Efficient Computation of Attributes and Saliency Maps on Tree-Based Image Representations

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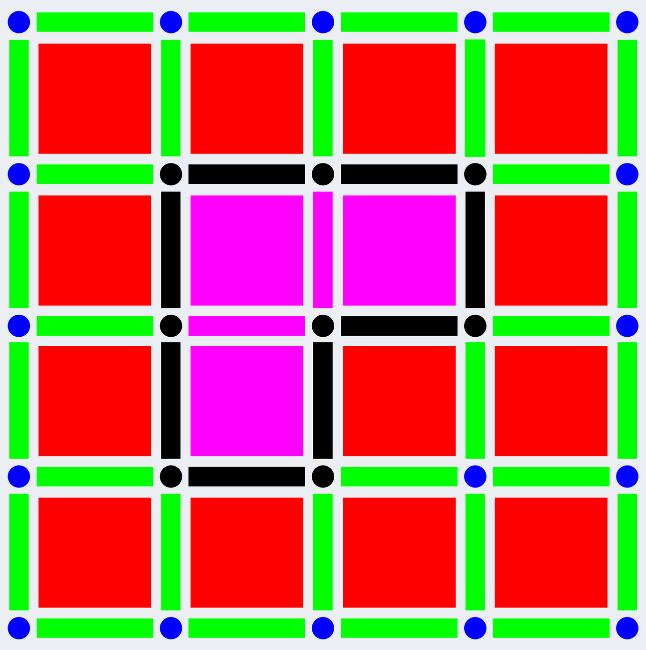
At a Glance

Issue Tree representations (many algorithms [1, 2, 3]) + attributes (very few explicit computation [4]): popular tools in MM and IP.

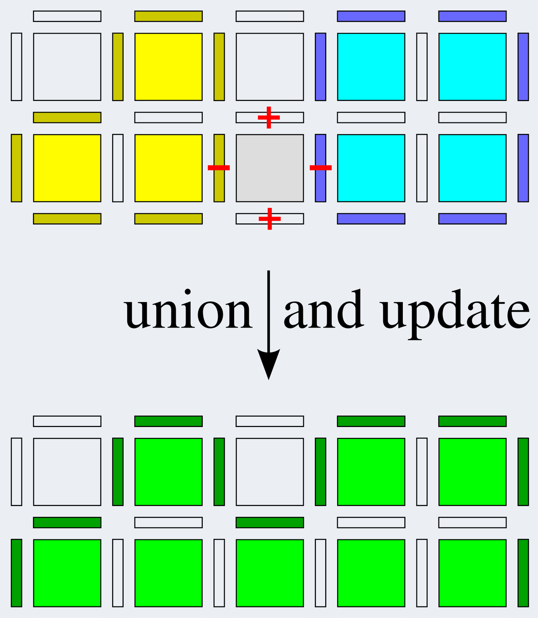
Goal Proposition of some efficient algorithms for computation of attributes and saliency maps.

Contribution A set of efficient algorithms for computation of:
- some accumulated information on region, contour, and context;
- extremal information along the contour (*e.g.*, NFA [5]);
- extinction-based saliency maps [6].

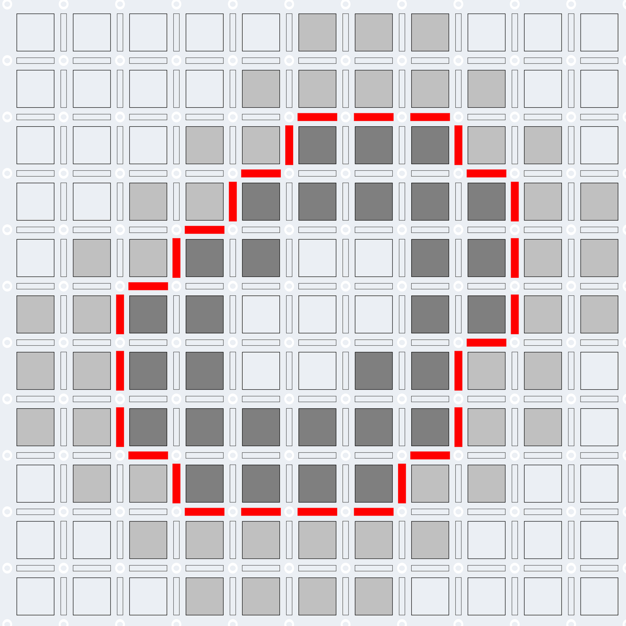
Basic structure and information to compute + some computation principles



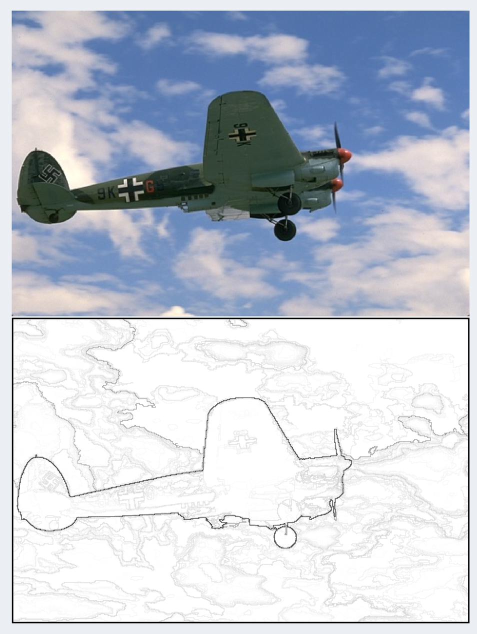
Khalimsky grid.



Updating contour.



Regional context.



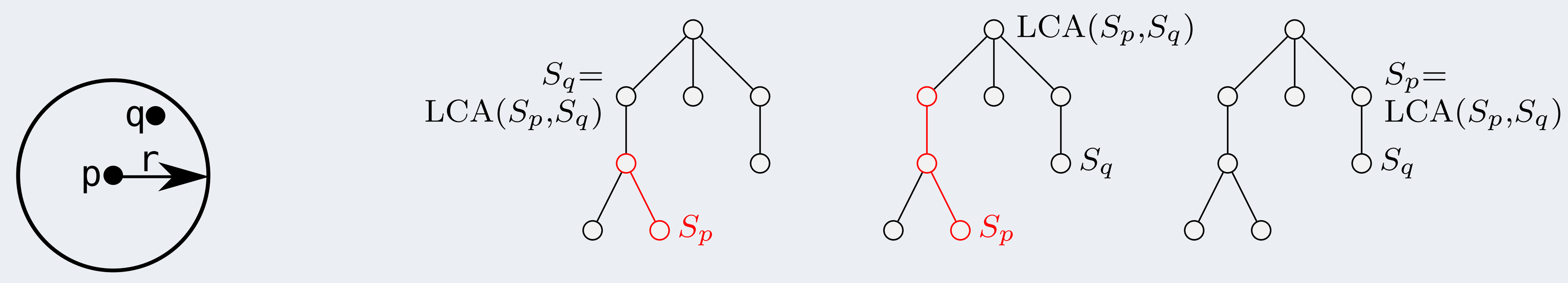
Extinction-based saliency map.

Algorithm computing incrementally some accumulated information

UNION_FIND($\mathcal{R}$) //black lines: union-find process; gray lines: for information on contour
for all p **do**
  $zpar(p) \leftarrow$ undef;
  $A(p) \leftarrow \hat{0}$, $L(p) \leftarrow \hat{0}$; $X^i(p) \leftarrow \hat{0}$, $X^e(p) \leftarrow \hat{0}$, $V_L(p) \leftarrow \widehat{M}$; $\mathcal{M}_\mathcal{E}(p) \leftarrow \hat{0}$;
for all e **do** $is\_boundary(e) \leftarrow$ false;
for $i \leftarrow n-1$ **to** 0 **do**
  $p \leftarrow \mathcal{R}[i]$, $parent(p) \leftarrow p$, $zpar(p) \leftarrow p$;
  $A(p) \leftarrow A(p) \hat{+} i_A(p)$;
  for all $n \in \mathcal{N}(p)$ **such as** $zpar(n) \neq$ undef **do**
    $r \leftarrow$ FIND_ROOT$(zpar, n)$; **if** $r \neq p$ **then**
      $parent(r) \leftarrow p$, $zpar(r) \leftarrow p$;
      $A(p) \leftarrow A(p) \hat{+} A(r)$; $L(p) \leftarrow L(p) \hat{+} L(r)$;
      $X^i(p) \leftarrow X^i(p) \hat{+} X^i(r)$, $X^e(p) \leftarrow X^e(p) \hat{+} X^e(r)$;
  for all $e \in \mathcal{N}_4(p)$ **do**
    if not $is\_boundary(e)$ **then**
      $is\_boundary(e) \leftarrow$ true;
      $L(p) \leftarrow L(p) \hat{+} i_L(e)$; //$i_L$: information on 1-faces
      //i_X^{tr} and i_X^{dl}: top-right and down-left context of 1-faces
      if e is above or on the right of p **then**
        $X^i(p) \leftarrow X^i(p) \hat{+} i_X^{dl}(e)$, $X^e(p) \leftarrow X^e(p) \hat{+} i_X^{tr}(e)$;
      else $X^i(p) \leftarrow X^i(p) \hat{+} i_X^{tr}(e)$, $X^e(p) \leftarrow X^e(p) \hat{+} i_X^{dl}(e)$;
      $appear(e) \leftarrow p$;
    else
      $is\_boundary(e) \leftarrow$ false;
      $L(p) \leftarrow L(p) \hat{-} i_L(e)$;
      if e is above or on the right of p **then**
        $X^i(p) \leftarrow X^i(p) \hat{-} i_X^{tr}(e)$, $X^e(p) \leftarrow X^e(p) \hat{-} i_X^{dl}(e)$;
      else $X^i(p) \leftarrow X^i(p) \hat{-} i_X^{dl}(e)$, $X^e(p) \leftarrow X^e(p) \hat{-} i_X^{tr}(e)$;
      $vanish(e) \leftarrow p$;
for all e **do**
  $N_a \leftarrow appear(e), N_v \leftarrow vanish(e)$;
  while $N_a \neq N_v$ **do**
    $V_L(N_a) \leftarrow$ update$(V_L(N_a), i_L(e))$; //**update**: either **min** or **max**
    $N_a \leftarrow parent(N_a)$;
return $parent$

Computation of information on region (in red), contour (in green), context (in blue), and extremal information along the contour (in magenta).

Principle of exact contextual information computation



$q \in B_r(p)$ $p \in$ interior and $q \in$ exterior of nodes $[S_p \rightsquigarrow LCA(S_p, S_q))$ (red path).

Algorithm computing exact contextual information

EXTERNAL_CONTEXT($parent$)
foreach *node* x **do** $X^e(x) \leftarrow \hat{0}$;
foreach point q in Ω **do**
  $DjVu \leftarrow \emptyset$;
  foreach point p in $B_\varepsilon(q)$ **do**
    $N_p \leftarrow$ getCanonical(p);
    $N_q \leftarrow$ getCanonical(q);
    $Anc \leftarrow$ LCA(N_p, N_q);
    while $N_p \neq Anc$ **do**
      if $N_p \notin DjVu$ **then**
        $X^e(N_p) \leftarrow X^e(N_p) \hat{+} i_X(q)$;
        $DjVu \leftarrow DjVu \cup \{N_p\}$;
      $N_p \leftarrow parent(N_p)$;
return X^e

INTERNAL_CONTEXT($parent$)
foreach *node* x **do** $X^i(x) \leftarrow \hat{0}$;
foreach point p in Ω **do**
  $DjVu \leftarrow \emptyset$;
  foreach point q in $B_\varepsilon(p)$ **do**
    $N_p \leftarrow$ getCanonical(p);
    $N_q \leftarrow$ getCanonical(q);
    $Anc \leftarrow$ LCA(N_p, N_q);
    while $N_p \neq Anc$ **do**
      if $N_p \notin DjVu$ **then**
        $X^i(N_p) \leftarrow X^i(N_p) \hat{+} i_X(p)$;
        $DjVu \leftarrow DjVu \cup \{N_p\}$;
      $N_p \leftarrow parent(N_p)$;
return X^i

$DjVu$: track the shapes for which the current point has already been considered.

Algorithm computing extremal information and saliency map

COMPUTE_SALIENCY_MAP(f)
$(\mathcal{T}, \mathcal{A}) \leftarrow$ COMPUTE_TREE(f);
$\mathcal{E} \leftarrow$ COMPUTE_EXTINCTION$(\mathcal{T}, \mathcal{A})$;
for all e **do** $\mathcal{M}_\mathcal{E}(e) \leftarrow 0$;
for all e **do**
  $N_a \leftarrow appear(e), N_v \leftarrow vanish(e)$;
  while $N_a \neq N_v$ **do**
    $\mathcal{M}_\mathcal{E}(N_a) \leftarrow$ **max**$(\mathcal{E}(N_a), \mathcal{M}_\mathcal{E}(e))$, $N_a \leftarrow parent(N_a)$;
for all 0-*face* o **do** $\mathcal{M}_\mathcal{E}(o) \leftarrow$ **max**$(\mathcal{M}_\mathcal{E}(e_1), \mathcal{M}_\mathcal{E}(e_2), \mathcal{M}_\mathcal{E}(e_3), \mathcal{M}_\mathcal{E}(e_4))$;
return $\mathcal{M}_\mathcal{E}$

Complexity analysis

Accumulated information on	Time complexity	Auxiliary memory cost
region	$O(n\,\alpha(n))$	0
contour	$O(n\,\alpha(n))$	$4n$
approximated context	$O(n\,\alpha(n))$	$4n$

Union-find process: $O(n\,\alpha(n))$;
n: number of pixels;
h: height of the tree;
ε: maximal distance to the contour for context.

Computation of	Time complexity	Auxiliary memory cost
exact contextual information	$O(n\varepsilon^2 h)$	$n+h$
extremal information along contour	$O(nh)$	$12n$ or $5n$
extinction-based saliency map	$O(nh)$	$12n$ or $5n$

References

[1] E. Carlinet and T. Géraud, "A comparative review of component tree computation algorithms," *IEEE Transactions on Image Processing*, vol. 23, pp. 3885–3895, Sept. 2014.

[2] P. Monasse and F. Guichard, "Fast computation of a contrast-invariant image representation," *IEEE Transactions on Image Processing*, vol. 9, no. 5, pp. 860–872, 2000.

[3] T. Géraud, E. Carlinet, S. Crozet, and L. Najman, "A quasi-linear algorithm to compute the tree of shapes of nD images," in *ISMM*, vol. 7883 of *Lecture Notes in Computer Science*, pp. 98–110, 2013.

[4] M. H. F. Wilkinson, H. Gao, W. H. Hesselink, J.-E. Jonker, and A. Meijster, "Concurrent computation of attribute filters on shared memory parallel machines," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 30, no. 10, pp. 1800–1813, 2008.

[5] F. Cao, P. Musé, and F. Sur, "Extracting meaningful curves from images," *Journal of Mathematical Imaging and Vision*, vol. 22, pp. 159–181, 2005.

[6] Y. Xu, T. Géraud, and L. Najman, "Two applications of shape-based morphology: Blood vessels segmentation and a generalization of constrained connectivity," in *ISMM*, vol. 7883 of *Lecture Notes in Computer Science*, pp. 390–401, 2013.