Liveness Analysis

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Liveness Analysis

Control Flow Graph

Control Flow Graph [Appel, 1998]

```
    a := 0
L1: b := a + 1
    c := c + b
    a := b * 2
    if a < N goto L1
    return c
```

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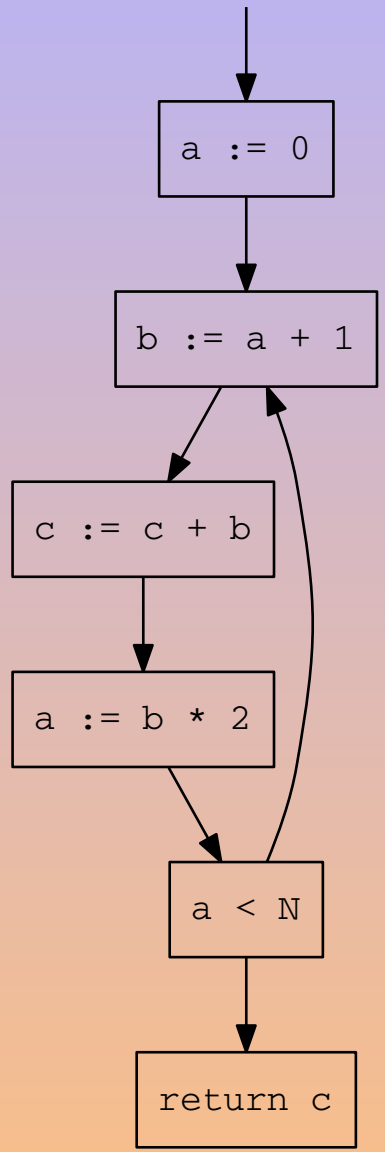


7.tig

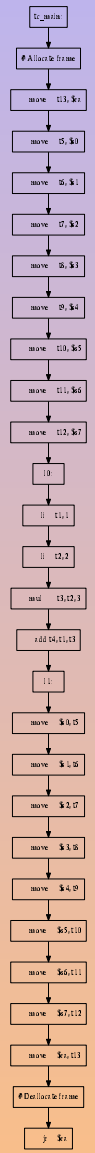
```
1 + 2 * 3
```

7's Pre-Assembly

```
tc_main:
# Allocate frame
        move    $x13, $ra
        move    $x5, $s0
        move    $x6, $s1
        move    $x7, $s2
        move    $x8, $s3
        move    $x9, $s4
        move    $x10, $s5
        move    $x11, $s6
        move    $x12, $s7
l0:
        li      $x1, 1
        li      $x2, 2
        mul     $x3, $x2, 3
        add     $x4, $x1, $x3
```

```
l1:
        move    $s0, $x5
        move    $s1, $x6
        move    $s2, $x7
        move    $s3, $x8
        move    $s4, $x9
        move    $s5, $x10
        move    $s6, $x11
        move    $s7, $x12
        move    $ra, $x13
# Deallocate frame
        jr      $ra
```

7's Flowgraph



7000.tig

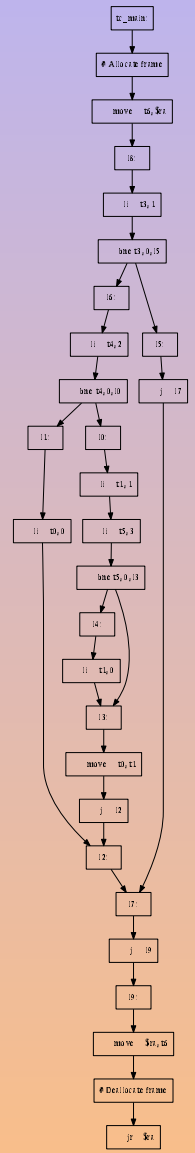
```
1 | 2 & 3
```

7000's Pre-Assembly

```
tc_main:
# Allocate frame
        move    $x6, $ra
18:
        li      $x3, 1
        bne     $x3, 0, 15
16:
        li      $x4, 2
        bne     $x4, 0, 10
11:
        li      $x0, 0
12:
17:
        j       19
```

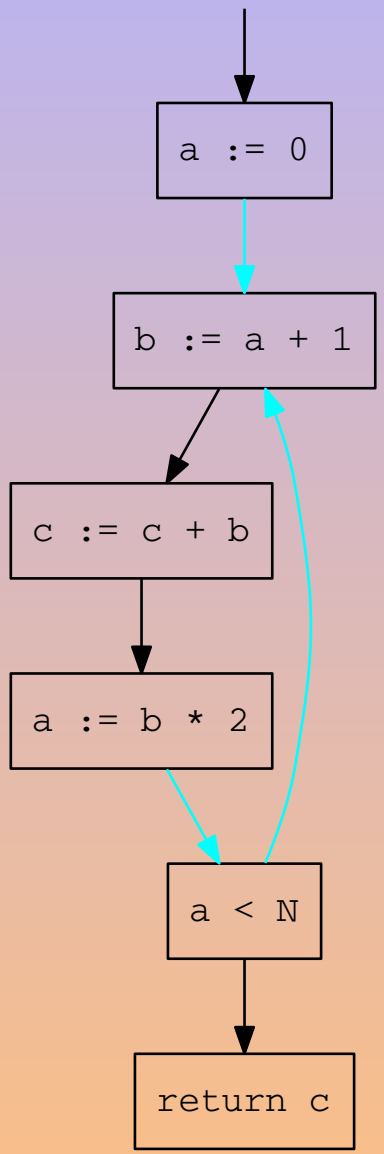
```
10:
        li      $x1, 1
        li      $x5, 3
        bne     $x5, 0, 13
14:
        li      $x1, 0
13:
        move    $x0, $x1
        j       12
15:
        j       17
19:
        move    $ra, $x6
# Deallocate frame
        jr      $ra
```

7000's Flowgraph

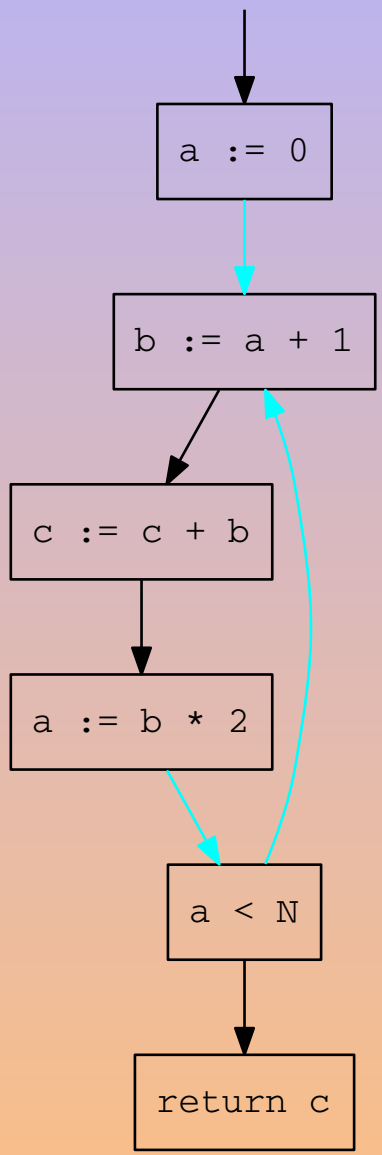


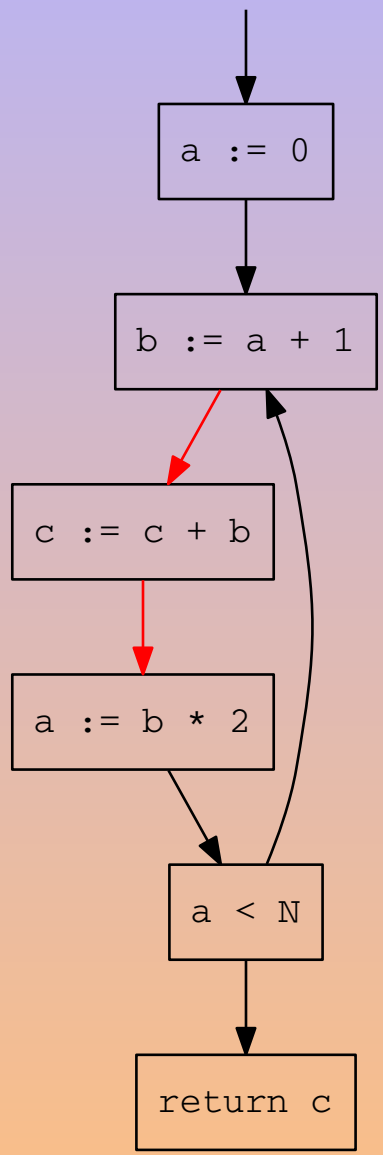
Liveness

Liveness

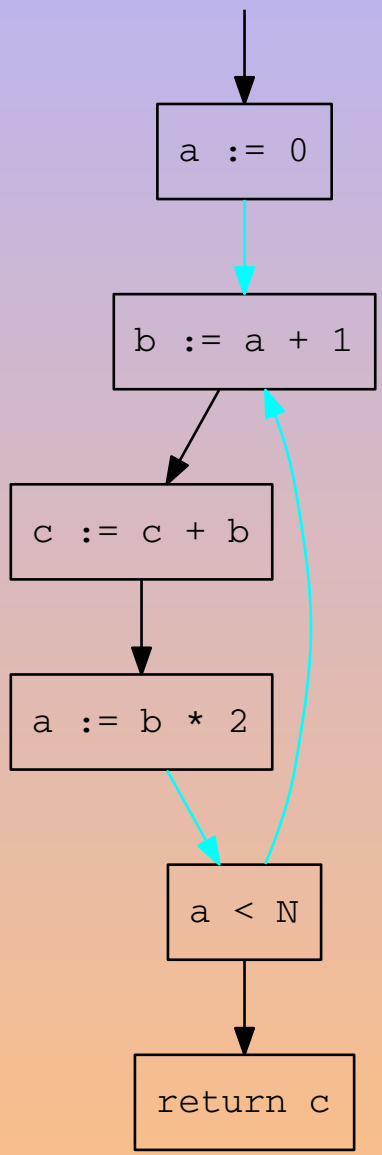


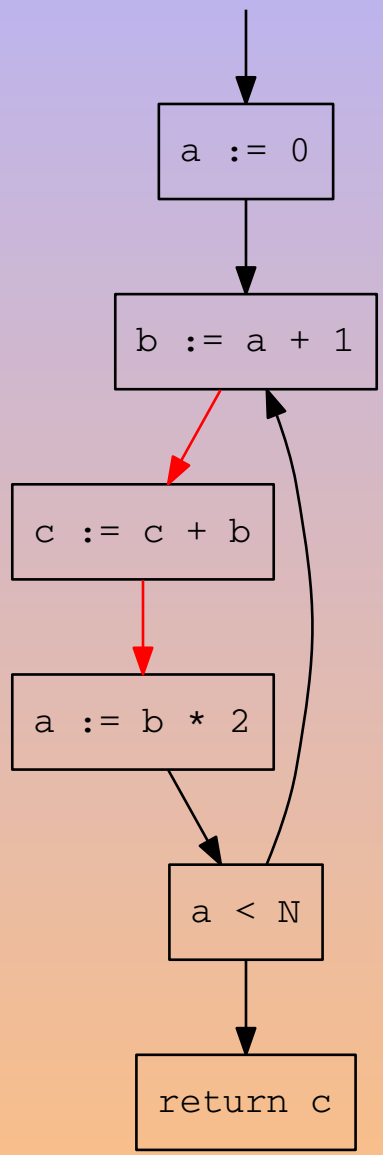
Liveness





Liveness


a := 0
b := a + 1
c := c + b
a := b * 2
a < N
return c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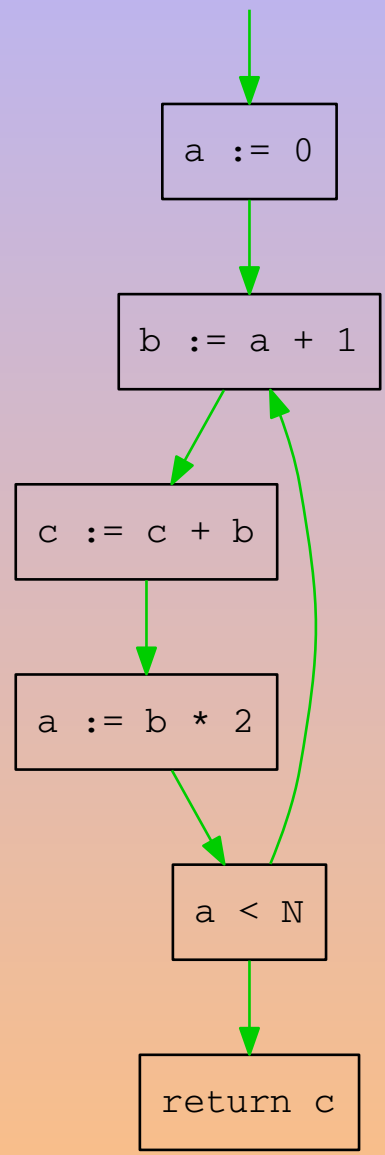
b := a + 1
c := c + b
a := b * 2
a < N
return c


a := 0
b := a + 1
c := c + b
a := b * 2
a < N
return c

Dataflow Equations for Liveness Analysis

$$
\begin{aligned}
\text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\
\text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s]
\end{aligned}
$$

Liveness Calculation

	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*	*in*	*out*
1		a								
2	a	b								
3	bc	c								
4	b	a								
5	a									
6	c									

	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*
1		a						
2	a	b						
3	bc	c						
4	b	a						
5	a							
6	c							

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

Liveness Calculation

1st step

	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*	*in*	*out*
1		a								
2	a	b	a							
3	bc	c	bc							
4	b	a	b							
5	a		a	a						
6	c		c							

	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*
1		a						
2	a	b						
3	bc	c						
4	b	a						
5	a							
6	c							

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

Liveness Calculation

			1st step		2nd step					
	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*	*in*	*out*
1		a				a				
2	a	b	a		a	bc				
3	bc	c	bc		bc	b				
4	b	a	b		b	a				
5	a		a	a	a	ac				
6	c		c		c					

	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*
1		a						
2	a	b						
3	bc	c						
4	b	a						
5	a							
6	c							

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

Liveness Calculation

			1st step		2nd step		3rd step			
	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*	*in*	*out*
1		a				a		a		
2	a	b	a		a	bc	ac	bc		
3	bc	c	bc		bc	b	bc	b		
4	b	a	b		b	a	b	a		
5	a		a	a	a	ac	ac	ac		
6	c		c		c		c			

	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*
1		a						
2	a	b						
3	bc	c						
4	b	a						
5	a							
6	c							

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

Liveness Calculation

			1st step		2nd step		3rd step		4th step	
	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*	*in*	*out*
1		a				a		a		ac
2	a	b	a		a	bc	ac	bc	ac	bc
3	bc	c	bc		bc	b	bc	b	bc	c
4	b	a	b		b	a	b	a	b	ac
5	a		a	a	a	ac	ac	ac	ac	ac
6	c		c		c		c		c	

	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*
1		a						
2	a	b						
3	bc	c						
4	b	a						
5	a							
6	c							

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

Liveness Calculation

			1st step		2nd step		3rd step		4th step	
	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*	*in*	*out*
1		a				a		a		ac
2	a	b	a		a	bc	ac	bc	ac	bc
3	bc	c	bc		bc	b	bc	b	bc	c
4	b	a	b		b	a	b	a	b	ac
5	a		a	a	a	ac	ac	ac	ac	ac
6	c		c		c		c		c	

			5th step					
	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*
1		a	c	ac				
2	a	b	ac	bc				
3	bc	c	bc	b				
4	b	a	bc	ac				
5	a		ac	ac				
6	c		c					

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

Liveness Calculation

			1st step		2nd step		3rd step		4th step	
	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*	*in*	*out*
1		a				a		a		ac
2	a	b	a		a	bc	ac	bc	ac	bc
3	bc	c	bc		bc	b	bc	b	bc	c
4	b	a	b		b	a	b	a	b	ac
5	a		a	a	a	ac	ac	ac	ac	ac
6	c		c		c		c		c	

			5th step		6th step			
	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*
1		a	c	ac	c	ac		
2	a	b	ac	bc	ac	bc		
3	bc	c	bc	b	bc	bc		
4	b	a	bc	ac	bc	ac		
5	a		ac	ac	ac	ac		
6	c		c		c			

$$\text{in}[n] = \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n])$$

$$\text{out}[n] = \bigcup_{s \in \text{succ}[n]} \text{in}[s]$$

Liveness Calculation

			1st step		2nd step		3rd step		4th step	
	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*	*in*	*out*
1		a				a		a		ac
2	a	b	a		a	bc	ac	bc	ac	bc
3	bc	c	bc		bc	b	bc	b	bc	c
4	b	a	b		b	a	b	a	b	ac
5	a		a	a	a	ac	ac	ac	ac	ac
6	c		c		c		c		c	

			5th step		6th step		7th step	
	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*
1		a	c	ac	c	ac	c	ac
2	a	b	ac	bc	ac	bc	ac	bc
3	bc	c	bc	b	bc	bc	bc	bc
4	b	a	bc	ac	bc	ac	bc	ac
5	a		ac	ac	ac	ac	ac	ac
6	c		c		c		c	

$$\text{in}[n] = \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n])$$

$$\text{out}[n] = \bigcup_{s \in \text{succ}[n]} \text{in}[s]$$

Liveness Calculation (Forward)

			1st step		2nd step		3rd step		4th step	
	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*	*in*	*out*
1		a				a		a		ac
2	a	b	a		a	bc	ac	bc	ac	bc
3	bc	c	bc		bc	b	bc	b	bc	c
4	b	a	b		b	a	b	a	b	ac
5	a		a	a	a	ac	ac	ac	ac	ac
6	c		c		c		c		c	

			5th step		6th step		7th step	
	use	*def*	*in*	*out*	*in*	*out*	*in*	*out*
1		a	c	ac	c	ac	c	ac
2	a	b	ac	bc	ac	bc	ac	bc
3	bc	c	bc	b	bc	bc	bc	bc
4	b	a	bc	ac	bc	ac	bc	ac
5	a		ac	ac	ac	ac	ac	ac
6	c		c		c		c	

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

Calculation done following forward control-flow edges.

Liveness Calculation (Backward)

	use	*def*	*out*	*in*	*out*	*in*	*out*	*in*
6	c							
5	a							
4	b	a						
3	bc	c						
2	a	b						
1		a						

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

Calculation done following *reverse* control-flow edges.

Liveness Calculation (Backward)

	use	*def*	1st step *out*	1st step *in*	*out*	*in*	*out*	*in*
6	c			c				
5	a		c	ac				
4	b	a	ac	bc				
3	bc	c	bc	bc				
2	a	b	bc	ac				
1		a	ac	c				

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

Calculation done following *reverse* control-flow edges.

Liveness Calculation (Backward)

			1st step		2nd step			
	use	*def*	*out*	*in*	*out*	*in*	*out*	*in*
6	c			c		c		
5	a		c	ac	ac	ac		
4	b	a	ac	bc	ac	bc		
3	bc	c	bc	bc	bc	bc		
2	a	b	bc	ac	bc	ac		
1		a	ac	c	ac	c		

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

Calculation done following *reverse* control-flow edges.

Liveness Calculation (Backward)

			1st step		2nd step		3rd step	
	use	*def*	*out*	*in*	*out*	*in*	*out*	*in*
6	c			c		c		c
5	a		c	ac	ac	ac	ac	ac
4	b	a	ac	bc	ac	bc	ac	bc
3	bc	c	bc	bc	bc	bc	bc	bc
2	a	b	bc	ac	bc	ac	bc	ac
1		a	ac	c	ac	c	ac	c

$$\begin{aligned} \text{in}[n] &= \text{use}[n] \cup (\text{out}[n] \setminus \text{def}[n]) \\ \text{out}[n] &= \bigcup_{s \in \text{succ}[n]} \text{in}[s] \end{aligned}$$

Calculation done following *reverse* control-flow edges.

Control Flow Graph [Appel, 1998]

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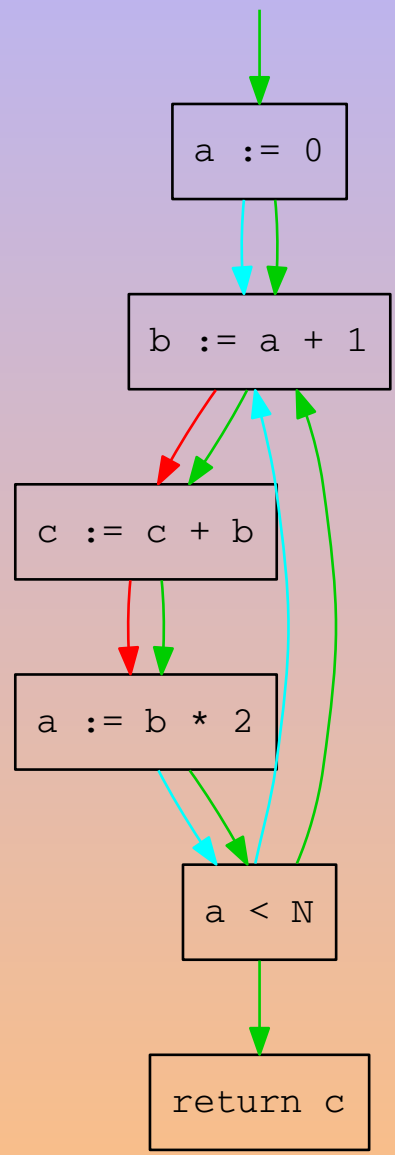
    a := 0
L1: b := a + 1
    c := c + b
    a := b * 2
    if a < N goto L1
    return c
```



Conservative Approximation

Suppose d a variable not used in the fragment of code

Another Solution

	use	*def*	*out*	*in*
1		a		
2	a	b		
3	bc	c		
4	b	a		
5	a			
6	c			

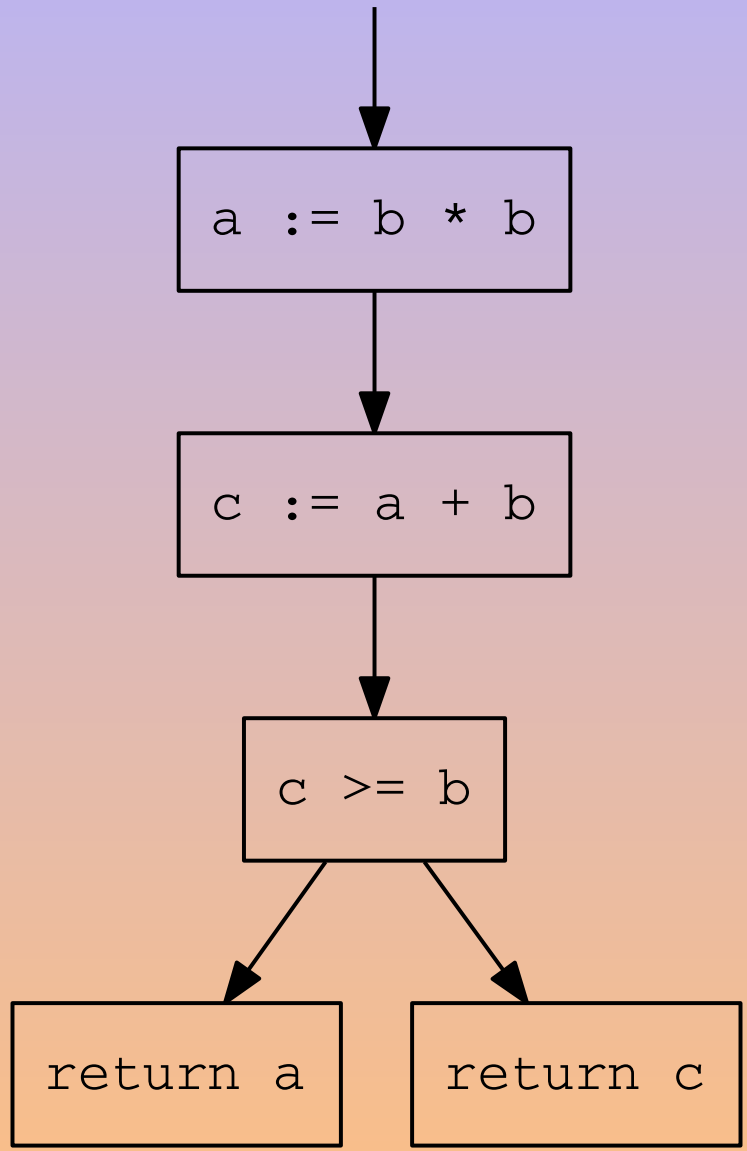
Conservative Approximation

Suppose d a variable not used in the fragment of code

Another Solution

	use	*def*	*out*	*in*
1		a	cd	acd
2	a	b	acd	bcd
3	bc	c	bcd	bcd
4	b	a	bcd	acd
5	a		acd	acd
6	c		c	

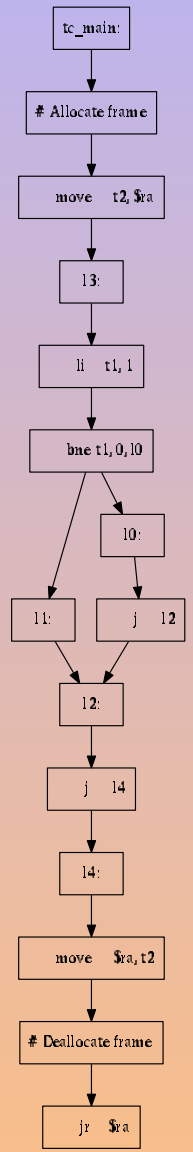
Conservative Approximation



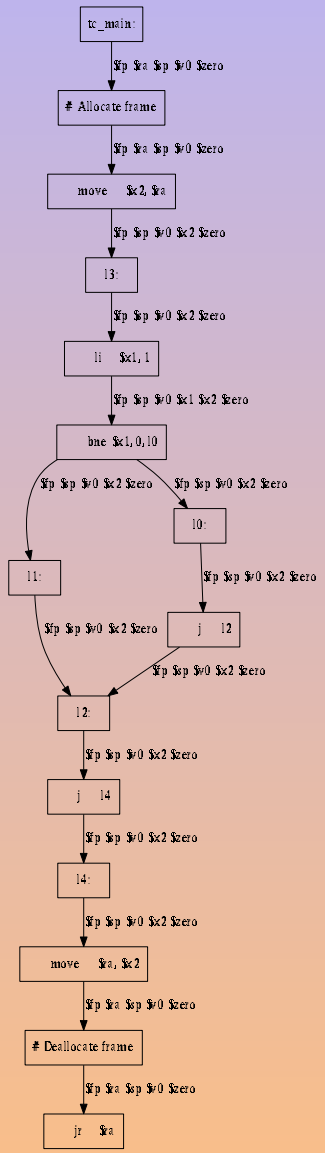
ors.tig

```
1 | 2
```

ors' Flowgraph



ors' Liveness Graph



Various Dataflow Analysis

Optimizing Compiler

- First step toward optimizing compilers
- How definitions and uses are related to each other
- What value a variable may have at a given point
- Constant propagation
- Common sub-expression elimination
- Copy propagation
- Dead Code Elimination

Constant propagation

An ambigous definition is a statement that might or not assign a temporary t. For instance, a call may sometimes modifies t and sometimes not.

We need to define the set of definitions that reach the begining and the end of each node.

- *gen*: when enter this statement, we know that we will reach the end of it
- kills: any statement that invalidates a *gen*
- begin[n]: which statements can reach the begining of statement n
- end[n]: which statements can reach the end of statement n

Reaching definition [Appel, 1998]

```
    a := 5
    c := 1
L1: if c > a goto L2
    c := c + c
    goto L1
L2: a := c - a
    c := 0
```

	gen	*kills*	*begin*	*end*	*begin*	*end*	*begin*	*end*
1	1	6						
2	2	4,7						
3								
4	4	2,7						
5								
6	6	1						
7	7	2,4						

$$\text{end}[n] = \text{gen}[n] \cup (\text{begin}[n] \setminus \text{kills}[n])$$

$$\text{begin}[n] = \bigcup_{p \in \text{pred}[n]} \text{end}[p]$$

			1st step					
	gen	*kills*	*begin*	*end*	*begin*	*end*	*begin*	*end*
1	1	6		1				
2	2	4,7	1	1,2				
3			1,2	1,2				
4	4	2,7	1,2	1,4				
5			1,4	1,4				
6	6	1	1,2	2,6				
7	7	2,4	2,6	6,7				

$$\text{end}[n] = \text{gen}[n] \cup (\text{begin}[n] \setminus \text{kills}[n])$$

$$\text{begin}[n] = \bigcup_{p \in \text{pred}[n]} \text{end}[p]$$

			1st step		2nd step			
	gen	*kills*	*begin*	*end*	*begin*	*end*	*begin*	*end*
1	1	6		1		1		
2	2	4,7	1	1,2	1	1,2		
3			1,2	1,2	1,2,4	1,2,4		
4	4	2,7	1,2	1,4	1,2,4	1,4		
5			1,4	1,4	1,4	1,4		
6	6	1	1,2	2,6	1,2,4	2,4,6		
7	7	2,4	2,6	6,7	2,4,6	6,7		

$$\text{end}[n] = \text{gen}[n] \cup (\text{begin}[n] \setminus \text{kills}[n])$$

$$\text{begin}[n] = \bigcup_{p \in \text{pred}[n]} \text{end}[p]$$

			1st step		2nd step		3rd step	
	gen	*kills*	*begin*	*end*	*begin*	*end*	*begin*	*end*
1	1	6		1		1		1
2	2	4,7	1	1,2	1	1,2	1	1,2
3			1,2	1,2	1,2,4	1,2,4	1,2,4	1,2,4
4	4	2,7	1,2	1,4	1,2,4	1,4	1,2,4	1,4
5			1,4	1,4	1,4	1,4	1,4	1,4
6	6	1	1,2	2,6	1,2,4	2,4,6	1,2,4	2,4,6
7	7	2,4	2,6	6,7	2,4,6	6,7	2,4,6	6,7

$$\text{end}[n] = \text{gen}[n] \cup (\text{begin}[n] \setminus \text{kills}[n])$$

$$\text{begin}[n] = \bigcup_{p \in \text{pred}[n]} \text{end}[p]$$

Constant Propagation

- If we have a statement $d_1 : t := c$, with c constant, and another statement d_2 that uses t.
- t is constant
- if d_1 reaches d_2 **and** no other definition of t reaches d_2
- then we can rewrite d_2

In the previous example, only one definition reaches statement 3 so we can replace $c > a$ by $c > 5$.

Copy Propagation

- If we have a statement $d_1 : t := z$, with z variable, and another statement d_2 that uses t.
- t is constant
- if d_1 reaches d_2 **and** no other definition of t reaches d_2 **and** the is no definition of z in all pathes between d_1 and d_2
- then we can rewrite d_2

Good register allocator will automatically detects some such cases.

Optimizing compiler

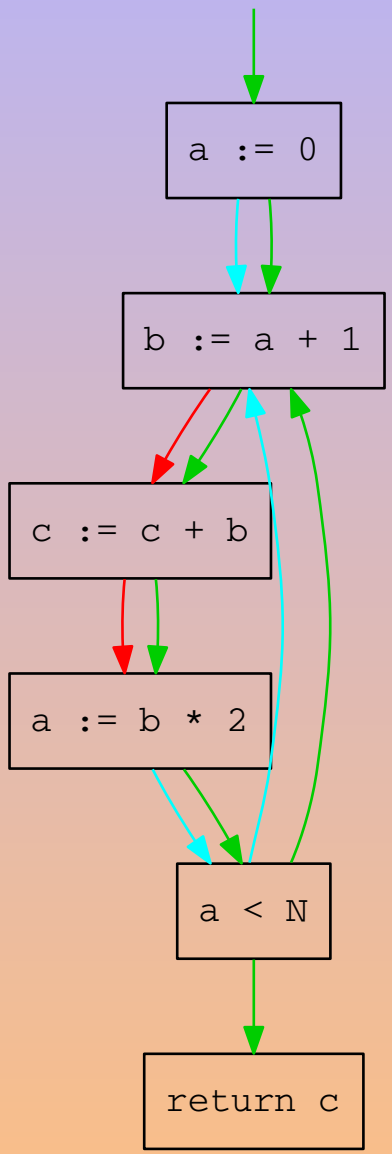
The removal of dead statements (or other optimizations) might introduce new dead statements.

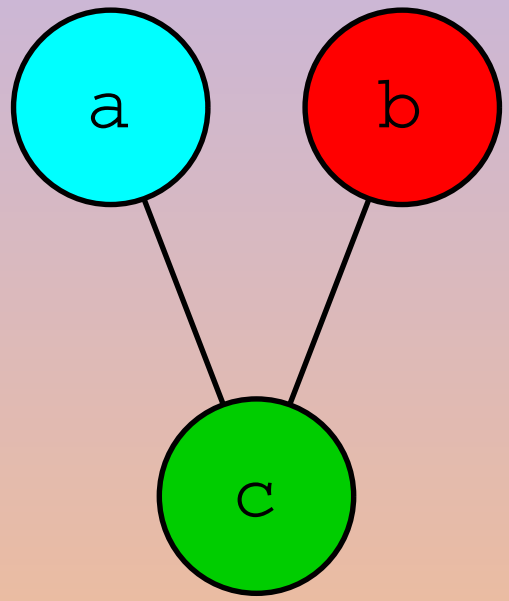
To avoid the need for repeated global calculation, several strategies exist:

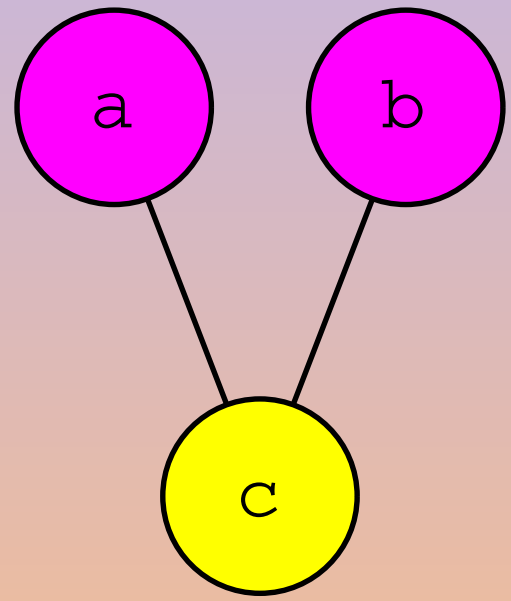
- Cutoff: perform no more than k round
- Cascading analysis: predict the cascade of effects of an optimization. Value numbering is a typical case of cascading analysis
- Incremental dataflow analysis: patch the dataflow after applying an optimization.

Interference Graph

Interference Graph

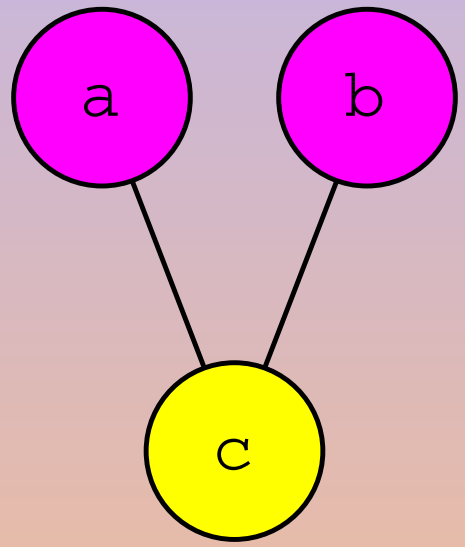






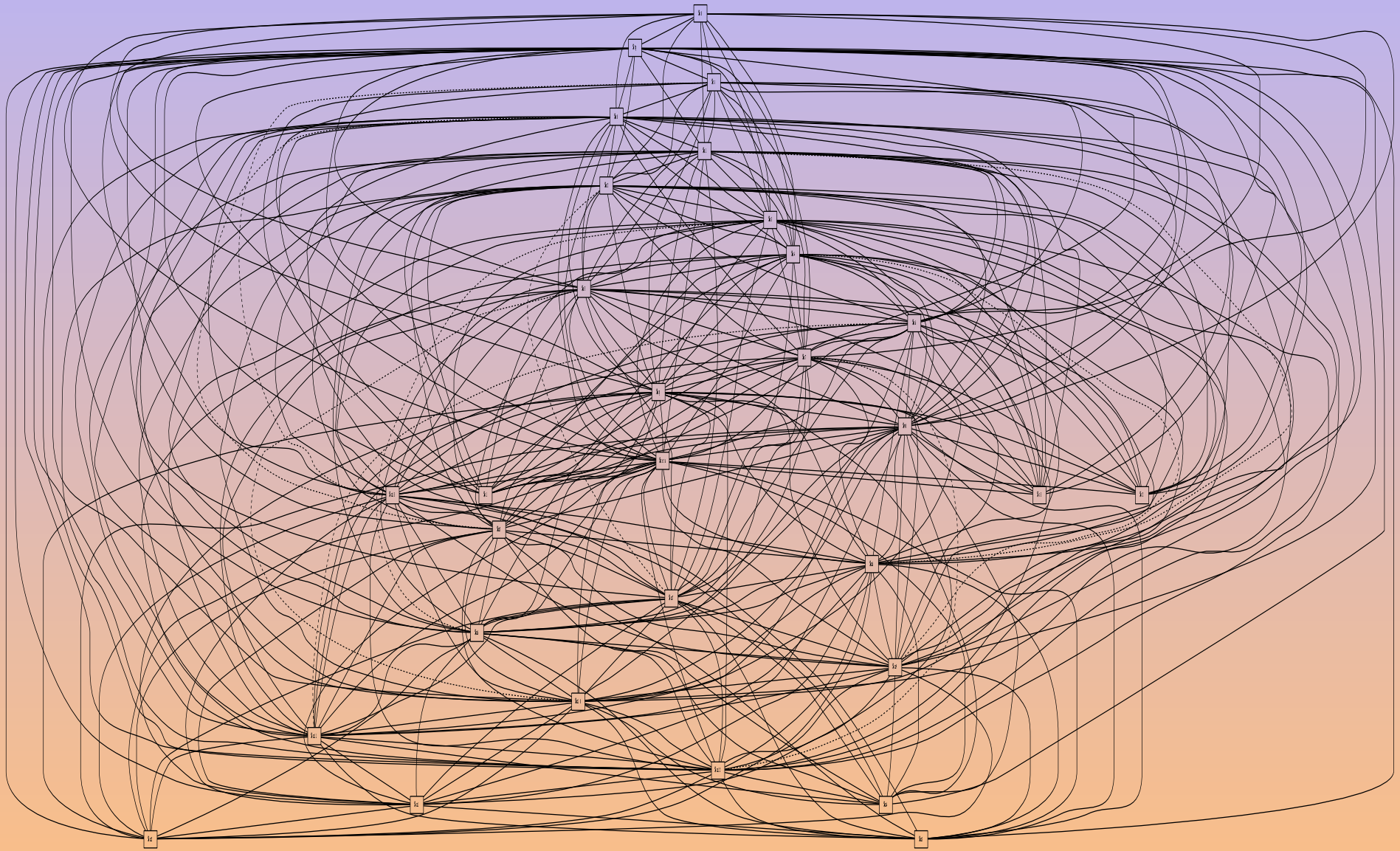
Register Allocation

```
     a := 0
L1:  b := a + 1
     c := c + b
     a := b * 2
     if a < N goto L1
     return c
```

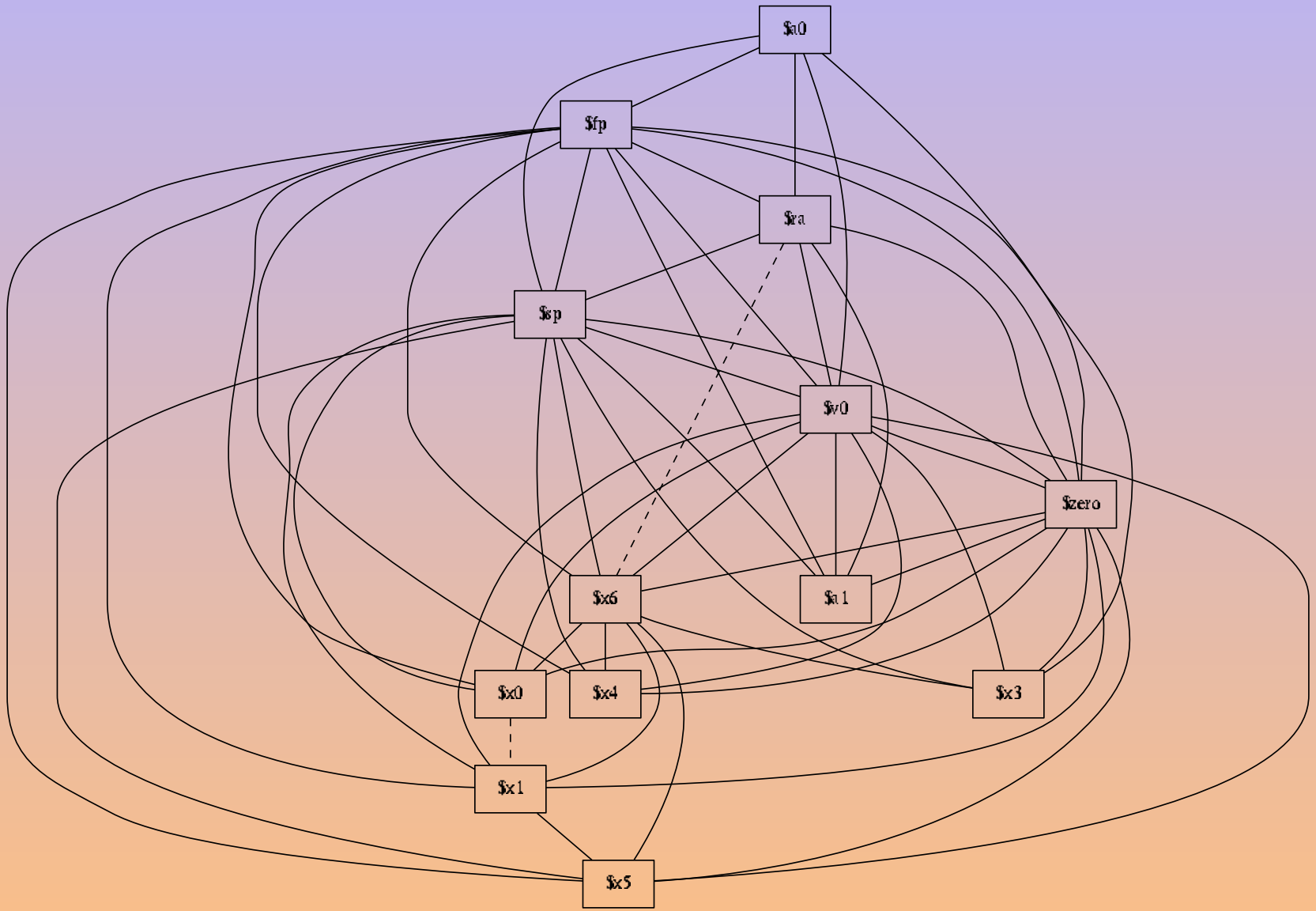



```
     r1 := 0
L1:  r1 := r1 + 1
     r2 := r2 + r1
     r1 := r1 * 2
     if r1 < N goto L1
     return r2
```

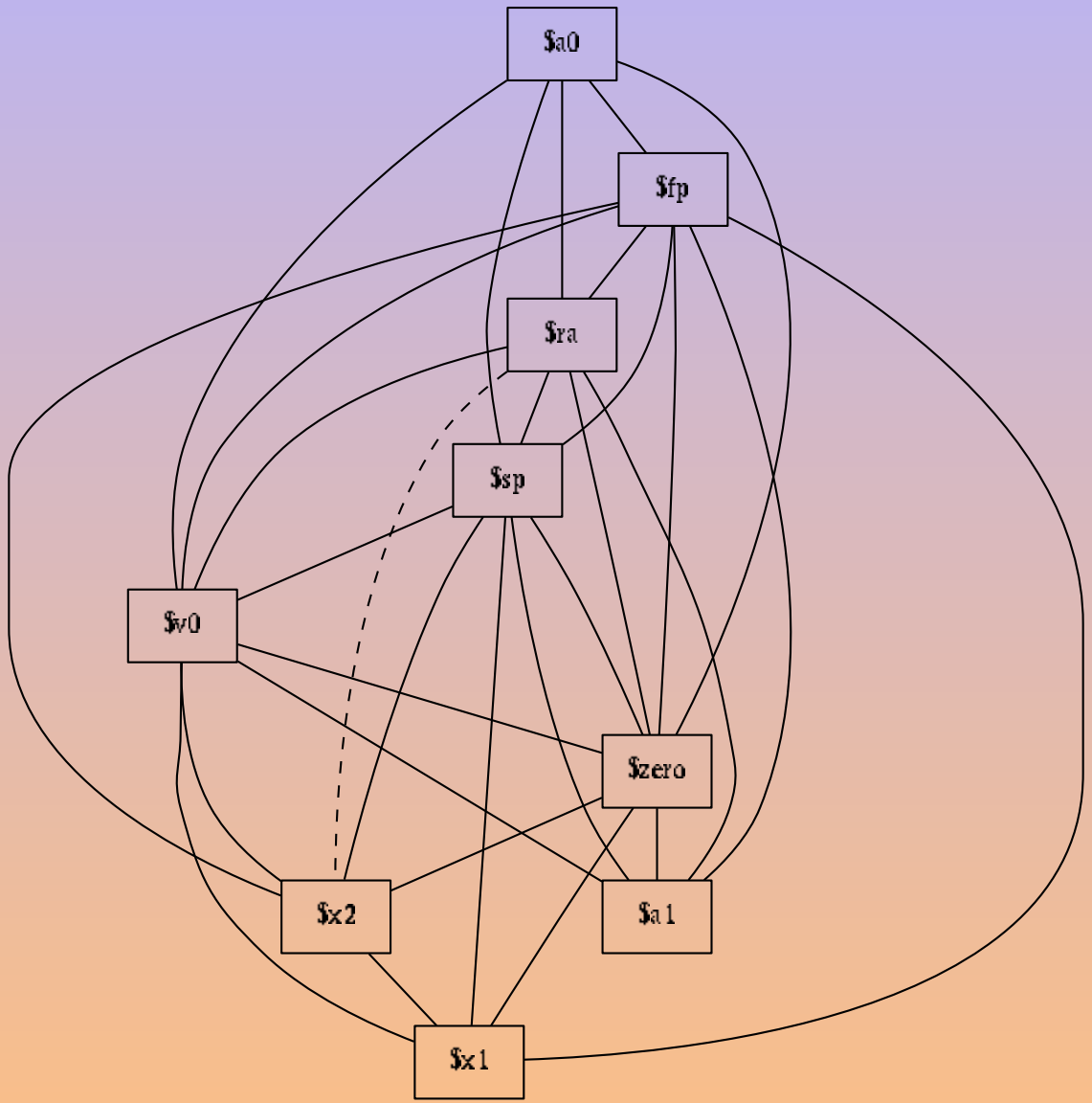
7's Interference Graph



7000's Interference Graph



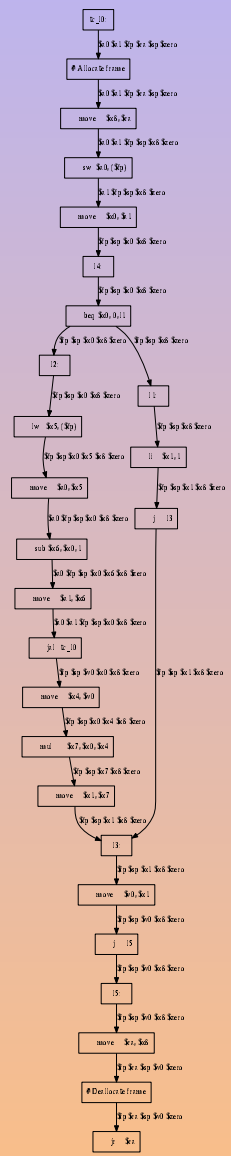
ors' Interference Graph



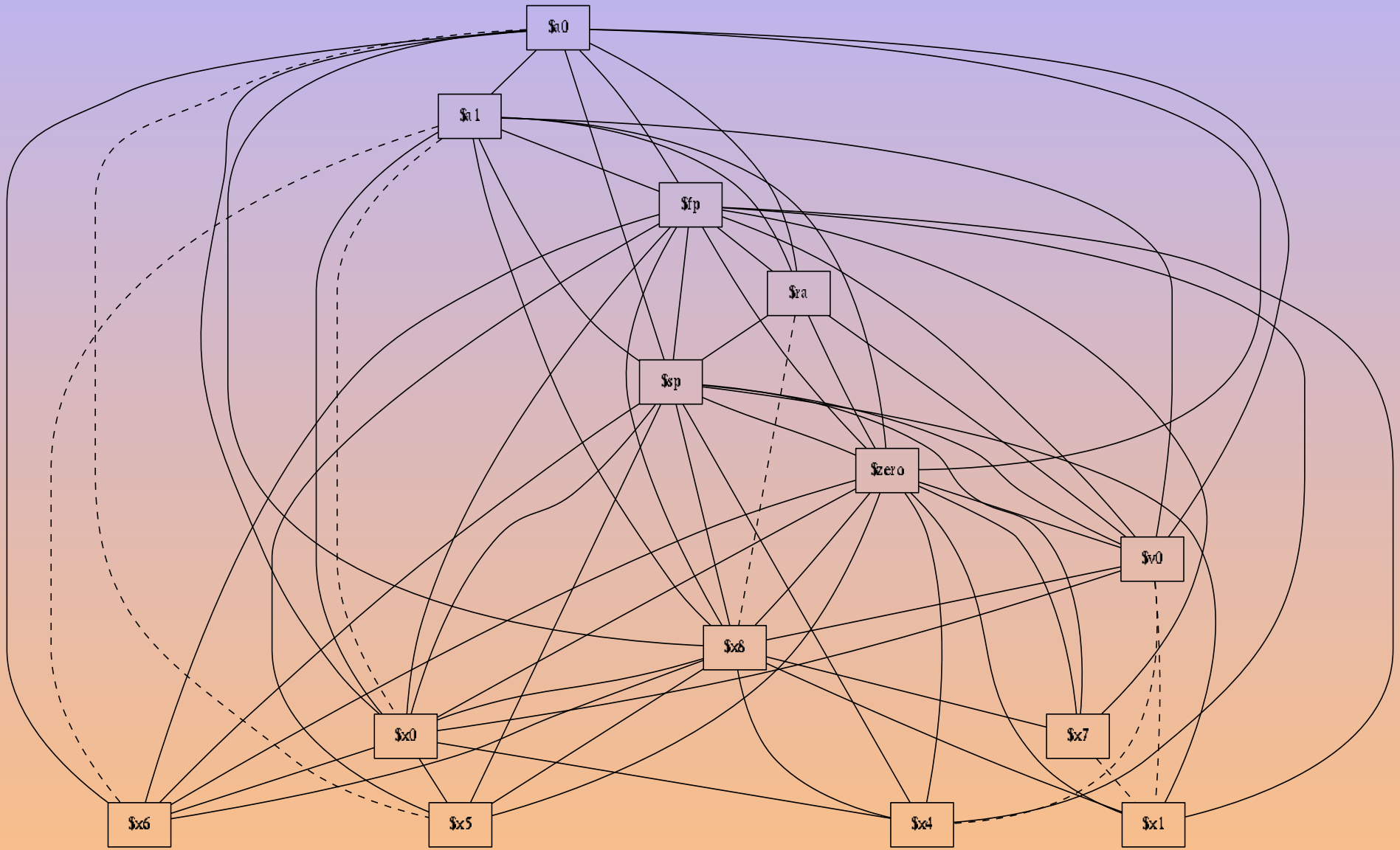
fact.tig

```
let function fact (n : int) : int =
      if n = 0 then
        1
      else
        n * fact (n - 1)
in
  fact (12)
end
```

fact's Liveness Graph



fact's Interference Graph



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