# A Quick Tour of Mathematical Morphology

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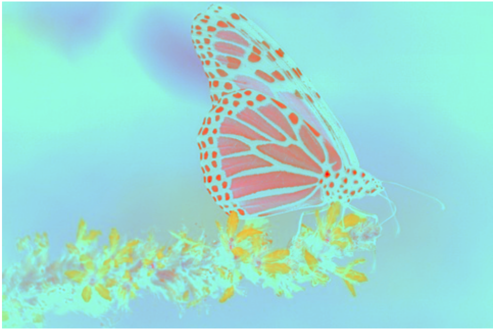
# Forewords

Which animal is it?

# Forewords



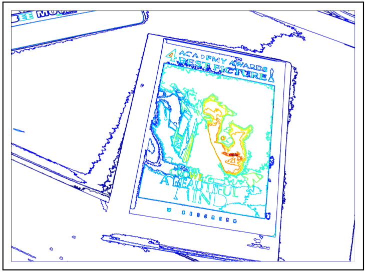
(Again:) Which animal is it?

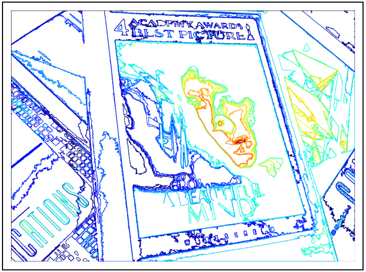
# Forewords

This question translates into:

- What makes you recognize this animal?
- What are the expected <u>invariants</u> we shall rely on?

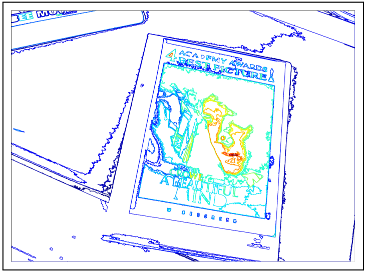
# Forewords

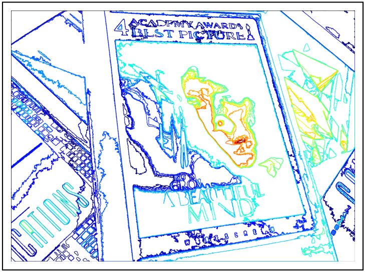




Some invariants are here *explicit*:
the geometrical ones that apply on the lines

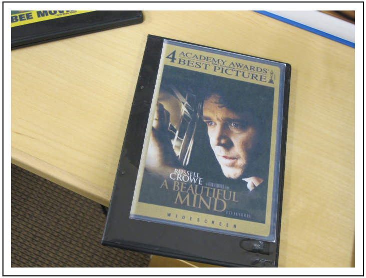
# Forewords

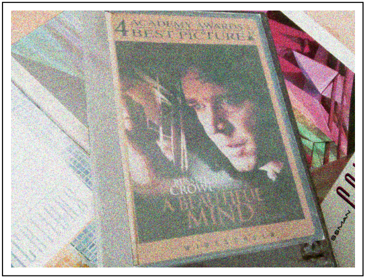



Some invariants are here *implicit*:
the ones behind the way we obtain the lines/shapes

# Forewords




Extracting lines/shapes shall be invariant, or at least robust, to variations of the image contents / pixel values...

# Forewords

Some key ideas:

- having poor contrast should not be a problem
- colors are often not so important
- we always want to be robust to noise...
- ...and to illumination changes

Actually:

- shapes matter a lot
- ensuring some strong “invariance” properties also matters a lot

F. Cao, J.-L. Lisani, J.-M. Morel, P. Musé and F. Sur, “A Theory of Shape Identification,” *Lecture Notes in Mathematics*, vol. 1948, Springer, 2008.

# Outline

- Mathematical Morphology (MM):
  from the very basics to recent results

- A tour of some applications
  (mainly of using some morphological trees)

# History of Mathematical Morphology (MM)

- *Mid 60's*
  Invention of MM by Georges Matheron and Jean Serra in CMM, France.
- *From 70's to 80's*
  Extension to sets (binary images) to functions (gray-level images).
- *End of the 80's*
  MM on graphs is defined (~~structural elements~~ neighborhood).
- *1995*
  Connected operators appear...
- *Beginning 2YK's*
  First attempts to get MM works on color images.
- *Since then*
  Adaptive filtering, optimization-related MM, etc.

# Mathematical Morphology (MM)

Mathematical Morphology is:

- a mathematical framework
  $\rightsquigarrow$ some common properties are expected
- a large toolkit
  $\rightsquigarrow$ some tools are very simple and powerful
- a way of thinking...

## Main idea

an image $f$ $\equiv$ a landscape where $f(x, y)$ is the elevation at $(x, y)$

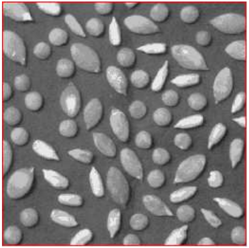
processing an image $\equiv$ modifying the landscape / function $f$

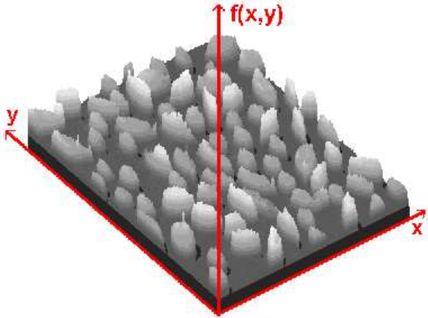
# Mathematical Morphology (MM)

This is a landscape:


so we have paths, mountains, peaks, valleys, flat zones, level lines, crest lines, passes, catchment basins...

# Mathematical Morphology (MM)






a 2D gray-scale image $\rightsquigarrow$ its corresponding landscape

$\Rightarrow$ we are going to think and act in terms of topography

# Simple maths (1/2)

Considering that $\subset$ is an ordering relation on sets (so over $\mathcal{P}(\Omega)$):

- set inclusion: $X_1 \leq X_2$
- set intersection: $X_1 \wedge X_2$ — $\wedge$ is infimum / minimum
- set union: $X_1 \vee X_2$ — $\vee$ is supremum / maximum
- complementation of $X$ in $\Omega$: $-X = \Omega \setminus X$ — $-$ is negation
- set minus: $X_1 - X_2$ — it is $X_1 \wedge -X_2$

Rationale:
this way, we also get the extension from sets to scalar functions...

# Simple maths (2/2)

An operator $\varphi$ on sets is:

- increasing iff $X_1 \leq X_2 \Rightarrow \varphi(X_1) \leq \varphi(X_2)$
- idempotent iff $\varphi \circ \varphi(X) = \varphi(X)$ we can write $\varphi\varphi = \varphi$
- extensive iff $\varphi(X) \geq X$ so $\varphi \geq \mathrm{id}$
- anti-extensive iff $\varphi(X) \leq X$

We say that:

- $\varphi$ and $\psi$ are dual iff $\psi(X) = -\phi(-X)$
- $\varphi$ is self-dual iff $\varphi(X) = -\varphi(-X)$ i.e., $\varphi$ and $-$ commute

# Structuring element

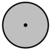
Structuring element:

- a set $B$ (usually small)
- it is the parameter of morphological operator
- given an operator, the filtering *effect* depends upon its shape
- and the filtering *strength* is related to its size

Extra stuff:

- translation of a set $X$ by $b$: $X_b = \{ x + b;\ x \in X \}$
- in the following, $B$ is centered and symmetrical

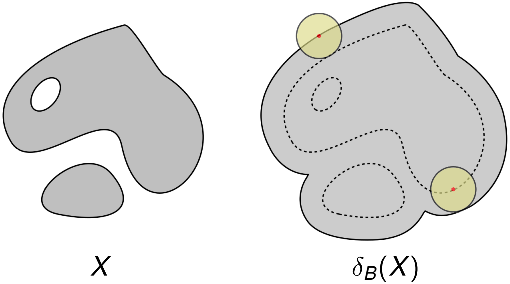
$$(0 \in B \quad \text{and} \quad b \in B \Rightarrow -b \in B)$$

In the next slides, $B =$  ...

# Dilation

Dilation of $X$ by $B$:

$$\begin{aligned}\delta_B(X) &= X \oplus B = \bigvee_{b \in B} X_b \\ &= \bigvee_{x \in X} B_x \qquad \text{—see the bottom-right } B_x \\ &= \{\, x + b;\ x \in X,\ b \in B \,\} \\ &= \{\, x;\ B_x \cap X \neq \emptyset \,\} \qquad \text{—see the top-left } B_x \rightsquigarrow \text{ impl.}\end{aligned}$$

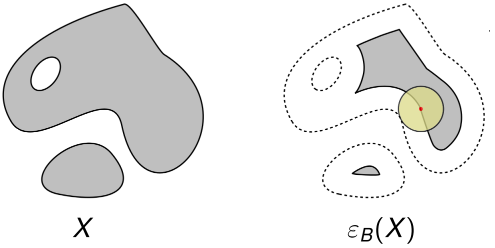


# Erosion

Erosion of $X$ by $B$:

$$\begin{aligned}\varepsilon_B(X) &= X \ominus B = \bigwedge_{b \in B} X_b \\ &= \{\, x;\ B_x \leq X \,\}\end{aligned}$$

—see below $B_x \rightsquigarrow$ impl.



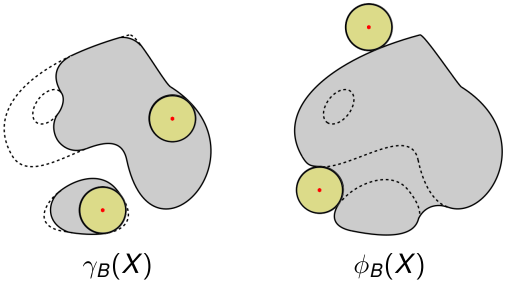
# Opening and Closing

Opening of $X$ by $B$:

$$\gamma_B(X) \;=\; \delta_B \circ \varepsilon_B\,(X) \;=\; \bigvee_{x,\,B_x \leq X} B_x$$

Closing of $X$ by $B$:

$$\phi_B(X) \;=\; \varepsilon_B \circ \delta_B\,(X)$$



# Properties

| operator | name | ext. | anti-ext. | idemp. |
|---|---|---|---|---|
| $\delta$ | dilation | x | | |
| $\varepsilon$ | erosion | | x | |
| $\phi$ | closing | x | | x |
| $\gamma$ | opening | | x | x |
| $\nu = \mathrm{med}(\gamma,\ \mathrm{id},\ \phi)$ | center | | | |
| $\varphi - \mathrm{id}$ | residue of $\varphi$ | ? | ? | |

($B$ is omitted here; horizontal separators shows duality)

and we have **many more** operators and **many more** properties...

# From sets to functions

From Mathematical Morphology on **sets**...

...we want to have Mathematical Morphology on **functions**

# Threshold Decomposition Principle

Let $f$ be a scalar function, i.e. a gray-level image.

Its *upper threshold set* (or *upper level set*) at a given gray-level $\lambda$ is:

$$[\, f \geq \lambda \,] \;=\; \{\, x;\; f(x) \geq \lambda \,\} \;\in\; \mathcal{P}(\Omega)$$

it is a set, i.e. a binary image.

From the family of sets $\{\, [\, f \geq \lambda \,] \,\}_\lambda$ we can reconstruct $f$:

$$f(x) \;=\; \arg\max_{\lambda} \{\, \lambda;\; x \in [\, f \geq \lambda \,] \,\}$$

This is the *threshold decomposition principle*.

Conclusion: $f$ and $\{\, [\, f \geq \lambda \,] \,\}_\lambda$ are the same.

# Operators on Functions

Actually

$$f(x) \;=\; \arg\max_{\lambda} \{\, \lambda;\; x \in [\, f \geq \lambda \,] \,\}$$

can be re-written like this:

$$(\mathrm{id}^{\mathrm{fun}}(f))(x) \;=\; \arg\max_{\lambda} \{\, \lambda;\; x \in \mathrm{id}^{\mathrm{set}}([\, f \geq \lambda \,]) \,\}$$

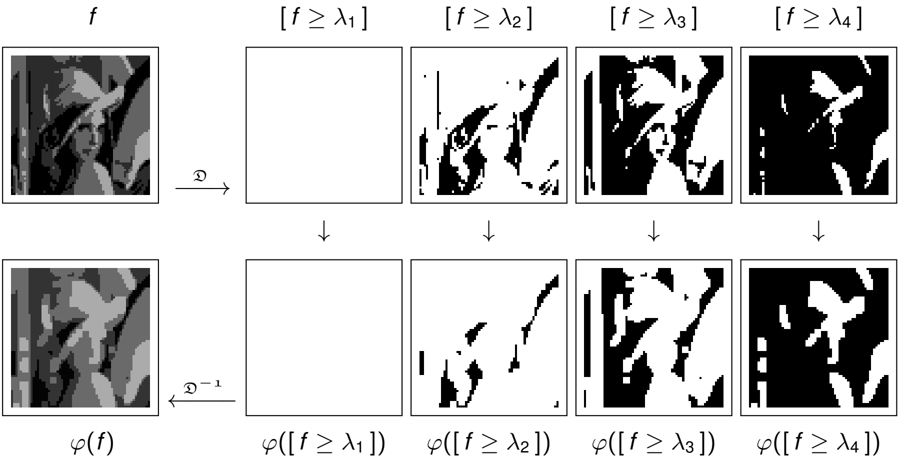
meaning that $\mathrm{id}^{\mathrm{fun}}$ is an op. on functions that maps the op. on sets $\mathrm{id}^{\mathrm{set}}$.

Given any operator $\varphi^{\mathrm{set}}$ on sets, we can then deduce its corresponding operator on functions:

$$(\varphi^{\mathrm{fun}}(f))(x) \;=\; \arg\max_{\lambda} \{\, \lambda;\; x \in \varphi^{\mathrm{set}}([\, f \geq \lambda \,]) \,\}.$$

$\Rightarrow$ We have a natural generalization of MM from sets to functions!

# Applying an Operator (the theoretical way)




with $\varphi = \delta_B$ in this example.

# Contrast Change Invariance

Consider:

- the ordered set of thresholds $S = \{ \lambda_1, ..., \lambda_Q \}$
- any non-decreasing function $g$

the set $\{ g(\lambda_1), ..., g(\lambda_Q) \}$ is ordered just like $S$
so $f$ and $g \circ f$ have the **same** upper threshold sets.

It means that:

## Invariance #1

any morphological operator $\varphi$ is invariant by any *contrast* change $g$

we have: $\varphi \circ g = g \circ \varphi$

**In mathematical morphology, contrast does not matter!**

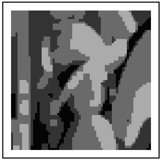
(Some explanations follow...)

# Contrast Change Invariance

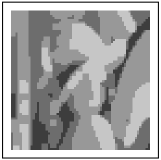
$f$ $\qquad\qquad$ $g(f)$

$\xrightarrow{g}$

$\varphi \downarrow$ $\qquad\qquad$ $\downarrow \varphi$



$\xrightarrow{g}$



$\varphi(f)$ $\qquad\qquad$ $g \circ \varphi(f) = \varphi \circ g(f)$

# A Consequence

Applying a morphological operator on these 3 images:

is equivalent (subject to $g$).

The right-most images are not more difficult to process than the left one!

"Contrast does not matter" means:

- values does not matter but...
- ...only the ordering of values matters!

# Contrast Inversion Invariance

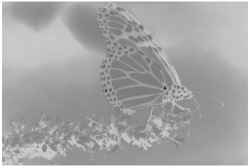
If, in addition, $\varphi$ is self-dual $(\varphi(-f) = -\varphi(f))$, we have:

**Invariance #2**

any self-dual morphological operator $\varphi$ is invariant by any *monotonic* change $h$

we have: $\varphi \circ h = h \circ \varphi$

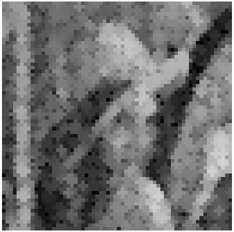
Applying a *self-dual* morphological operator on these 3 images:



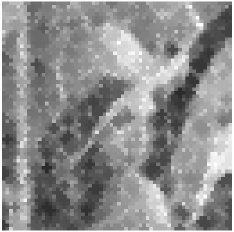
is equivalent (subject to $h$).

# Duality vs Self-Duality

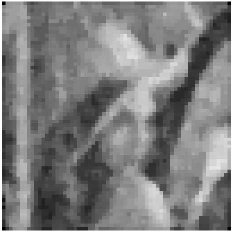
$f$



$\gamma_B(f)$



$\phi_B(f)$

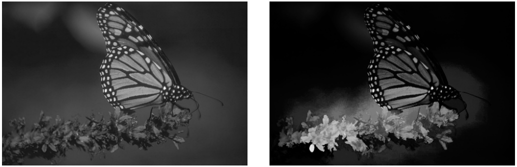


$\varphi_B^{\text{median}}(f)$

$\leftarrow$ self-dual

# Local Illumination Change Invariance

That would be great to have operators that behave the same way on these 2 images:



We are missing this:

**Invariance #3**

some class of operators $\varphi$ is invariant by any *local illumination change* $\ell$

we have: $\varphi \circ \ell = \ell \circ \varphi$

# Temporary conclusion (1/2)

About Mathematical Morphology:

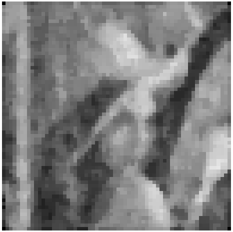
- there are many tools [...]
- they have sound mathematical foundations

  so we can understand what we have to do
  and we can interpret the results we obtain
- their invariants are important for PR + CV

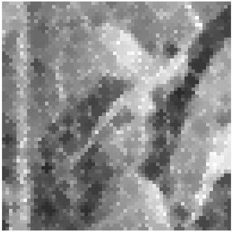
but MM with structuring elements shift contours :−(

# Pick your favorites...

$\gamma_B(f)$



$\varphi_B^{\text{median}}(f)$



$\phi_B(f)$

$\gamma_{(\mathcal{A},\alpha)}(f)$

$\nu_{(\mathcal{A},\alpha)}(f)$

$\phi_{(\mathcal{A},\alpha)}(f)$

# Temporary conclusion (2/2)

Reminder:

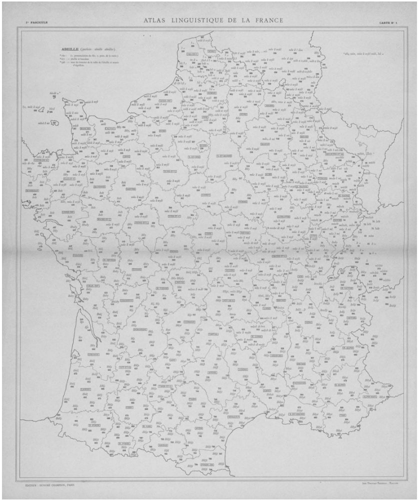
- image processing $=$ pixel (+ around) level
- Pattern Recognition $=$ object (primitive, region) level
- Computer Vision $=$ image level (scene)

Mathematical morphological operators deal with:

- threshold sets, that are, binary images
- so actually the connected components of threshold sets [...]

MM has the ability of having IP operators do some PR tasks
and so ease some CV tasks...

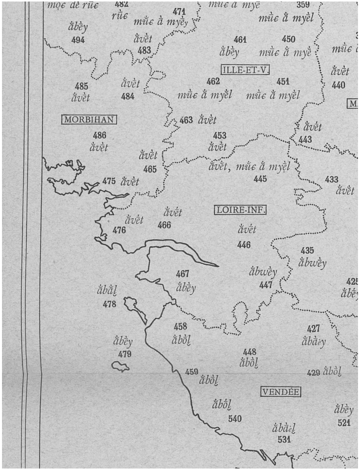
# A case study: a tiny set of ancient maps




including:

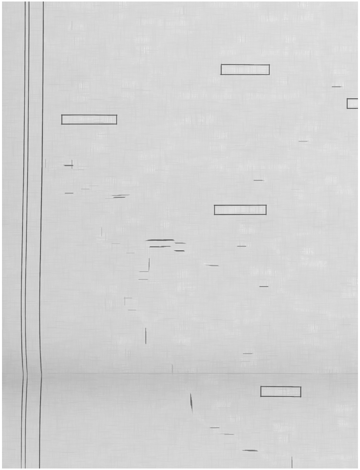
- different texts
- rectangles
- dots for dept. limits
- coast lines
- frames
- uneven background

# A case study




how to obtain the rectangles?

# A case study



with $\phi_{B_{\text{horizontal}}} \wedge \phi_{B_{\text{vertical}}}$

# A case study

A few remarks:

- this is MM *with structuring elements*
- this is a **much more valuable** input to find rectangles than the original image is
- actually we can go much further with MM...

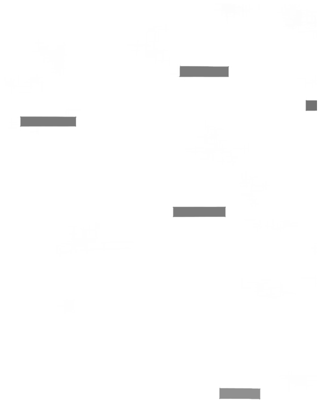
# A case study

that's better for the rectangles!

# A case study

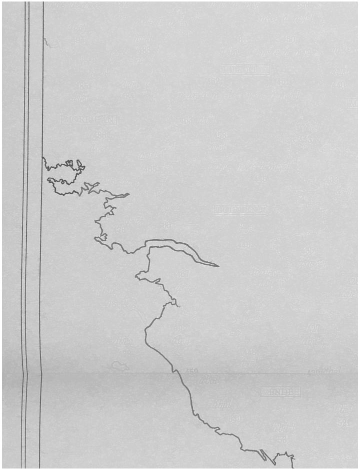
but we also can get the background...

# A case study



to keep only the objects of interest! (we're not done...)

# A case study



let's retrieve the coast and frames

# A case study

and why not department frontiers? (ok stop!)

# A case study

Case study conclusion:

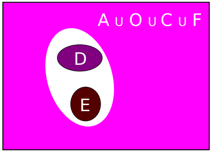
- we have mainly used openings and closings *
- without any structuring element
- so without getting any contour-shift effect (!)
- this case study is just a trivial exercise with MM...

* Class of openings (anti-extensive) and closings (extensive):
  - increasing
  - idempotent
  - invariant by contrast change
  - translation invariant
  - ...

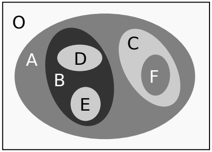
# Morphological dual trees

When thresholding $f$ at level $\lambda$, we get:

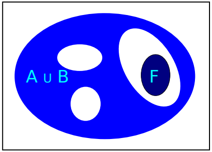
- an upper level set: $[f \geq \lambda] \; = \; \{ \, x; \; f(x) \geq \lambda \, \}$
- a lowel level set: $[f < \lambda] \; = \; \{ \, x; \; f(x) < \lambda \, \}$



a upper level set



$f$



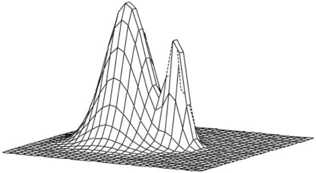
a lower level set

Considering the connected components ($\mathcal{CC}$) of threshold sets
$\rightsquigarrow$ we obtain two trees:

- the **max-tree**: $\mathcal{T}_{\text{max}}(f) \; = \; \{ \, \Gamma \in \mathcal{CC}( \, [f \geq \lambda] \, ) \, \}_\lambda$
- the **min-tree**: $\mathcal{T}_{\text{min}}(f) \; = \; \{ \, \Gamma \in \mathcal{CC}( \, [f < \lambda] \, ) \, \}_\lambda$

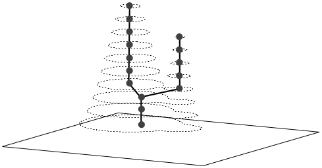
# Morphological dual trees

Example with the max-tree:



$f$

$\equiv$

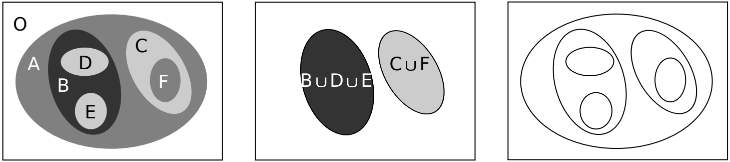


$\{\ [f \geq \lambda]\ \}_\lambda$

We have the duality:

$$\mathcal{T}_{\min}(-f) \; = \; \mathcal{T}_{\max}(f)$$

# The morphological tree of shapes (ToS)




$f$ two shapes of $f$ level lines of $f$

Using the cavity-fill-in operator Sat, we have the **tree of shapes**:

$$\mathfrak{S}(f) \ = \ \{\, \mathrm{Sat}(\Gamma);\ \Gamma \in \mathcal{CC}([f < \lambda]) \cup \mathcal{CC}([f \geq \lambda])\,\}_\lambda$$

$\rightsquigarrow$ The tree of shapes is also the inclusion tree of the level lines.

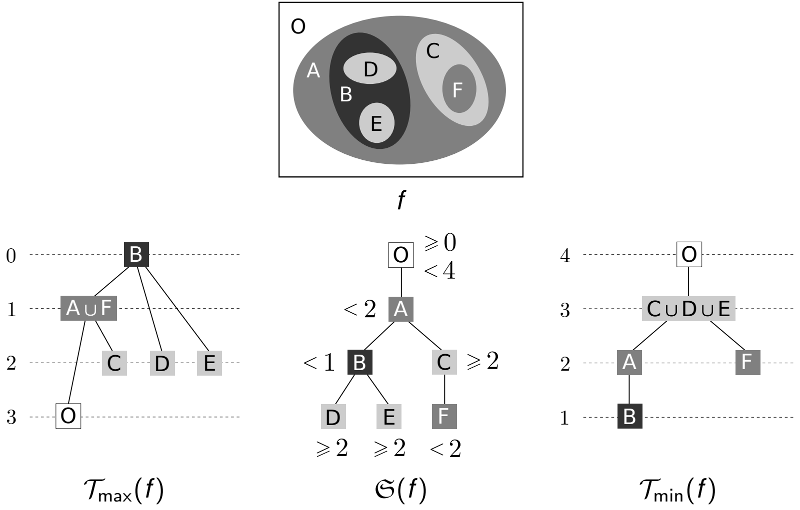
It is a **self-dual** tree:

$$\mathfrak{S}(-f) \ = \ \mathfrak{S}(f)$$

# Illustration of a ToS

every 15 levels only *and* without grain less than 3 pixels

# Morphological component trees



# Invariances

With $g$ strictly increasing:

$$\mathcal{T}_{\max}(g \circ f) \;=\; \mathcal{T}_{\max}(f) \quad \text{and} \quad \mathcal{T}_{\min}(g \circ f) \;=\; \mathcal{T}_{\min}(f)$$

With $h$ strictly monotonic:

$$\mathfrak{S}(h \circ f) \;=\; \mathfrak{S}(f)$$

These trees do not care about contrast changes,
and
the ToS does not care about contrast inversion
and...

# Invariance #3


...the ToS is invariant by *local illumination* changes!!!

# Advertising

Having a tree structure is great:

- simple structure
- easy to browse and do some iterative / recursive computations
- easy to transform (think about IP filtering...)
- and very versatile

With morphological trees, a **node** represents a **component** of the image $\Rightarrow$ we are at PR level!

# Connected Operators

**Definition**

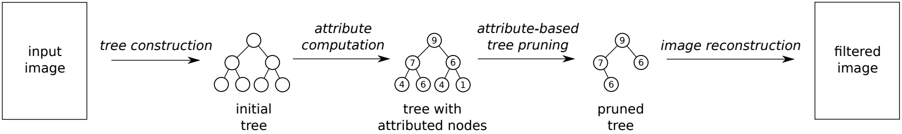
A morphological operator $\varphi$ is a *connected operator* iff:

$$\forall f,\ \forall x \mathcal{N} x',\ \ \varphi(f)(x) \neq \varphi(f)(x') \ \Rightarrow\ f(x) \neq f(x').$$

A very interesting class of operators:

- not based on structuring elements (no $B$ involved)
- do not shift contours; do not create new contours
- intuitive, powerfull, and efficient
- can be implemented as tree filtering

# Pruning-based Connected Operators




Pruning based on:

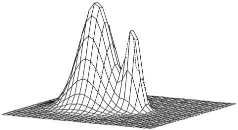
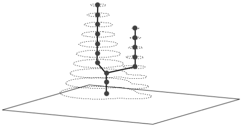
- an increasing attribute $\mathcal{A}$ = *effect* of the filtering
- and a threshold $\alpha$ = *strength* of the filtering

The *type* of filtering depends of the tree:

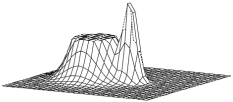
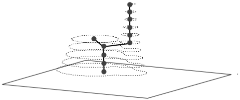
$$
\begin{array}{lcrl}
\mathcal{T}_{\max}(f) & \rightsquigarrow & \text{openings} & \gamma_{(\mathcal{A},\alpha)}(f) \\
\mathcal{T}_{\min}(f) & \rightsquigarrow & \text{closings} & \phi_{(\mathcal{A},\alpha)}(f) \\
\mathfrak{S}(f) & \rightsquigarrow & \text{grain filters} & \nu_{(\mathcal{A},\alpha)}(f)
\end{array}
$$

# Pruning-based Connected Operators

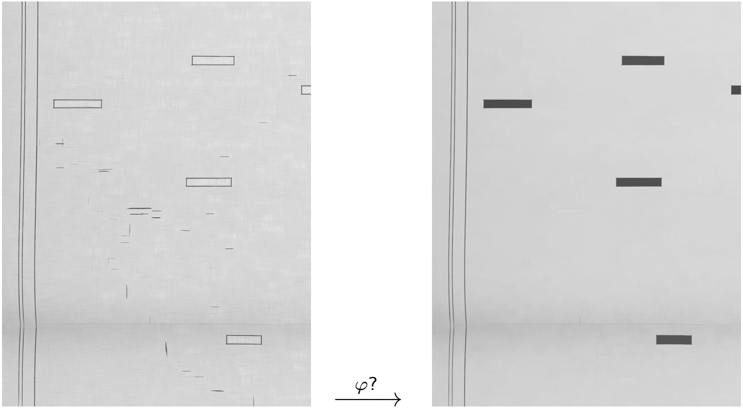
Remember that “MM operating = landscape modifying”:

$f$ =  $\equiv$ 

$\downarrow$

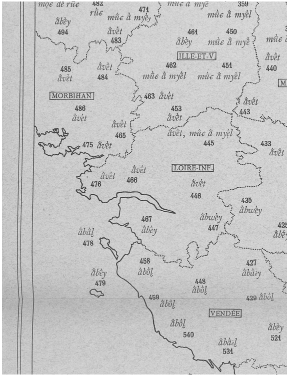
$\varphi(f)$ =  $\equiv$ 

# Illustrations




Left as exercise: What is the transform?

# Illustrations




Left as exercise: What are the transforms?

# Temporary conclusion

Morphological connected operators are powerful...
...but they only work on scalar functions / gray-level images.

*Getting threshold sets (and trees) means ordering pixel values.*

A lot of data are **not** scalar, but multi-variate data:

- color images
- multi-modal medical images
- multi-spectral and hyper-spectral satellite images
- ...

We face a major issue: how to get a *sensible* ordering for vectors?

Hum... red is greater than green, or is it the contrary?

# ToS for multi-variate data

Consider a function having $n$ components:

$$\mathbf{f} \;=\; (f_1,\, \ldots,\, f_n)$$

so every $f_i$ is a scalar function, i.e., a gray-level image.

Consider several monotonic transforms $\ell_i$:

$$\mathbf{l} \;=\; (\ell_1,\, \ldots,\, \ell_n)$$

that apply on $\mathbf{f}$:

$$\mathbf{l} \circ \mathbf{f} \;=\; (\ell_1 \circ f_1,\, \ldots,\, \ell_n \circ f_n)$$

What if we find a way to compute a tree for multi-variate functions so that we have:

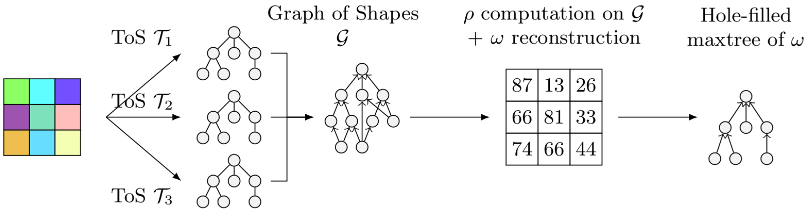
$$\mathfrak{S}(\mathbf{l} \circ \mathbf{f}) \;=\; \mathfrak{S}(\mathbf{f})$$

...

# ToS for multi-variate data

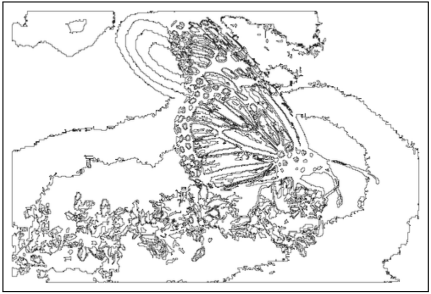
it would mean then we have a “tree of shapes” for multi-variate data...
...without a need for ordering vectors!

and actually, we did it:



# ToS for multi-variate data

**f**



Level lines of $\mathfrak{S}(\mathbf{f})$

E. Carlinet and TG, "MToS: A tree of shapes for multivariate images," *IEEE Transactions on Image Processing*, vol. 24, num. 12, pp. 5330–5342, 2015. [PDF]

# Some “how-to” extra references

E. Carlinet and TG, “A comparative review of component tree computation algorithms,” *IEEE Transactions on Image Processing*, vol. 23, num. 9, pp. 3885–3895, 2014. [PDF]

$\rightsquigarrow$ a very dense and effective representation for component trees

TG, E. Carlinet, S. Crozet, and L.W. Najman, “A quasi-linear algorithm to compute the tree of shapes of $n$-D images,” *in: Proc. of ISMM*, LNCS, vol. 7883, pp. 98–110, Springer, 2013. [PDF]

$\rightsquigarrow$ an efficient algorithm to compute the ToS

S. Crozet and TG, “A first parallel algorithm to compute the morphological tree of shapes of $n$D images,” *in: Proc. of ICIP*, pp. 2933–2937, 2014. [PDF]

$\rightsquigarrow$ and its parallel version

# Temporary conclusion

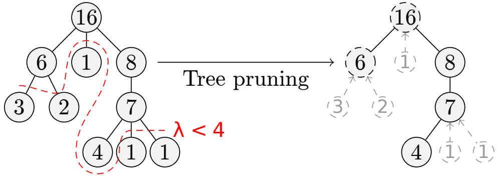
- There are many morphological tools...
- Why not using them?!
- Now let us see some applications...

# Some applications based on morphological trees

Morphological trees can support:

- grain filter
- shaping (filtering in shape space)
- object detection
- simplification / segmentation
- hierarchy of segmentations
- object picking / classification

# App: Grain filter

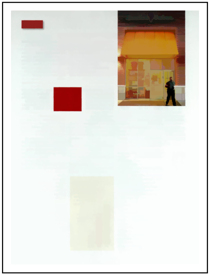



1. Compute an increasing attribute $\mathcal{A}$ over the tree.
2. Threshold to prune, and reconstruct.

$\mathcal{A}$ can be: area, diameter, width and/or height...

# App: Grain filter





↑

getting this low-contrasted box is possible
and we will have very precise contours

# App: Grain filter

AFRICA

## Emotional Intelligence

*Soul City*, South Africa's wildly popular soap opera, spreads public-health messages across the continent

By PETER HAWTHORNE JOHANNESBURG

IT'S 8:30 ON A TUESDAY NIGHT, AND MILlions of South Africans are tuning in to find out if the white social worker, Karen, will be able to adopt AIDS orphan Benni—"People who love me always end up dying," the child says. No, it's not *Melrose Place* or *E.R.* or any of the other imported dramatic series that fill the country's airwaves. It's a show about South Africans themselves. *Soul City*, the local soap opera that began as a good deed, is now an award-winning, multi-media business internationally recognized for its role in Third World development. It is also one of the top three most widely watched progams on South African TV.

which today include the European Union, Britain's Department for International Development, UNICEF, British Petroleum, the mobile-phone network MTN and the South African government's Department of Health—to put up the money, and *Soul City* was launched nationwide on the South African Broadcasting Corporation's TV service in 1994. "We realized that to be effective we had to go out at prime time, we had to keep it going and we had to be as good as or even better than any other production," says Japhet. Since then, *Soul City* has collected a bundle of prizes. On June 14, the show received a special award from Britain's One World Broadcasting Trust for its role in "illuminating the concerns of developing societies."

Africa's official languages, and the series is subtitled in English. It has been shown in Zimbabwe, Zambia, Nigeria, Namibia, Malawi and Barbados and has been dubbed into French and Portuguese for showing in Ivory Coast, Mozambique and other countries. "This concept has got legs," says Japhet. "It moves."

It has also spun off *Soul Buddyz*, a show for younger viewers, which began a 26-episode run on SABC-TV last year. Much as *Soul City* does for adults, *Soul Buddyz* looks at AIDS and other problems as they affect the young. The show is already being sought by countries outside South Africa.

What makes *Soul City* so successful is not merely its authentic portrayal of township life but also its intimacy with viewers. Many of the actors live in the townships. "That sometimes presents a problem," says Lebo Ramafoko, producer of the latest series. "Our actors are seen as real people. They have to become the advocates and counselors they portray on the screen." Some of the cast members have been discovered through *Soul City's* annual "Search for a Star" initiative. Among them

**IT'S A WRAP:** Actors on the set take a breather from filming after a successful take

**HIGH DRAMA:** Confronting prejudices toward HIV is one of Soul City's main objectives

**EDUCATIONAL FOCUS:** Benni tries his hand learning sign language at the township clinic

CLEAR MEDIA AND CURIOUS PICTURES (3)

*Soul City* reflects the life and hard times of people in a typical South African urban black township. In fact, most of the series' location shots are taken in Alexandra, one of the oldest and most squalid of South Africa's black slums, just outside Johannesburg. And *Soul City's* clinic, where much of the drama is centered, is based on an actual clinic in Alexandra.

That medical theme is not surprising, since the idea for the show came from two young white South African doctors, Garth Japhet and Shereen Usdin, who had studied and worked in township hospitals and clinics, including in Alexandra. They envisioned a dramatic program that would highlight the social and health problems—particularly HIV-AIDS—in the townships. They convinced a long list of donors—

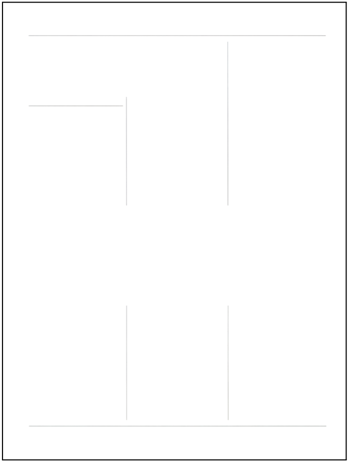
Because AIDS is a huge problem in South Africa—almost 10% of the population is infected with HIV—the subject is featured in every installment of the show. Other topics include violence against women, drunkenness, rape, child abuse, sexual conduct, occupational health, disability, setting up a business, even buying and selling a used car. "This is not just about reality in life, it's about real-life skills," says Japhet, who is executive director of the nongovernmental Institute for Health and Development Communications, which produces the show. *Soul City* also has a radio version broadcast daily in nine of South Africa's 11 official languages, as well as weekly newspaper columns and a series of educational booklets.

Characters speak in several of South

is Doreen Kotsedi-Pitse, a blind woman who will play the part of a radio talk-show coordinator. Kotsedi-Pitse lost her sight as a result of domestic violence, an issue frequently raised by *Soul City*. "It made us all aware how uncomfortably close we are to real life," says Ramafoko.

*Soul City* is affecting the way South Africans lead their real lives. A study sponsored by the E.U. says there is evidence that the program has played "a major role" in increasing public knowledge of HIV-AIDS and safer sexual behavior. Researchers say, for instance, that there is a "significant" association of the use of condoms with people who watch *Soul City*. The survey did not say whether Benni would find a happy new home. For more about that, tune in to next week's episode. ■

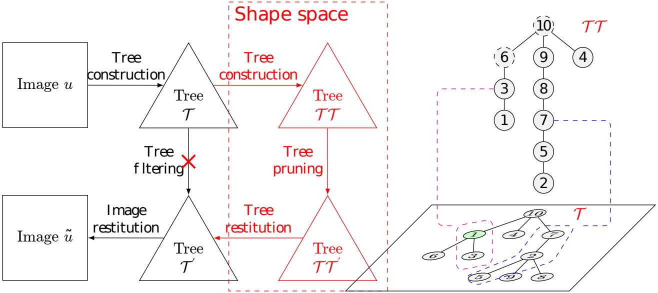
38 TIME, JULY 2, 2001



Left as an exercise (so DIY!)

# App: Shaping

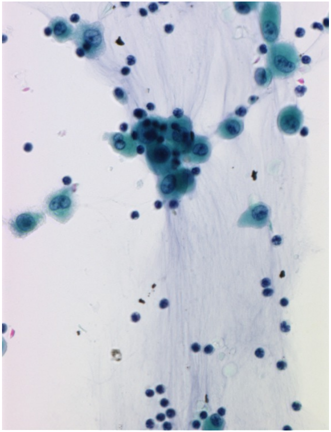
When $\mathcal{A}$ is not increasing:

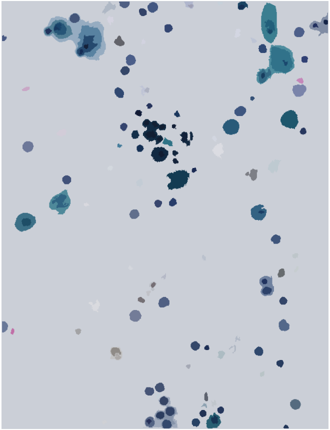



it is now not a pruning.

Y. Xu, TG, and L. Najman, "Connected filtering on tree-based shape-spaces," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 38, num. 6, pp. 1126–1140, 2016. [PDF]

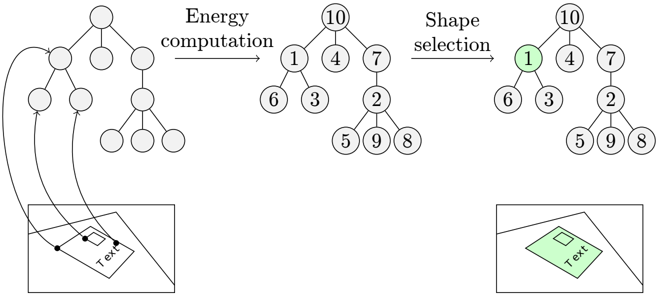
# App: Shaping





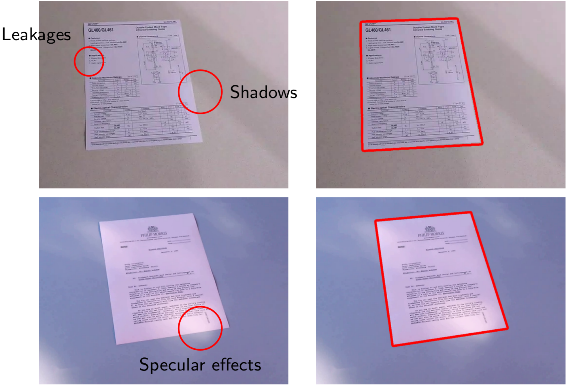
Nodes with poor circularity are filtered out.

# App: Object detection



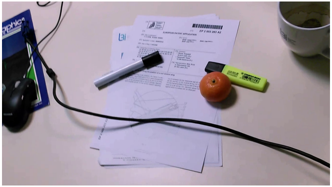

1. Valuate an energy adpated to the object(s) to detect
2. Retrieve the shape(s) with minimal energy

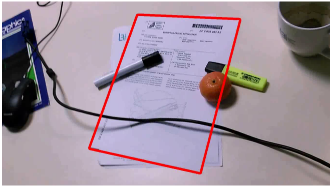
# App: Object detection




the detection (yet not impressive) is very precise...

# App: Object detection

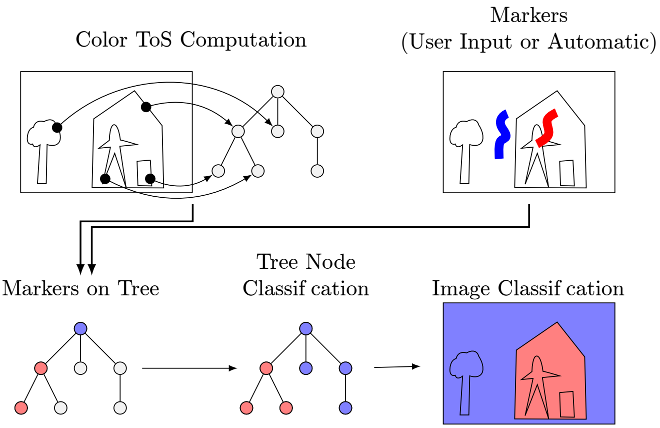




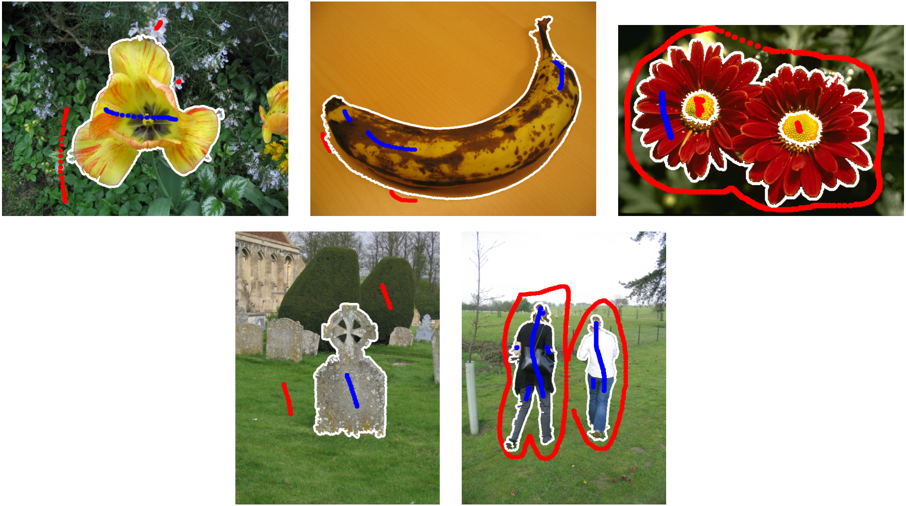
...and we can handle difficult cases

winner of the SmartDoc competition at:
*Intl. Conf. on Document Analysis and Recognition (ICDAR) 2015*

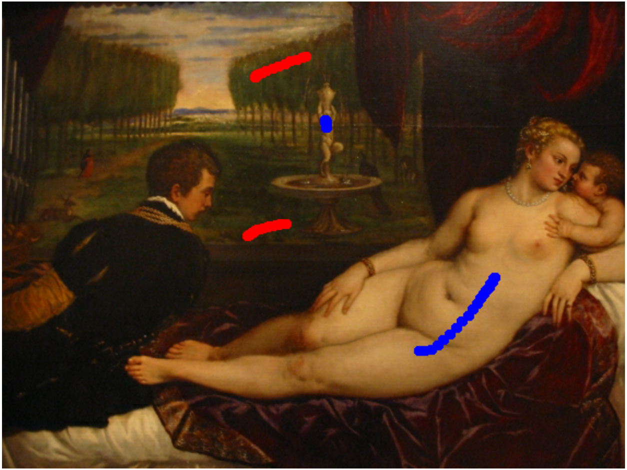
# App: Object picking



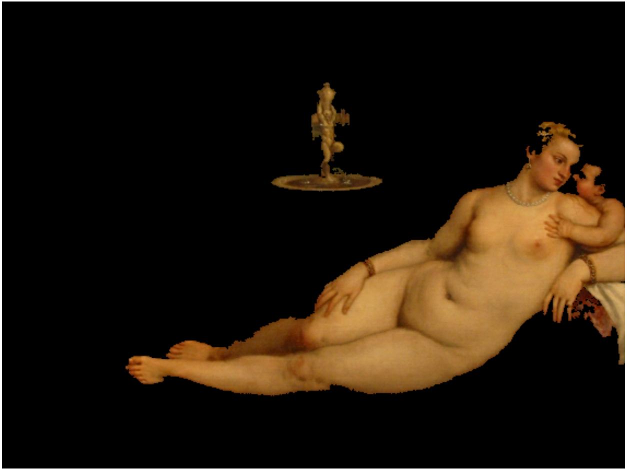
# App: Object picking



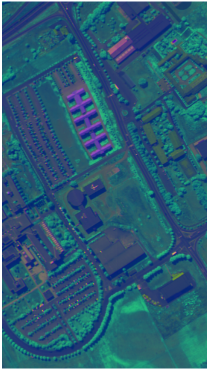
# App: Object picking



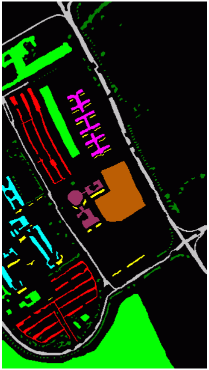
# App: Object picking



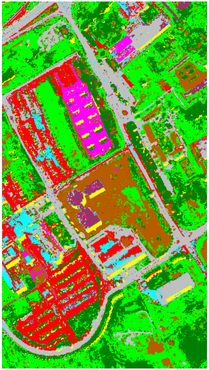
# App: Object picking $\rightarrow$ classification



3 PCA components



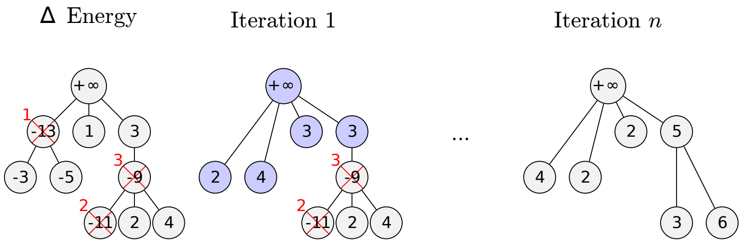
GT



result

G. Cavallaro, M. Dalla Mura, E. Carlinet, TG, N. Falcon, and J.A. Benediktsson, "Region-Based Classification of Remote Sensing Images with the Morphological Tree of Shapes," *Proc. of the IEEE Intl. Geoscience and Remote Sensing Symposium (IGARSS)*, pp. 5087–5090, 2016.

# App: Simplification / segmentation

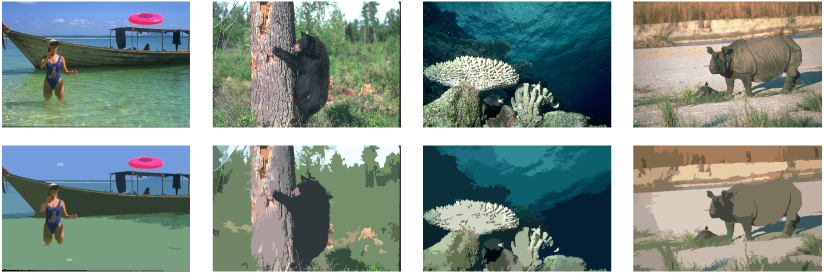



1. Sort nodes by increasing meaningfulness
2. Compute $\Delta E$ on the tree
3. For each node
   - if removing it makes the energy decrease ($\Delta E < 0$)
     remove it and update the local $\Delta$ energy of its relatives
   - otherwise stop

# App: Simplification / segmentation

based on the cartoon model of the Mumford-Shah functional

# App: Simplification / segmentation



# App: From a direct approach to a hierarchical one

For both filtering and segmenting $\rightsquigarrow$ we set a strength value

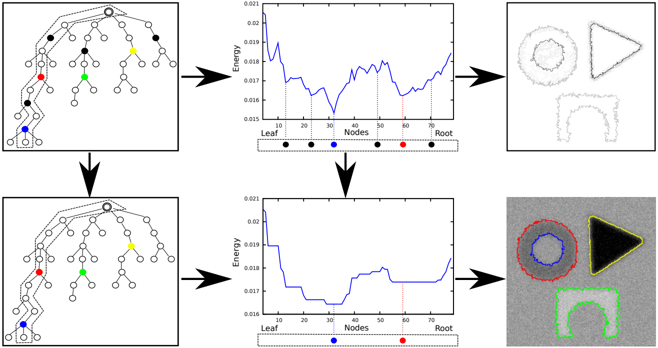
$\Rightarrow$ we can analyze what happens when this strength is varying

*we are interesting in the* **saliency** *of components* = *their persistence w.r.t. this strength*

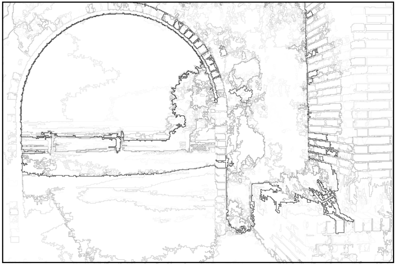
Y. Xu, E. Carlinet, TG, and L. Najman, "Hierarchical segmentation using tree-based shape spaces," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 39, num. 3, pp. 457–469, 2017. [PDF]

# App: From a direct approach to a hierarchical one

Illustration with ToS + contour meaningfulness energy + shaping:



# App: Saliency map

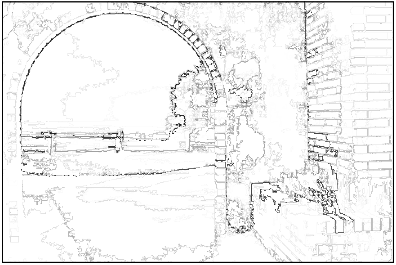


saliency map

original image

there is an underlying hierarchy of segmentations

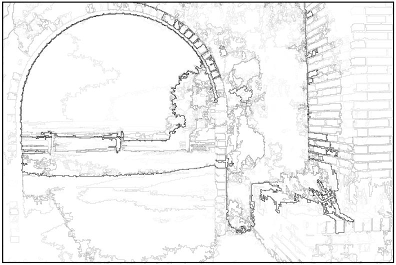
# App: Saliency map


saliency map

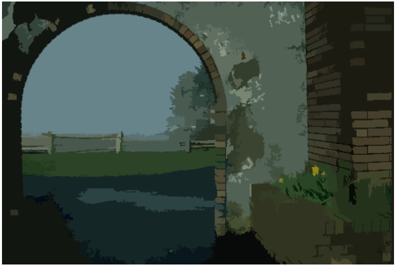

low threshold

there is an underlying hierarchy of segmentations

# App: Saliency map



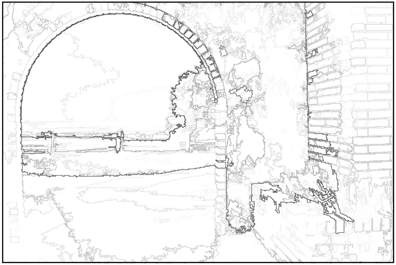
saliency map



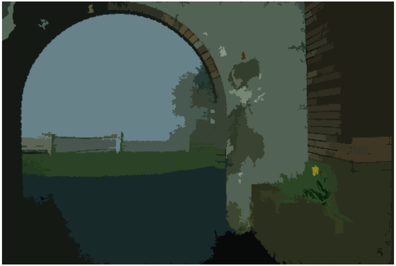
mid threshold

there is an underlying hierarchy of segmentations

# App: Saliency map



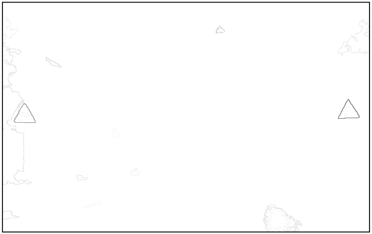
saliency map



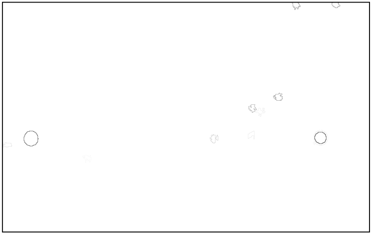
high threshold

there is an underlying hierarchy of segmentations

# App: Saliency map



# App: Saliency map

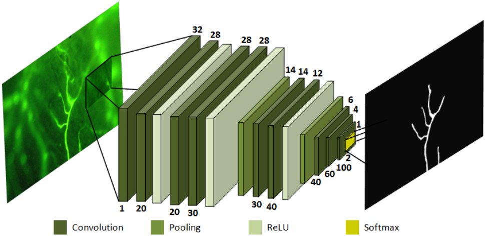


# Some other apps

How Mathematical Morphology (MM)
and
Convolutional Neural Network (CNN)
can mix?

# Example #1

Y. M. Kassim *et al.*, "Microvasculature Segmentation of Arterioles using Deep CNN," *Proc. of the IEEE Intl. Conf. on Image Processing (ICIP)*, pp. 580–584, 2017. [PDF]




ABSTRACT:
Segmenting microvascular structures is an important requirement in understanding angioadaptation by which vascular networks remodel their **morphological** structures. [...] In this work, we utilize a deep **convolutional neural network** framework for obtaining robust segmentations of microvasculature from epifluorescence microscopy imagery of mice dura mater. [...]

# Example #1

and the presenter said:

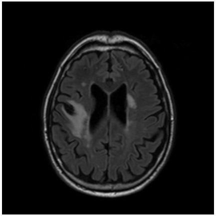
"After publication in ICIP 2017, we found a way to get better results;
we just pre-process the images with some morphological operators!"

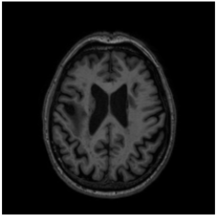
# Example #2

*White Matter Hyperintensities (WMH) Segmentation Challenge*

at the 20th Intl. Conf. on Medical Image Computing and Computer Assisted Intervention (MICCAI), September 2017

Our erstwhile solution:



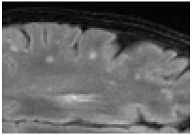


FLAIR $f$ & T1 $\xrightarrow{\text{CNN}}$ result

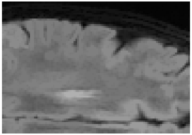
# What about MM and CNN?

Observation:
we miss a lot of small WMH regions

Idea: help the CNN to retrieve them



$f$



$\gamma_{\text{area}}(f)$

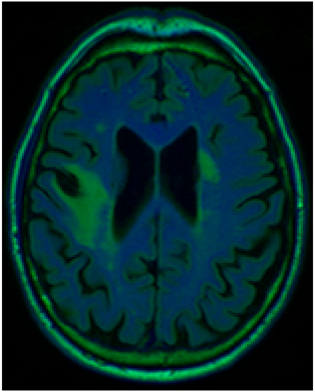
$\text{TH} = f - \gamma_{\text{area}}(f)$

Result:

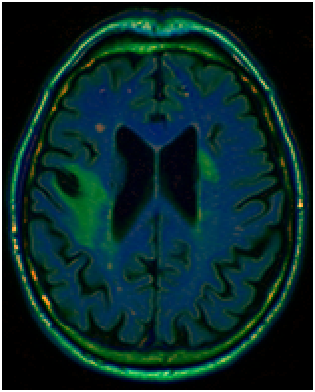
| | **Detection** | **F-measure** |
|---|---|---|
| with FLAIR & T1 only | 0.39 | 0.48 |
| when adding TH | 0.61 | 0.63 |

# What about MM and CNN?

Input comparison:



FLAIR (as green) & T1 (as blue)



TH (as additional red)

Competition result: 1st rank (out of 20 competitors)
w.r.t. the average volume difference (≈20%)

# Conclusion

- We have see some basic stuff (with structuring elements)...
- ...and some more advanced stuff:
  - connected operators and trees
  - the multi-variate tree of shapes
- There is actually many more morphological things to see!

# Going further...

There is a dedicated conference every 2 years:

- International Symposium on Mathematical Morphology (ISMM)

and there are books:

- *Image Analysis and Mathematical Morphology—Vol. 1.*
  J. Serra. Academic Press, **1982**.
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# The end

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Thanks for your attention; any questions?