Morphological Filtering in Shape Spaces: Applications using Tree-Based Image Representations

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Abstract

Connected operators are filtering tools that act by merging elementary regions of an image. A popular strategy is based on tree-based image representations: for example, one can compute an attribute on each node of the tree and keep only the nodes for which the attribute is sufficiently strong. This operation can be seen as a thresholding of the tree, seen as a graph whose nodes are weighted by the attribute. Rather than being satisfied with a mere thresholding, we propose to expand on this idea, and to apply connected filters on this latest graph. Consequently, the filtering is done not in the space of the image, but on the space of shapes build from the image.

Such a processing is a generalization of the existing tree-based connected operators. Indeed, the framework includes classical existing connected operators by attributes. It also allows us to propose a class of novel connected operators from the leveling family, based on shape attributes. Finally, we also propose a novel class of self-dual connected operators that we call morphological shapings.

1. Introduction

Mathematical morphology, as originally developed by Matheron and Serra [10], proposes a set of morphological operators based on structuring elements. Later, Salembier and Serra [8], followed by Breen and Jones [1], proposed morphological operators based on attributes, rather than on elements. Such operators rely on transforming the image into an equivalent representation, generally a tree of components (*e.g.* level sets) of the image; such trees are equivalent to the original image in the sense that the image can be reconstructed from its associated tree. Filtering then involves the design of a shape attribute that weights how much a node of the tree fits a given shape. Two different approaches for filtering the tree (and hence the image) have been proposed: the more evolved approach consists in pruning the tree by removing whole branches of the tree, and is easy to apply if the attribute is increasing on the tree (*i.e.*, if the attribute is always stronger for the ancestors of a node). The process is illustrated in Figure 1 by the black path.

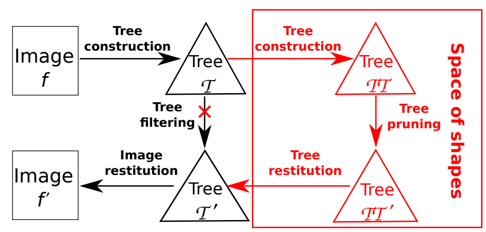


Figure 1. Classical connected operators (black path) and our proposed process (by adding the red path).

However, most shape attributes are not increasing. When the attribute is not increasing, three strategies have been proposed (min, max, Viterbi; see [9] for more details). They all choose a particular node on which to take the decision, and remove the whole subtree rooted in this node. While it may give interesting results in some cases, it does not take into account the possibility that several relevant objects can have some inclusion relationship, which means that they are on the same branch of the tree (for instance a ring object in a tree of shapes, see Figure 5 (a)).

In the simplest approach, one simply removes the nodes of the tree for which the attribute is lower than a given threshold [11]. Such a thresholding does not take into account the intrinsic parenthood relationship of the tree. Moreover it is often impossible to retrieve all expected objects with one unique threshold.

The founding idea of this paper is to apply connected filters on the space made by all the components of the image, such space being structured into a graph by the parenthood relationship (i.e., a node has for neighbors its children and its parent). This is the red path in Figure 1. This surprising and simple idea has several deep consequences, that form the main contributions of this paper. First, we show that the framework encompasses the usual attribute filtering operators. Second, novel connected filters based on non-increasing criterion can be proposed, and we show that such filters are new morphological levelings [4]. And third, we propose a novel family of self-dual connected filters that we call *morphological shapings*.

The rest of this paper is organized as follows. Our proposed shape-based morphology is explained in Section 2. In Section 3, we depict some experimental results. Finally we conclude in Section 4.

2. Shape-based morphology

As stated in the introduction, one way to compute connected operators is to represent the input image f by a component tree $\mathcal{T}$, either a min-tree (given by the inclusion relationship of the connected components of the lower level sets), a max-tree [7] (upper level sets), or a tree of shapes [5]. The connected components are given thanks to a neighborhood, usually C4 or C8. Let us remark that the tree $\mathcal{T}$ with its nodes valued with an attribute $\mathcal{A}$ can be seen as a node-weighted graph where vertices are tree nodes, adjacency (graph edges) is parenthood. Such a graph is a shape-space, as any node is a component of the original image. As an edge of the graph represents an inclusion relationship between components, the neighborhood of a node is a kind of "context" of the node (component) in the image.

If, for example, $\mathcal{A}$ encodes the probability for a component to be of a given type, the minima of the space of shapes are the components that are the less probable to be of that type, compared to their parents and children. As a node-weighted graph, the space $\mathcal{T}$ can be represented by a component tree $\mathcal{TT}$ (explicitly and for clarity a min-tree in the sequel of this paper). Minima of $\mathcal{A}$ on $\mathcal{T}$ are leaves of $\mathcal{TT}$. Pruning or removing some branches of the new tree $\mathcal{TT}$ thus removes the parts of $\mathcal{T}$ that are less likely to be of the type favored by $\mathcal{A}$. Remark that with this strategy, we can remove two different "objects" located in the same branch of $\mathcal{T}$, but represented by two different branches of $\mathcal{TT}$.

Let us now compare our approach with the state of the art in connected filtering. The regular case is when the attribute $\mathcal{A}$ is increasing, *i.e.* when $\mathcal{A}$ is lower for a

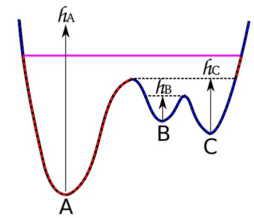


Figure 2. Illustration of the extinction values of three minima.

component than for its parent. In that case, the min-tree $\mathcal{TT}$ is equal to $\mathcal{T}$. Taking the red path in Figure 1 is equivalent to taking the black path. In other words, the proposed approach encompasses the classical one.

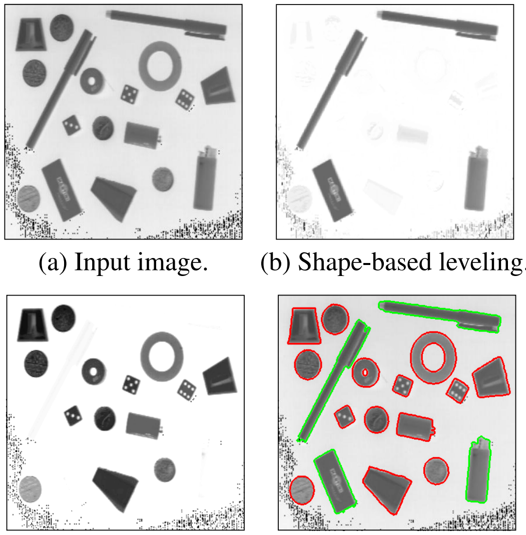
A shape-attribute $\mathcal{A}$ is more often non-increasing. In such a case, $\mathcal{TT}$ is different from $\mathcal{T}$. Figure 2 shows an example of a non-increasing attribute in a branch of $\mathcal{T}$. Pruning $\mathcal{TT}$ is equivalent to the thresholding of $\mathcal{A}$ on $\mathcal{T}$. In Figure 2, such a pruning removes the nodes whose $\mathcal{A}$ is under the purple line.

Another classical strategy is to compute an attribute $\mathcal{AA}$ on $\mathcal{TT}$, and to prune the tree according to this attribute in shape-space. In that case,

- when the tree $\mathcal{T}$ is a min-tree or a max-tree, such a morphological filtering in shape space is a leveling of the original image (recall that an operator is a leveling if and only if it preserves the order $\leq$ and $\geq$ between neighboring pixels). We call such a filtering *shape-based leveling*.
- when the tree $\mathcal{T}$ is a tree of shape, then the order between neighboring pixels is no longer preserved by the filtering, and it is thus no longer a leveling. We call this new family of self-dual morphological connected filters the *morphological shapings*.

Of course, $\mathcal{AA}$ can be as simple as the height of a component in the shape space. But other attributes $\mathcal{AA}$ can be computed based on $\mathcal{A}$ or even the image domain: for example, the area of a connected component of $\mathcal{TT}$ can be the number of pixels of this set of components in the original image.

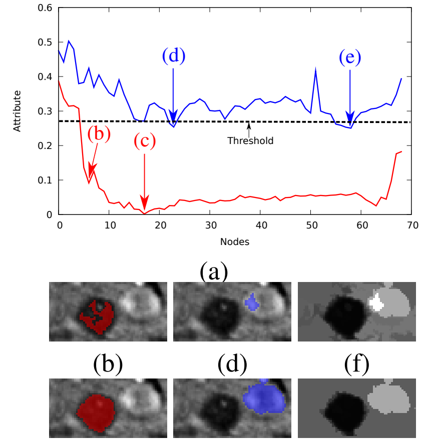
Let us mention another interesting variant for the filtering strategy. It is based on the use of extinction value [12] of minima. Let $\prec$ be a strict total order on the set of minima $m_1 \prec m_2 \prec \ldots$, such that $m_i \prec m_{i+1}$ whenever the altitude of m_i is lower than the altitude of m_{i+1}. Let CC be the lowest connected component that contains both m_{i+1} and a minimum m_j with $j < (i+1)$. The extinction value for the minimum m_{i+1} is defined as the difference between the altitude of CC and the altitude of m_{i+1}. Figure 2 shows an example of the extinction value for three minima. The order is $A \prec C \prec B$. The filtering strategy is to preserve (or remove) only the blobs determined by a minimum whose extinction value is higher than a given value. The ad-



(a) Input image. (b) Shape-based leveling.

(c) is (a) minus (b). (d) Object detection.

Figure 3. Illustration of shape-based leveling and object detection.



(a)

(b) (d) (f)

(c) (e) (g)

Figure 4. (a) Evolution of "circularity" on two branches of $\mathcal{T}$; (b to e): Some shapes; (f) Attribute thresholding; (g) Shaping.

vantage of this strategy is that it preserves only those shapes which are meaningful enough compared to their context. For example, the red part in Figure 2 is preserved for the value given by the purple line, but the blobs corresponding to minima B and C are removed.

3. Some illustrations

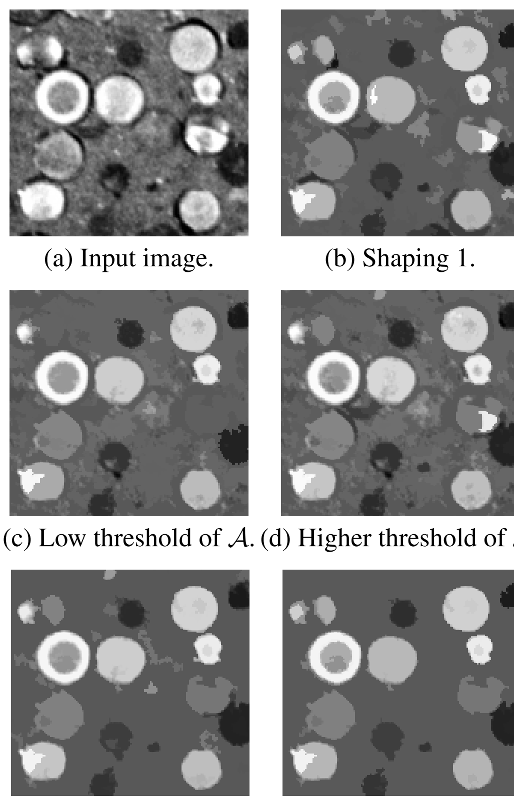
In this section, we present some illustrations of our new filters, both for image filtering (section 3.1) and for object detection (section 3.2).

3.1. Object filtering

In Figure 3, we present a round shape-based leveling filter. For that, $\mathcal{T}$ is the min-tree of the input image (a) and $\mathcal{A}$ is a circularity attribute. The tree simplification $\mathcal{T}\mathcal{T} \rightarrow \mathcal{T}\mathcal{T}'$ is performed by a morphological closing of $\mathcal{A}\mathcal{A}$. The reconstruction gives the filtered image (b), which is a leveling of the input image. The image (c) is a top-hat (subtraction of the leveling from the input image) that depicts the objects removed by the filter.

Figure 4 shows an example of evolution of a shape attribute, the circularity on a simple image. The light round shape and the dark one are both meaningful round objects compared to their context. However, their attribute values are very different. In order to obtain the light one, a higher threshold is applied, but this makes some non desired shapes appear, the shapes presented in the background in Figure 4 (f).

In Figure 5, we compare our extinction-based self-dual shaping approach with a variant of the state-of-



(a) Input image. (b) Shaping 1.

(c) Low threshold of $\mathcal{A}$. (d) Higher threshold of $\mathcal{A}$.

(e) Threshold strategy. (f) Shaping 2.

Figure 5. Comparison of extinction-based shapings with attribute-thresholding. (b-d): Using one shape attribute; (e-f): Using a combination of shape attributes.

the-art thresholding approach [11]. To process both upper and lower level sets, we propose to use a tree of shapes $\mathcal{T}$. In this illustration $\mathcal{A}$ is the circularity of image shapes for (b) to (d). When the threshold of $\mathcal{A}$ is low, we loose some objects (see (c)). To be able to get all expected objects, we have to set a high threshold; however, we keep too many unwanted objects (see (d)). With our shaping filter, we obtain all the expected objects as depicted in (b). The results can be improved by combining some shape attributes. In (e) and (f), we use a combination of circularity and the I/A^2 [11], the moment of inertia divided by the square of area. The combination of shape attributes improves significantly the results. Still, our shaping in (f) performs much better than the threshold strategy in (e).

3.2. Object detection

Some authors have proposed to rely on image representation based on morphological trees to extract *meaningful* objects. In [6], objects are selected when their level lines match some criterion based on compactness and contrast. In [2], objects are spotted as minima of the number of false alarms estimated on every level lines thanks to an *a contrario* model. We have introduced in [13] an energy functional, computed on image components of the tree of shapes and dedicated to object detection. This functional mixes a snake-like term and a contextual Mumford-Shah term, and its minima correspond to potential meaningful objects. Put differently $\mathcal{T}$ is the tree of shapes and $\mathcal{A}$ is the energy functional. To extract the significant minima (so the proper objects), we compute the min-tree $\mathcal{TT}$ of the space of shapes $\mathcal{T}$ valued with attribute $\mathcal{A}$; a morphological closing with attribute $\mathcal{AA}$ provides a simplified tree $\mathcal{TT}'$; the only minima remaining in $\mathcal{TT}'$ are the relevant objects.

This strategy is valid for any general-purpose energy functional, including any shape attribute, or a combination of shape attributes. As such, we are able to extract objects based on their shape features. In Figure 3 (d), red and green contours correspond to components with significant minima of respectively a circularity attribute and an elongation attribute.

4. Conclusion

This paper has presented two new classes of morphological connected filters: new levelings (obtained with $\mathcal{T}$ being the min-tree or the max-tree) and self-dual shapings (obtained with $\mathcal{T}$ being the tree of shapes [5]). Those connected operators filter image components based on some non-increasing shape criterion. We have also shown that they encompass the usual attribute filtering operators. Properties of the morphological shaping (such as conditions for idempotence) will be studied in a forthcoming extended version.

We have implemented those filters [1] using our C++ image processing library [3], available on the Internet as free software. The present paper also demonstrates the interest of applying image processing algorithms to different types of graphs, in other words, the interest of genericity for image processing.

Acknowledgments

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