SELF-DUALITY AND DIGITAL TOPOLOGY:
Links between the morphological tree of shapes and well-composed gray-level images

Thierry Géraud, Edwin Carlinet*, Sébastien Crozet

EPITA Research and Development Laboratory (LRDE), France

thierry.geraud@lrde.epita.fr

* also with Université Paris-Est, Laboratoire d'Informatique Gaspard-Monge (LIGM), Équipe A3SI, ESIEE Paris, France

At a Glance

Problem statement:

- digital topology $\Rightarrow$ using a pair of connectivities (c_α, c_β) is required,
- actually self-duality is *impure* (see below), so we want to fix this.

Why it is interesting:

- values can be independant from the underlying graph structure,
- we can have a really pure self-dual representation.

What our solution achieves:

- a new representation of images,
- some interesting (?) theoretical results.

What follows from our solution:

- the companion paper [4] has nice extra results,
- and we are happy \o/

Self-dual operators

process the same way the image contents whatever the contrast...

...except for their connectivity:

$$u \xrightarrow{\varphi} \varphi_{(c_\alpha,c_\beta)}(u)$$

complementation ↓ complementation ↓

$$\complement u \xrightarrow{\psi \neq \varphi} \complement\, \varphi_{(c_\alpha,c_\beta)}(u) = \varphi_{(c_\beta,c_\alpha)}(\complement u)$$

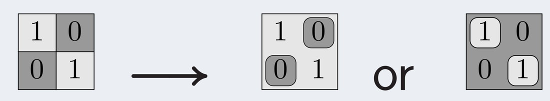
Tree of shapes [2]

a representation of the image contents which is self-dual...

...except for the connectivity:

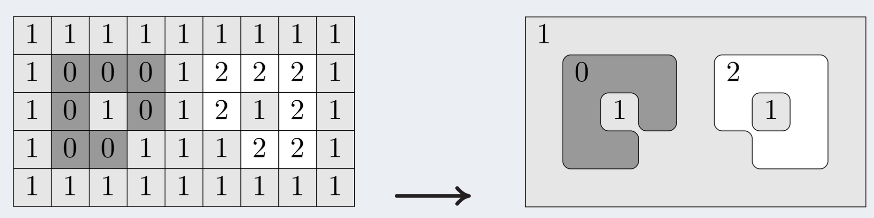
$$\mathfrak{S}_{(<,c_\alpha)}(\complement u) = \mathfrak{S}_{(<,c_\beta)}(u)$$

Flaws in self-duality



Arbitrary Choice $\longrightarrow$ (c_4, c_8) or (c_8, c_4)

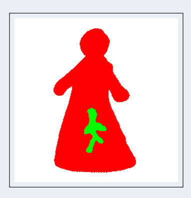
A gray-level image



Asymmetry $\longrightarrow$ $(<, c_4)$ so $(>, c_8)$

woman as foreground

The paradigm "foreground v. background" should be reconsidered →

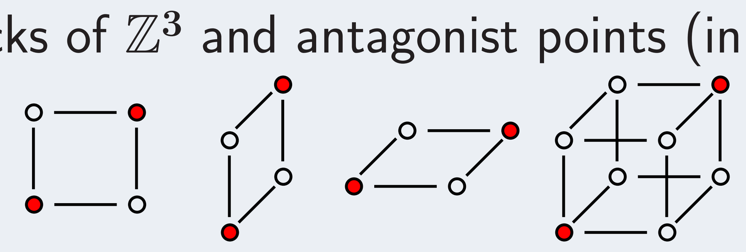


child as foreground so... woman as background

Proposed solution (Evangelization from the Church of Mathematical Morphology)

From Boutry et al. [3, 4]:

Blocks of $\mathbb{Z}^3$ and antagonist points (in red)



A critical configuration is either a set of two antagonist points $\{p, p'\}$ of a block S or a set $S \setminus \{p, p'\}$.

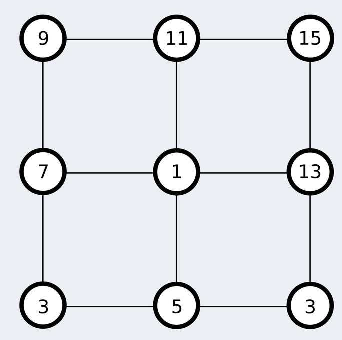
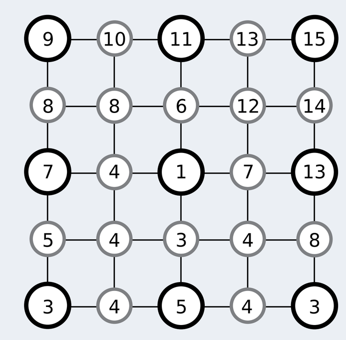
A set is *digitally well-composed* (DWC) iff it does not contain any critical configuration.

If a set is DWC, then its $2n$-components are identical to its $(3^n - 1)$-components.

so all connectivities are equivalent!

A gray-level image is said DWC iff all its threshold sets are DWC.

Our proposal: making an image DWC by interpolation

$u =$  $\mapsto$ $u_{\text{DWC}} =$ 

where $u : \mathcal{D} \subset \mathbb{Z}^n \longrightarrow \mathbb{Z}$ and u_{DWC} is defined on $\left(\frac{\mathbb{Z}}{2}\right)^n$.

The HOW-TO

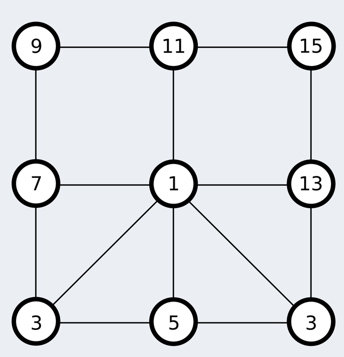
Turning a self-dual operator φ into a *pure* self-dual one φ^*

$$u \xrightarrow{\text{interpolation}} u_{\text{DWC}}$$

φ^* ↓ ↓ φ

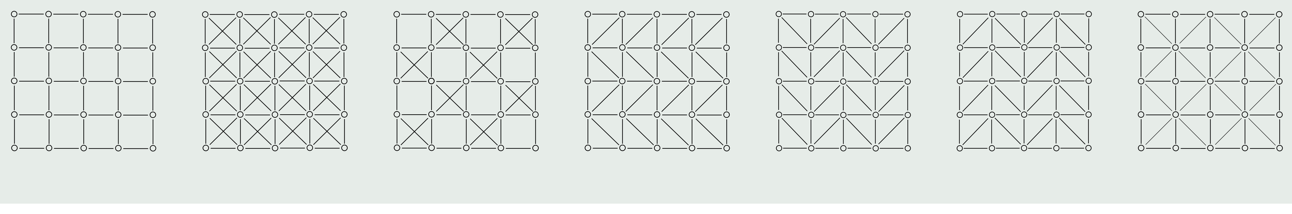
$$\varphi^*(u) = \varphi(u_{\text{DWC}})|_{\mathcal{D}} \xleftarrow{\text{un-interpolation}} \varphi(u_{\text{DWC}})$$

Actually it is *as if* we had this purely self-dual representation for u:



(the components of the threshold sets of u are the ones of u_{DWC} restricted to $\mathcal{D}$).

Quizz

Name these grids: 

Extra results from this paper

Theorem. If a gray-level nD image u is digitally well-composed, then the components of $\mathfrak{S}_{(<,c_{2n})}(u)$ form a purely self-dual tree of shapes.

Proposition. The only "morphological" digitally well-composed self-dual 2D interpolation is based on the median operator.

The proofs are provided in the paper at no extra charge...

Selected Bibliography

[1] R. Levillain, T. Géraud, and L. Najman, "Why and how to design a generic and efficient image processing framework: The case of the Milena library," in *Proc. of the IEEE International Conference on Image Processing (ICIP)*, 2010, pp. 1941–1944.

[2] T. Géraud, E. Carlinet, S. Crozet, and L. Najman, "A quasi-linear algorithm to compute the tree of shapes of n-D images," in *Proc. of the International Symposium on Mathematical Morphology (ISMM)*, vol. 7883 of LNCS, pp. 98–110, Springer, 2013.

[3] N. Boutry, T. Géraud, and L. Najman, "On making nD images well-composed by a self-dual local interpolation," in *Proc. of Discrete Geometry for Computer Imagery (DGCI)*, vol. 8668 of LNCS, pp. 320–331, Springer, 2014.

[4] N. Boutry, T. Géraud, and L. Najman, "How to make well-composed images in nD in a self-dual way with a front propagation algorithm," in *Proc. of the International Symposium on Mathematical Morphology (ISMM)*, vol. 9082 of LNCS, pp. 561–572, Springer, 2015.