SELF-DUALITY AND DIGITAL TOPOLOGY: Links between the morphological tree of shapes and well-composed gray-level images

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At a Glance
Problem statement:
• digital topology ⇒ using a pair of connectivities \( (c_A, c_B) \) is required, actually self-duality is impure (see below), so we want to fix this.
Why it is interesting:
• values can be independent from the underlying graph structure,
• we can have a really pure self-dual representation.

What our solution achieves:
• a new representation of images,
• some interesting (?) theoretical results.
What follows from our solution:
• the companion paper [4] has nice extra results,
• and we are happy \( \backslash o / \)

Self-dual operators
process the same way the image contents whatever the contrast...
...except for their connectivity:
\[
\begin{align*}
\mathcal{E}(c, c') & : \mathbb{Z}^n \to \mathbb{Z}^n \\
\mathcal{E}(c, c')(u) & = \mathcal{E}(c, c')(u)
\end{align*}
\]

Tree of shapes [2]
a representation of the image contents which is self-dual...
...except for the connectivity:
\[
\mathcal{E}(c, c')(u) = \mathcal{E}(c, c')(u)
\]

Proposed solution
(Evangelization from the Church of Mathematical Morphology)
From Boutry et al. [3, 4]:
Blocks of \( \mathbb{Z}^2 \) and antagonist points (in red)
A critical configuration is either a set of two antagonist points \( \{p, p'\} \) of a block \( S \) or a set \( S \setminus \{p, p'\} \).

A set is digitally well-composed (DWC) if it does not contain any critical configuration.
If a set is DWC, then its \( 2n \)-components are identical to its \( (3^n - 1) \)-components.
so all connectivities are equivalent!
A gray-level image is said DWC iff all its threshold sets are DWC.

Our proposal: making an image DWC by interpolation
\[
\mathcal{u}_{\text{DWC}} : \mathcal{D} \hookrightarrow \mathbb{Z}^n \quad \text{where} \quad \mathcal{u} \in \mathcal{D} \subset \mathbb{Z}^n 
\]

Flaws in self-duality

The paradigm "foreground v. background"
should be reconsidered
child as foreground so...
woman as background

Extra results from this paper
Theorem. If a gray-level \( n \)-D image \( \mathcal{u} \) is digitally well-composed, then the components of \( \mathcal{E}(c, c')(\mathcal{u}) \) form a purely self-dual tree of shapes.
Proposition. The only "morphological" digitally well-composed self-dual 2D interpolation is based on the median operator.
The proofs are provided in the paper at no extra charge...

Selected Bibliography