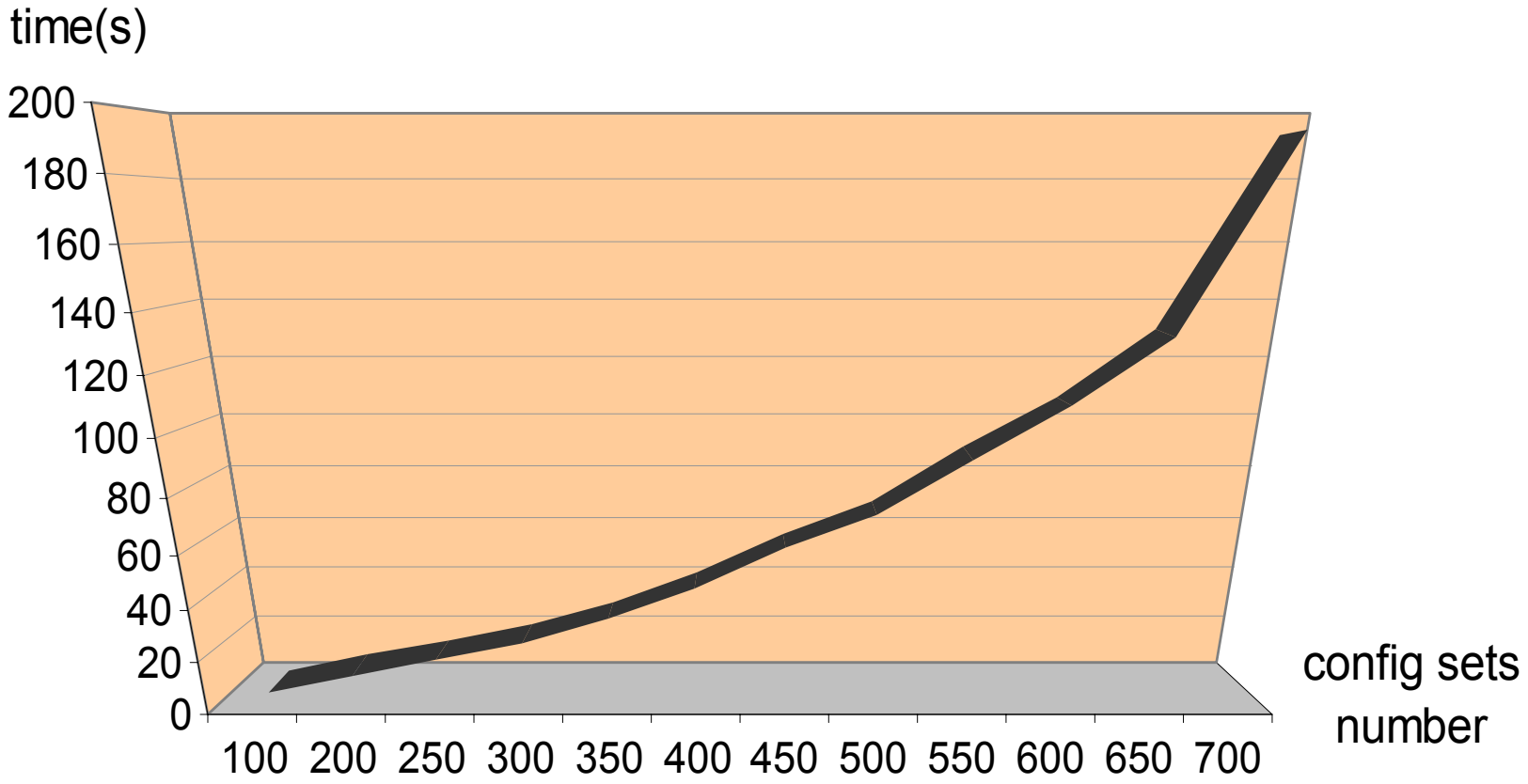
Evidenz characteristics

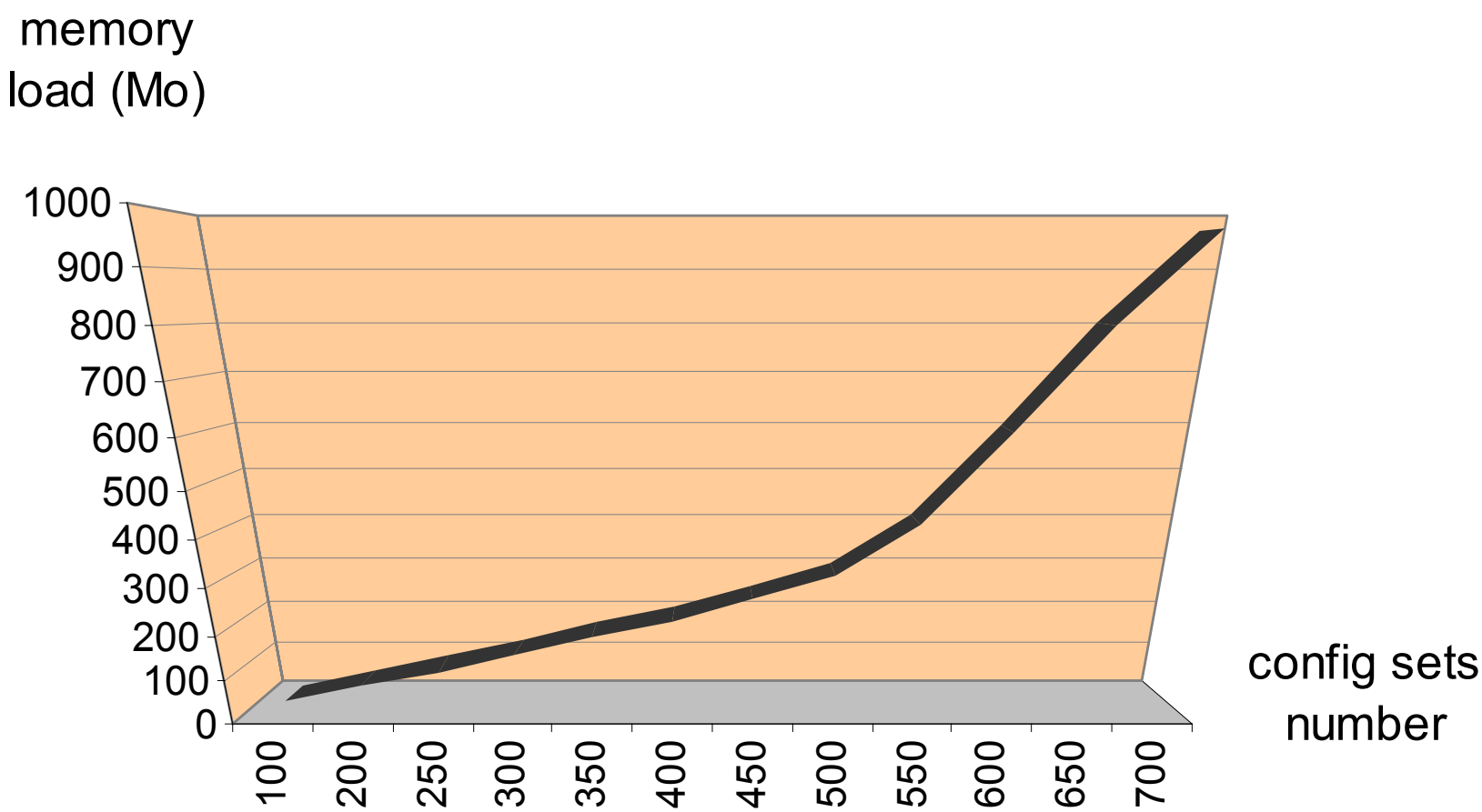
- TBM exact modeling (Smets et al., 1992)
- written in C++ (dynamic version)

- focal elements represented by bitsets (Boost)
- generic towards configurations representation
- potentials represented by balanced trees (STL maps)

Performances

- Study of $\neq$ factors influence on execution **time** and **memory load**:
  - ▷ variables
  - ▷ variable realisations
  - ▷ focal elements per potential
  - ▷ configurations per focal element
  - ▷ **one combination of 2 potentials randomly generated**
- example, on **focal elements per potential** influence:
  - ▷ 2 variables with 500 realisations
  - ▷ 100 configurations per focal element
  - ▷ 100 to 700 focal elements per potential


time(s)
200
180
160
140
120
100
80
60
40
20
0
100 200 250 300 350 400 450 500 550 600 650 700
config sets number

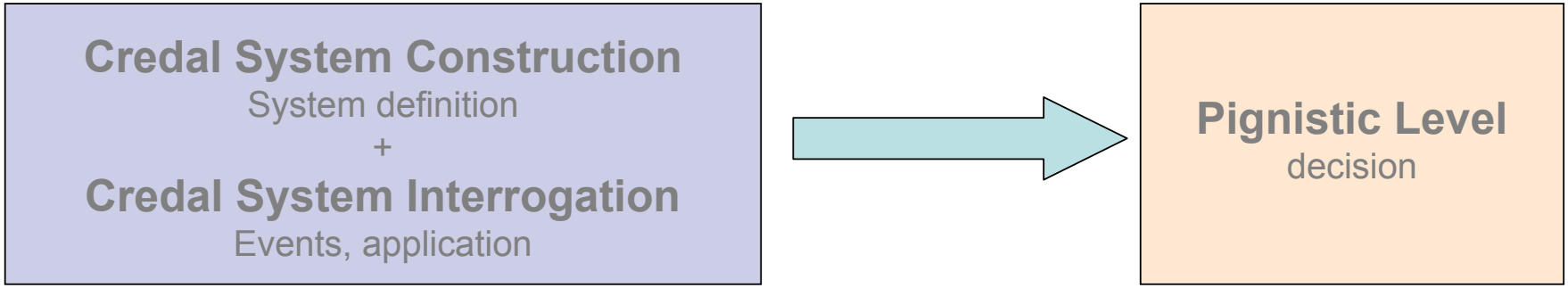

memory
load (Mo)
1000
900
800
700
600
500
400
300
200
100
0
100
200
250
300
350
400
450
500
550
600
650
700
config sets
number

- **Performance study conclusions**
  - ▷ Time and memory load evolutions are **exponential** towards **all** the factors
  - ▷ Problems become quickly infeasible !
  - ▷ In practice, applications are **much more** reasonable...

Evidenz engine future improvements

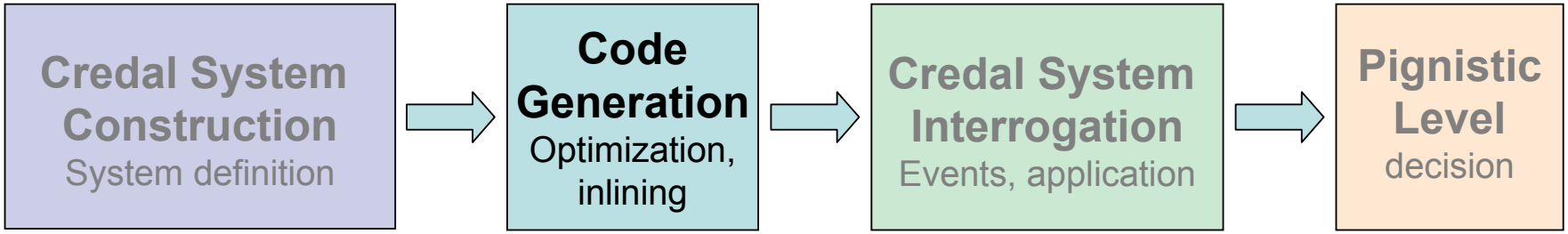
- Heuristics for **optimizing elimination sequences** (using **Join trees**)
- Decision algorithms, like **decision trees**
- **Memoizing** (set intersections and mass functions)

- **2 use cases:**
  - **Need of quick, unique application**:



  - **Approximations**

- **Need of multiple applications on different events**:

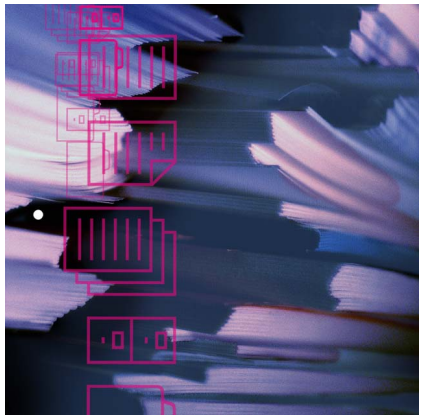


- **code generation**: generates an optimal system at the end of credal definition phase

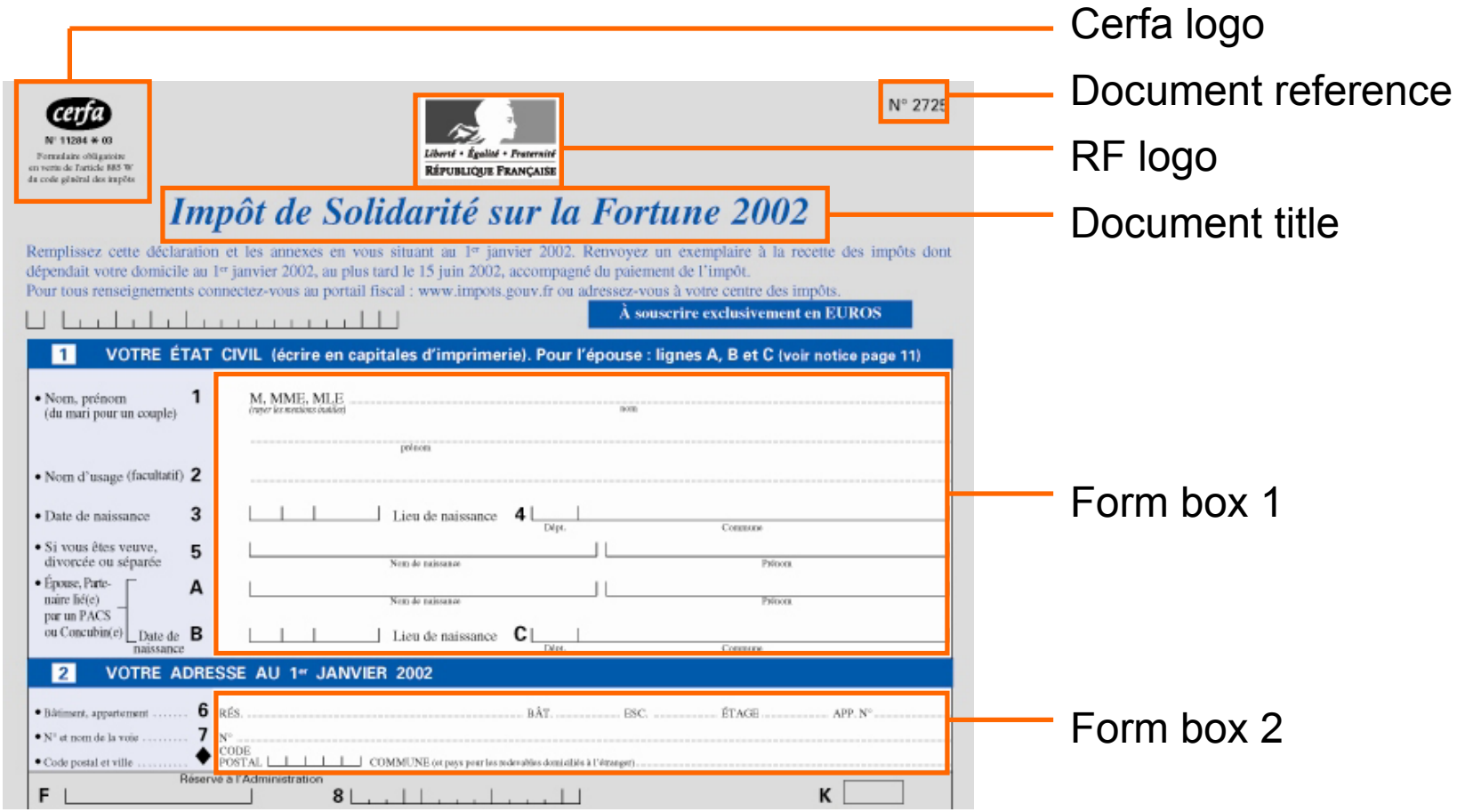
Applications

- Document classification
- Cytology (Adhoc^2)
- Satellite image classification

Document classification

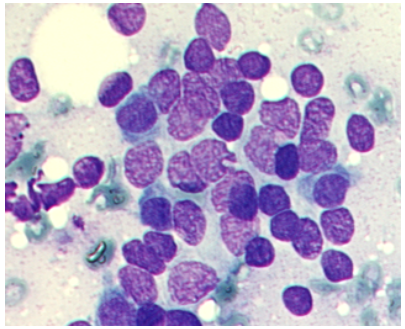


- Automatic classification and data extraction
  - ▷ to associate a model to a document
- A document contains **anchors**: logos, text boxes...
- Each anchor corresponds to one or several models

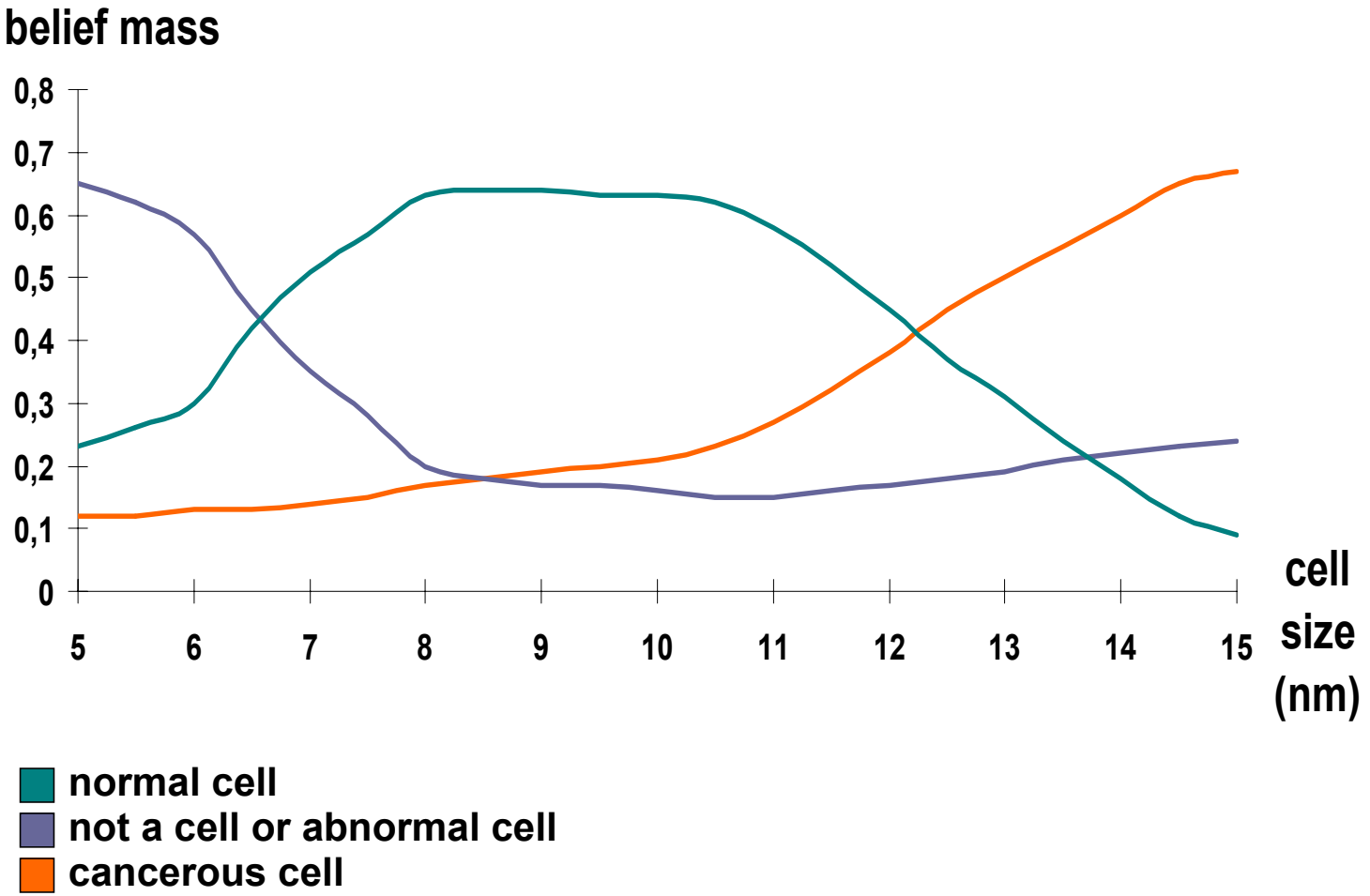

Cerfa logo
Document reference
RF logo
Document title
Form box 1
Form box 2
cerfa
N° 11284 * 03
Liberté • Égalité • Fraternité
RÉPUBLIQUE FRANÇAISE
N° 2725
Impôt de Solidarité sur la Fortune 2002
Remplissez cette déclaration et les annexes en vous situant au 1er janvier 2002. Renvoyez un exemplaire à la recette des impôts dont dépendait votre domicile au 1er janvier 2002, au plus tard le 15 juin 2002, accompagné du paiement de l'impôt.
Pour tous renseignements connectez-vous au portail fiscal : www.impots.gouv.fr ou adressez-vous à votre centre des impôts.
À souscrire exclusivement en EUROS
1 VOTRE ÉTAT CIVIL (écrire en capitales d'imprimerie). Pour l'épouse : lignes A, B et C (voir notice page 11)
• Nom, prénom (du mari pour un couple) 1
M, MME, MLE
• Nom d'usage (facultatif) 2
• Date de naissance 3
Lieu de naissance 4
• Si vous êtes veuve, divorcée ou séparée 5
• Épouse, Partenaire lié(e) par un PACS ou Concubin(e) A
Date de naissance B
Lieu de naissance C
2 VOTRE ADRESSE AU 1er JANVIER 2002
• Bâtiment, appartement 6
• N° et nom de la voie 7
• Code postal et ville
Réservé à l'Administration
F
8
K

- Modelisation:
  - ▷ 1 variable: $document\_model = \{model_1, model_2, model_3, ..., model_k\}$
  - ▷ potentials = anchors
- Anchor recognition ➔ belief masses for model sets
  - ▷ combining anchor masses ➔ global belief for each document model

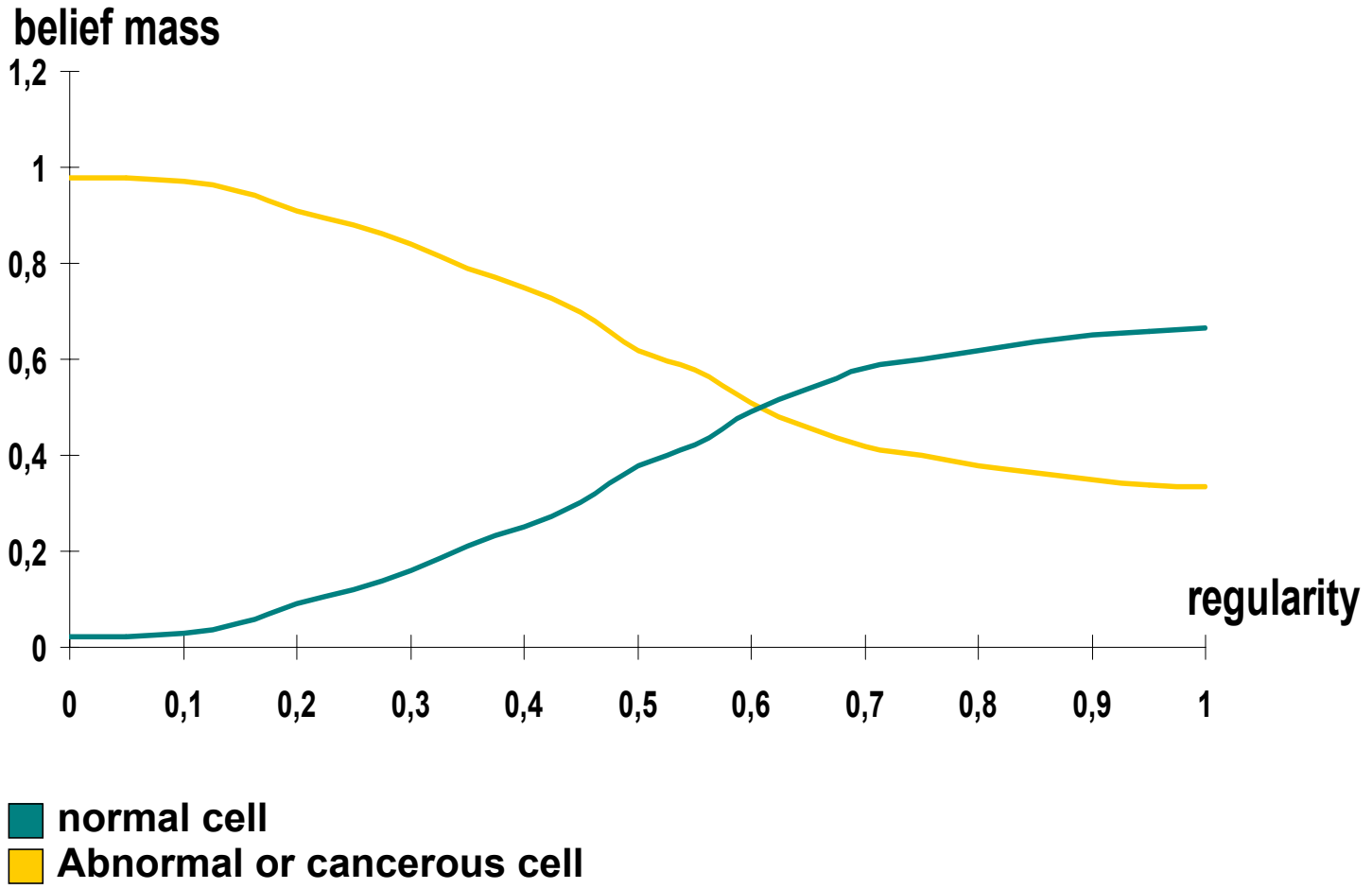
Cytology (Adhoc^2)



- Cancerous cells detection
  - ▷ diagnosis help
  - ▷ $\neq$ factors:
    - $*$ cell/nucleus size
    - $*$ cell/nucleus regularity
    - $*$ color...
- **Fuzzy**, **subjective** decision rules


belief mass
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
5
6
7
8
9
10
11
12
13
14
15
cell size (nm)
normal cell
not a cell or abnormal cell
cancerous cell

Size factor



Regularity factor

- Modelisation:
  - ▷ 1 variable: $cell\_type = \{normal, abnormal, not\_a\_cell, cancerous\}$
  - ▷ potentials = criters: size, regularity, color...

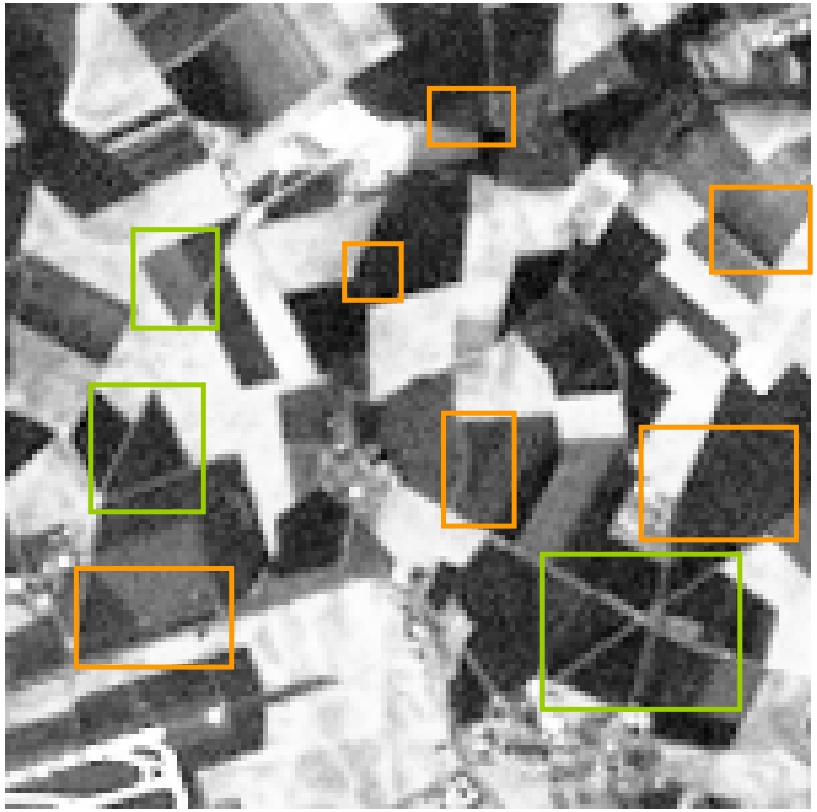
- Cell characteristics ➔ belief masses for cell type sets
  - ▷ combining criters ➔ global belief for each cell type
  - ▷ justified decision, quantified ignorance...

Satellite image classification

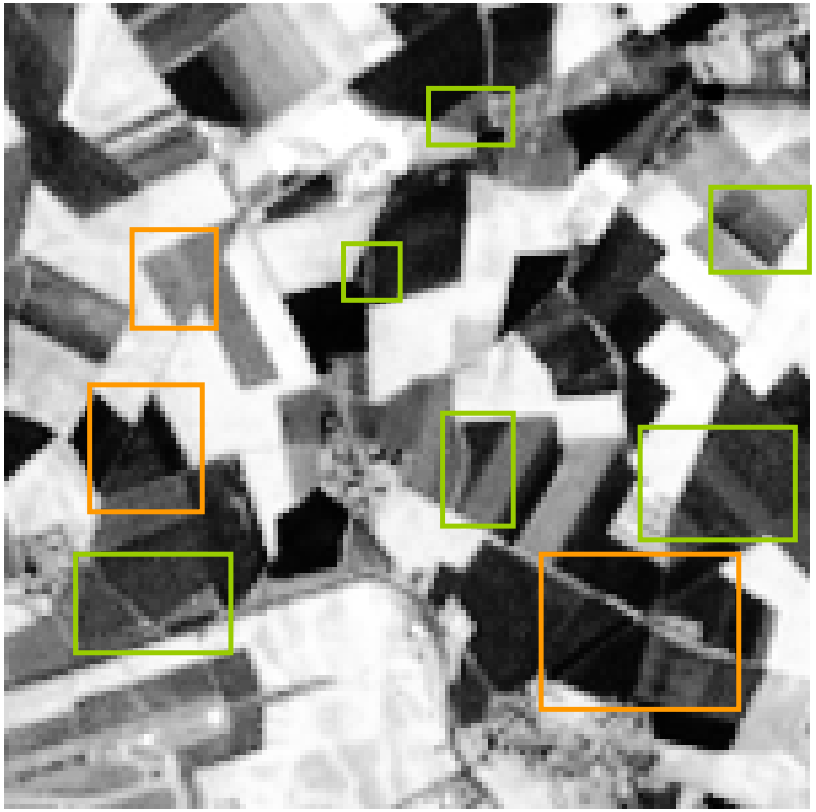
- Satellite images composed of several layers:
  - 7 layers according to wave length
  - Different visions of the same image ➔ different informations

Layer x

Layer y



Layer x



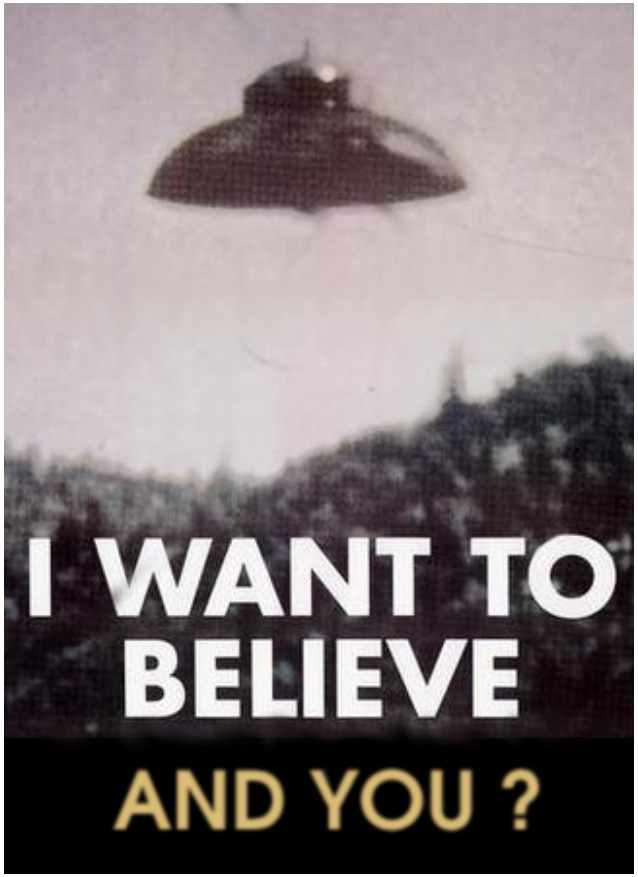
Layer y

- Modelisation:
  - ▷ 1 variable: $pixel\_type = \{forest, water, road, building, ...\}$
  - ▷ potentials = layers
- Pixel gray level (**context**) ➔ belief masses for pixel type sets
  - ▷ combining layers ➔ global belief for each pixel type

Conclusion

- Theory designed for dealing with uncertainty
  - ▷ Modelisation power
  - ▷ Computational complexity
- Evidenz engine
  - ▷ Code generation and algorithmic extension/improvement
- Future research applications:
  - ▷ Adhoc^2 cytology project
  - ▷ Satellite image classification

Questions?



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