

# Compiler Construction

~ Liveness Analysis ~

# Goals

How to precisely catch the liveness of each variable?

If a variable is in register \$1 then we can reuse this register as soon as the variable is no longer used

# Scopes vs. Liveness

## Scopes

- Front-end analysis
- Detect names visibility according to textual rules

## Liveness

- Back-end analysis
- Focus on all generated variables (even temporaries)
- Exact computation of which variables are used at the same time

# Liveness Definition

## Definition

A variable is **live** if it holds a value that may be needed in the future.

# Example

a := 0	1
L1: b := a + 1	2
c := c + b	3
a := b * 2	4
if a < N goto L1	5
return c	6

## Question:

What are the liveness of  $a$ ,  $b$  and  $c$ ?

# Flow Graph

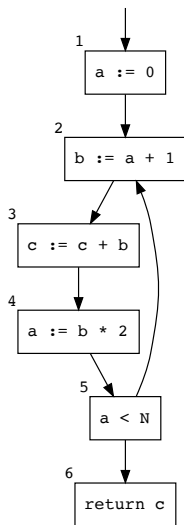
Liveness computation requires an adequate data-structure.

## Control Flow graph (CFG):

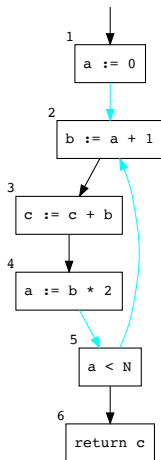
A representation, using graph notation, of all paths that might be traversed through a program during its execution.

*Remark:  $\text{pred}[n]$  (resp.  $\text{succ}[n]$ ) denotes the predecessors (resp. successors) of node  $n$*

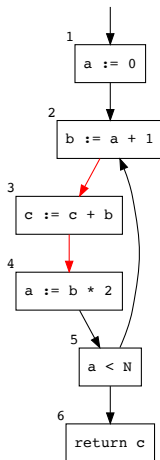
# Flow Graph for the example



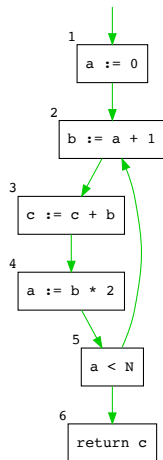
# Liveness



Liveness for  $a$



Liveness for  $b$



Liveness for  $c$

# Terminology

- **defs:** nodes that define a variable, i.e. left (lhs) part of assignment nodes.
- **uses:** nodes that use (read) a variable (rhs).
- **live-in:** a variable is live-in of a node  $n$ , if it lives on any in-edges of that node.
- **live-out:** a variable is live-out of a node  $n$ , if it lives on any out-edges of that node.

# Liveness computation

- 1 If a variable is in  $use[n]$  then it is live-in at node  $n$ .
- 2 If a variable is live-in at node  $n$  then it is live-out at all nodes  $m$  in  $pred[m]$ .
- 3 If a variable is live-out at node  $n$  and not in  $def[n]$ , then it is live-in at node  $n$ .

# Dataflow Equations for Liveness Analysis

$$\begin{aligned}\text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s]\end{aligned}$$

## Possible Implementation (quadratic)

```
foreach  $n$   
   $\text{in}[n] \leftarrow \{ \}$   
   $\text{out}[n] \leftarrow \{ \}$   
  
repeat  
  foreach  $n$   
     $\text{in\_t}[n] \leftarrow \text{in}[n]$   
     $\text{out\_t}[n] \leftarrow \text{out}[n]$   
     $\text{in}[n] \leftarrow \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n])$   
     $\text{out}[n] \leftarrow \bigcup_{s \in \text{succ}[n]} \text{in}[s]$   
until  $\text{in\_t}[n] = \text{in}[n]$  and  $\text{out\_t}[n] = \text{out}[n]$  ( $\forall n$ )
```

# Liveness Calculation

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2								
2	a	b	3								
3	bc	c	4								
4	b	a	5								
5	a		2,6								
6	c										

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2						
2	a	b	3						
3	bc	c	4						
4	b	a	5						
5	a		2,6						
6	c								

$$\begin{aligned}
 \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\
 \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s]
 \end{aligned}$$

# Liveness Calculation

1st step

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2								
2	a	b	3	a							
3	bc	c	4	bc							
4	b	a	5	b							
5	a		2,6	a	a						
6	c			c							

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2						
2	a	b	3						
3	bc	c	4						
4	b	a	5						
5	a		2,6						
6	c								

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

# Liveness Calculation

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	1st step		2nd step		<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
				<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>				
1		a	2				a				
2	a	b	3	a		a	bc				
3	bc	c	4	bc		bc	b				
4	b	a	5	b		b	a				
5	a		2,6	a	a	a	ac				
6	c			c		c					

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2						
2	a	b	3						
3	bc	c	4						
4	b	a	5						
5	a		2,6						
6	c								

$$\text{in}[n] = \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n])$$

$$\text{out}[n] = \bigcup_{s \in \text{succ}[n]} \text{in}[s]$$

# Liveness Calculation

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	1st step		2nd step		3rd step			
				<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2				a		a		
2	a	b	3	a		a	bc	ac	bc		
3	bc	c	4	bc		bc	b	bc	b		
4	b	a	5	b		b	a	b	a		
5	a		2,6	a	a	a	ac	ac	ac		
6	c			c		c		c			

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2						
2	a	b	3						
3	bc	c	4						
4	b	a	5						
5	a		2,6						
6	c								

$$\text{in}[n] = \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n])$$

$$\text{out}[n] = \bigcup_{s \in \text{succ}[n]} \text{in}[s]$$

# Liveness Calculation

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	1st step		2nd step		3rd step		4th step	
				<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2				a		a		ac
2	a	b	3	a		a	bc	ac	bc	ac	bc
3	bc	c	4	bc		bc	b	bc	b	bc	b
4	b	a	5	b		b	a	b	a	b	ac
5	a		2,6	a	a	a	ac	ac	ac	ac	ac
6	c			c		c		c		c	

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2						
2	a	b	3						
3	bc	c	4						
4	b	a	5						
5	a		2,6						
6	c								

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

# Liveness Calculation

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	1st step		2nd step		3rd step		4th step	
				<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2				a		a		ac
2	a	b	3	a		a	bc	ac	bc	ac	bc
3	bc	c	4	bc		bc	b	bc	b	bc	b
4	b	a	5	b		b	a	b	a	b	ac
5	a		2,6	a	a	a	ac	ac	ac	ac	ac
6	c			c		c		c		c	

5th step											
<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>		
1		a	2	c	ac						
2	a	b	3	ac	bc						
3	bc	c	4	bc	b						
4	b	a	5	bc	ac						
5	a		2,6	ac	ac						
6	c			c							

$$\begin{aligned}
 \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\
 \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s]
 \end{aligned}$$

# Liveness Calculation

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	1st step		2nd step		3rd step		4th step	
				<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2				a		a		ac
2	a	b	3	a		a	bc	ac	bc	ac	bc
3	bc	c	4	bc		bc	b	bc	b	bc	b
4	b	a	5	b		b	a	b	a	b	ac
5	a		2,6	a	a	a	ac	ac	ac	ac	ac
6	c			c		c		c		c	

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	5th step		6th step			
				<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2	c	ac	c	ac		
2	a	b	3	ac	bc	ac	bc		
3	bc	c	4	bc	b	bc	bc		
4	b	a	5	bc	ac	bc	ac		
5	a		2,6	ac	ac	ac	ac		
6	c			c		c			

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

# Liveness Calculation

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	1st step		2nd step		3rd step		4th step	
				<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2				a		a		ac
2	a	b	3	a		a	bc	ac	bc	ac	bc
3	bc	c	4	bc		bc	b	bc	b	bc	b
4	b	a	5	b		b	a	b	a	b	ac
5	a		2,6	a	a	a	ac	ac	ac	ac	ac
6	c			c		c		c		c	

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	5th step		6th step		7th step	
				<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2	c	ac	c	ac	c	ac
2	a	b	3	ac	bc	ac	bc	ac	bc
3	bc	c	4	bc	b	bc	bc	bc	bc
4	b	a	5	bc	ac	bc	ac	bc	ac
5	a		2,6	ac	ac	ac	ac	ac	ac
6	c			c		c		c	

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

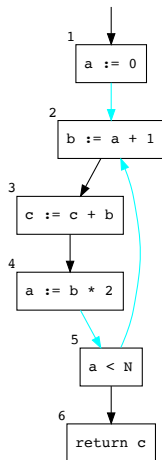
# Liveness Calculation (Forward)

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	1st step		2nd step		3rd step		4th step	
				<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2				a		a		ac
2	a	b	3	a		a	bc	ac	bc	ac	bc
3	bc	c	4	bc		bc	b	bc	b	bc	b
4	b	a	5	b		b	a	b	a	b	ac
5	a		2,6	a	a	a	ac	ac	ac	ac	ac
6	c			c		c		c		c	

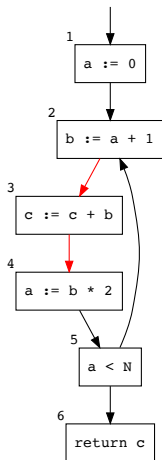
<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	5th step		6th step		7th step	
				<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1		a	2	c	ac	c	ac	c	ac
2	a	b	3	ac	bc	ac	bc	ac	bc
3	bc	c	4	bc	b	bc	bc	bc	bc
4	b	a	5	bc	ac	bc	ac	bc	ac
5	a		2,6	ac	ac	ac	ac	ac	ac
6	c			c		c		c	

$$\begin{aligned}
 \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\
 \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s]
 \end{aligned}$$

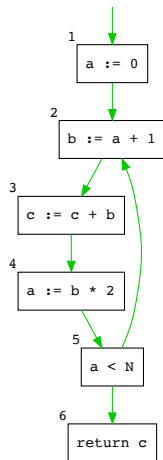
# Liveness



Liveness for  $a$



Liveness for  $b$



Liveness for  $c$

# Liveness Calculation (Backward)

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>
6	c								
5	a		2,6						
4	b	a	5						
3	bc	c	4						
2	a	b	3						
1		a	2						

$$\text{in}[n] = \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n])$$

$$\text{out}[n] = \bigcup_{s \in \text{succ}[n]} \text{in}[s]$$

Calculation done following *reverse* control-flow edges.

# Liveness Calculation (Backward)

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	1st step		<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>
				<i>out</i>	<i>in</i>				
6	c				c				
5	a		2,6	c	ac				
4	b	a	5	ac	bc				
3	bc	c	4	bc	bc				
2	a	b	3	bc	ac				
1		a	2	ac	c				

$$\text{in}[n] = \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n])$$

$$\text{out}[n] = \bigcup_{s \in \text{succ}[n]} \text{in}[s]$$

Calculation done following *reverse* control-flow edges.

# Liveness Calculation (Backward)

<i>n</i>	<i>use</i>	<i>def</i>	<i>succ</i>	1st step		2nd step		<i>out</i>	<i>in</i>
				<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>		
6	c				c		c		
5	a		2,6	c	ac	ac	ac		
4	b	a	5	ac	bc	ac	bc		
3	bc	c	4	bc	bc	bc	bc		
2	a	b	3	bc	ac	bc	ac		
1		a	2	ac	c	ac	c		

$$\text{in}[n] = \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n])$$

$$\text{out}[n] = \bigcup_{s \in \text{succ}[n]} \text{in}[s]$$

Calculation done following *reverse* control-flow edges.

# Liveness Calculation (Backward)

$n$	$use$	$def$	$succ$	1st step		2nd step		3rd step	
				$out$	$in$	$out$	$in$	$out$	$in$
6	c				c		c		c
5	a		2,6	c	ac	ac	ac	ac	ac
4	b	a	5	ac	bc	ac	bc	ac	bc
3	bc	c	4	bc	bc	bc	bc	bc	bc
2	a	b	3	bc	ac	bc	ac	bc	ac
1		a	2	ac	c	ac	c	ac	c

$$in[n] = use[n] \cup (out[n] \setminus def[n])$$


$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

Calculation done following *reverse* control-flow edges.


# Summary



Liveness



Dataflow  
Equations



Live-in  
Live-out



Control Flow  
Graph