## Register Allocation

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## Register Allocation

(1) Interference Graph
(2) Coloring by Simplification
(3) Alternatives to Graph Coloring

## Interference Graph

(1) Interference Graph

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## Interference Graph



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## Register Allocation

$$
\text { L1: } \begin{aligned}
& \mathrm{a}:=0 \\
& \mathrm{c}:=\mathrm{a}+1 \\
& \mathrm{a}:=\mathrm{c}+\mathrm{b} * 2 \\
& \text { if } \mathrm{a}<\mathrm{N} \text { goto L1 } \\
& \text { return } \mathrm{c}
\end{aligned}
$$

## Register Allocation

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## Coloring by Simplification

(1) Interference Graph
(2) Coloring by Simplification

- Spilling
- Coalescing
- Precolored Nodes
- Implementation
(3) Alternatives to Graph Coloring


## Interference Graph [Appel, 1998]

Four registers: r1, r2, r3, r4.

$$
\begin{aligned}
& \text { live in }: k j \\
& g:=[j+12] \\
& h:=k-1 \\
& f:=g * h \\
& e:=[j+8] \\
& m:=[j+16] \\
& b:=[f] \\
& c:=e+8 \\
& d:=c \\
& k:=m+4 \\
& j:=b \\
& \text { live out }: d k j
\end{aligned}
$$



## Interference Graph: Simplify 0



## Interference Graph: Simplify 1



## Interference Graph: Simplify 2



三

## Interference Graph: Simplify 3



## Interference Graph: Simplify 4



## Interference Graph: Simplify 5



## Interference Graph：Simplify 6



## Interference Graph: Simplify 7

b
f
e
j
d
$k$
h
g


## Interference Graph: Simplify 8



## Interference Graph: Simplify 9

m
c
b
f
e
j
d
k
$h$
g


## Interference Graph: Color 9




## Interference Graph: Color 8



## Interference Graph：Color 7



## Interference Graph: Color 6



## Interference Graph: Color 5



클

## Interference Graph: Color 4



## Interference Graph: Color 3



## Interference Graph: Color 2



## Interference Graph: Color 1



## Interference Graph: Color 0



## Result

$$
\begin{aligned}
& \text { live } \text { in }: k j \\
& g:=[j+12] \\
& \mathrm{h}:=\mathrm{k}-1 \\
& \mathrm{f}:=\mathrm{g} * \mathrm{~h} \\
& \mathrm{e}:=[j+8] \\
& \mathrm{m}:=[j+16] \\
& \mathrm{b}:=[\mathrm{f}] \\
& \mathrm{c}:=\mathrm{e}+8 \\
& \mathrm{~d}:=\mathrm{c} \\
& \mathrm{k}:=\mathrm{m}+4 \\
& \mathrm{j}:=\mathrm{b} \\
& \text { live out }: \mathrm{d} k
\end{aligned}
$$

$$
\begin{aligned}
\text { live } & \text { in }: r 1 ~ r 3 \\
r 4 & :=[r 3+12] \\
r 2 & :=r 1-1 \\
r 2 & :=r 4 * r 2 \\
r 4 & :=[r 3+8] \\
r 1 & :=[r 3+16] \\
r 2 & :=[r 2] \\
r 3 & :=r 4+8 \\
r 4 & :=r 3 \\
r 1 & :=r 1+4 \\
r 3 & :=r 2
\end{aligned}
$$

live out: r4 r1 r3

## Simple Register Allocation


build the conflict graph from the program
the nodes with insignificant degree
while rebuilding the graph

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simplify the nodes with insignificant degree
select (or color) while rebuilding the graph.
Based on:
A.B. Kempe. On the Geographical problem of the four colors, Am. J. Math 2, 193-200, 1879.
[Appel, 1998, Matz, 2003]

## Yes, but What Color? [Matz, 2003]

- Usually, first-fit (registers are ordered).

Biased Coloring. [Briggs, 1992] Use a color already unavailable to ou neighbors.

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A map can always be colored with 4 colors...

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But for graph coloring, there is no reason for:

- this simple heuristics to always find a solution,


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But for graph coloring, there is no reason for:

- this simple heuristics to always find a solution,
- a solution to always exist...


## Spilling

- Not enough registers t1 := t1 + t2


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- So use the stack
$[\mathrm{sp}+4]:=[\mathrm{sp}+4]+[\mathrm{sp}+8]$
But use temporaries to do so! -Why should it solve the problem?


## Spilling

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\mathrm{t} 1:=\mathrm{t} 1+\mathrm{t} 2
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- So use the stack

$$
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- Why should it solve the problem?


## Register Allocation with Spills


spill when one cannot simplify, the (uses of the) temporary must be rewritten using the stack.
rebuild but then, the conflict graph is to be rewritten
[Appel, 1998, Matz, 2003]

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so "first spilled, last served"


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## Optimistic Coloring

- We miss many opportunities to avoid the stack

- Handle spills as if they were simplified (potential spills)
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## Coalescing

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- While allocating registers, we try to remove moves
live-out: t 1


## Coalescing

- Some low-level form of copy propagation
- While building traces we tried to remove jumps
- While allocating registers, we try to remove moves live-in: t2
t1 := ...
t2 := t1 + t2
t3 := t2
$\mathrm{t} 4:=\mathrm{t} 1+\mathrm{t} 3$
$\mathrm{t} 2:=\mathrm{t} 3+\mathrm{t} 4$
t1 := t2 - t4
live-out: t 1


## Coalescing Improves the Coloralibility



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## Interference Graph [Appel, 1998]

Four registers: $\mathrm{r} 1, \mathrm{r} 2, \mathrm{r} 3, \mathrm{r} 4$.

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\begin{aligned}
\text { live } & \text { in: } k j \\
g & :=[j+12] \\
h & :=k-1 \\
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e & :=[j+8] \\
m & :=[j+16] \\
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d & :=c \\
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## Interference Graph: Simplify 0



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## Interference Graph: Simplify 9



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## Interference Graph: Simplify 3



## Interference Graph: Simplify 2



## Interference Graph: Simplify 1



## Interference Graph: Simplify 0



## Interference Graph: Result

$$
\begin{aligned}
& \text { live } \text { in }: \mathrm{k} \\
& \mathrm{~g}:=[j+12] \\
& \mathrm{h}:=\mathrm{k}-1 \\
& \mathrm{f}:=\mathrm{g} * \mathrm{~h} \\
& \mathrm{e}:=[j+8] \\
& \mathrm{m}:=[j+16] \\
& \mathrm{b}:=[\mathrm{f}] \\
& \mathrm{c}:=\mathrm{e}+8 \\
& \mathrm{~d}:=\mathrm{c} \\
& \mathrm{k}:=\mathrm{m}+4 \\
& \mathrm{j}:=\mathrm{b} \\
& \text { live out }: \mathrm{d} k
\end{aligned}
$$

## Precolored Nodes

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## Callee \& Caller Save Registers

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Caller Save Def'd by calls.
Callee Save Def'd at entry, used at exit of functions.

- Register pressure will push temporaries live across calls into callee save.


## Conflicts

Minimize the conflicts ("pressure") with hard regs. Source and sink.
\# Routine: fact

10:
move $\$ x 11, \$$ s 0
move $\$ \mathrm{x} 12$, $\$$ s 1
16:

$$
\text { move } \$ \text { s0, } \$ x 11
$$

$$
\text { move } \$ \mathrm{~s} 1, \$ \mathrm{x} 12
$$

...
\# def \$s0, \$s1...
\# def: \$x11 use: \$s0
\# def: \$x12 use: \$s1
\# def: \$s0 use: \$x11
\# def: \$s1 use: \$x12
\# use: \$fp, \$ra, \$sp,
\# ... \$v0, \$zero

## Example [Appel, 1998]

```
int
f (int a, int b)
{
    int d = 0;
    int e = a;
    do
    {
        d += b;
        --e;
    } while (e > 0);
    return d;
}
```

$$
\begin{aligned}
& \text { enter: } \\
& \text { c }:=r 3 \\
& \mathrm{a}:=\mathrm{r} 1 \\
& \mathrm{~b}:=\mathrm{r} 2 \\
& \mathrm{~d}:=0 \\
& \mathrm{e}:=\mathrm{a} \\
& \text { loop }: \\
& \mathrm{d}:=\mathrm{d}+\mathrm{b} \\
& \mathrm{e}:=\mathrm{e}-1 \\
& \text { if e > } 0 \text { goto loop } \\
& \mathrm{r} 1:=\mathrm{d} \\
& \mathrm{r} 3 \\
& \text { return } \\
& \text { re } \\
& \text { liveout: r1, r3 }
\end{aligned}
$$

## Example

$$
\begin{aligned}
& \text { enter: } \\
& c:=r 3 \\
& \mathrm{a}:=\mathrm{r} 1 \\
& \text { b := r2 } \\
& \text { d }:=0 \\
& \text { e := a } \\
& \text { loop: } \\
& d:=d+b \\
& \text { e := e - } 1 \\
& \text { if e > } 0 \text { goto loop } \\
& \text { r1 := d } \\
& \text { r3 := c } \\
& \text { return } \\
& \text { \# liveout: r1, r3 }
\end{aligned}
$$



## Interference Graph: Simplify 0



## Interference Graph: Simplify 1



## Interference Graph: Simplify 2



## Interference Graph: Simplify 3



## Interference Graph: Simplify 4



## Interference Graph: Simplify 4



## Interference Graph: Simplify 3



## Interference Graph: Simplify 2



## Interference Graph: Simplify 1



## Interference Graph: Simplify 0



## Spilling

$$
\begin{aligned}
& \text { enter: } \\
& \quad \mathrm{c}:=\mathrm{r} 3 \\
& \mathrm{a}:=\mathrm{r} 1 \\
& \mathrm{~b}:=\mathrm{r} 2 \\
& \mathrm{~d}:=0 \\
& \mathrm{e}:=\mathrm{a} \\
& \text { loop }: \\
& \mathrm{d}:=\mathrm{d}+\mathrm{b} \\
& \mathrm{e}:=\mathrm{e}-1 \\
& \text { if e > o goto loop } \\
& \mathrm{r} 1 \text { := d } \\
& \text { r3 }:=\mathrm{c} \\
& \text { return } \\
& \# \text { liveout }: ~ r 1, ~ r 3
\end{aligned}
$$

## Example

$$
\begin{aligned}
& \text { enter : } \\
& \text { c1 := r3 } \\
& \text { [sp+8] := c1 } \\
& \text { a } \quad:=r 1 \\
& \text { b }:=r 2 \\
& \text { d }:=0 \\
& \text { e := a } \\
& \text { loop: } \\
& \text { d }:=d+b \\
& \text { e }:=e-1 \\
& \text { if e > } 0 \\
& \text { goto loop } \\
& \text { r1 := d } \\
& \text { c2 }:=[\mathrm{sp}+8] \\
& \text { r3 := c2 } \\
& \text { return } \\
& \text { \# liveout: r1, r3 }
\end{aligned}
$$



## Interference Graph: Simplify 0

c1\&r3


## Interference Graph: Simplify 1



## Interference Graph: Simplify 2



## Interference Graph: Simplify 3



## Interference Graph: Simplify 4



## Interference Graph: Simplify 5



## Interference Graph: Simplify 5



## Interference Graph: Simplify 4



## Interference Graph: Simplify 3



## Interference Graph: Simplify 2



## Interference Graph: Simplify 1



## Interference Graph: Simplify 0



## Result

## enter:

c1 := r3
[sp+8] := c1
a $:=r 1$
b $:=r 2$
d $\quad:=0$
e := a
loop:
d $:=d+b$
e $\quad:=$ e - 1
if e > 0
goto loop
r1 $:=d$
c2 $:=[\mathrm{sp}+8]$
r3 := c2
return
\# liveout: r1, r3
enter:

$$
r 3:=r 3
$$

$$
[s p+8]:=r 3
$$

$$
r 1:=r 1
$$

$$
\text { r2 }:=r 2
$$

$$
\text { r3 }:=0
$$

$$
r 1:=r 1
$$

loop:

$$
r 3:=r 3+r 2
$$

$$
\text { r1 }:=r 1-1
$$

$$
\text { if } r 1>0
$$

goto loop

$$
r 1:=r 3
$$

r3 := [sp+8]
r3 := r3
return
\# liveout: r1, r3
enter:

$$
[s p+8]:=r 3
$$

$$
\text { r3 }:=0
$$

loop:

$$
\begin{aligned}
& r 3 \quad:=r 3+r 2 \\
& r 1 \quad:=r 1-1 \\
& \text { if } r 1>0 \\
& \quad \text { goto loop } \\
& r 1 \quad:=r 3 \\
& r 3 \quad:=[s p+8]
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$$

return
\# liveout: r1, r3

## Implementation

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## Implementation

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- Existence of an edge between two nodes, hence bit matrix.
- For more information, see [Appel, 1998]


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- Naive implementation is quadratic
- Lower with heavy use of worklists
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Use both!

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## Alternatives to Graph Coloring

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## Register Allocation for Trees

Can be done during instruction selection with maximal munch function SimpleAlloc ( t )
for each nontrivial tile $u$ child of $t$
SimpleAlloc (u)
for each nontrivial tile $u$ child of $t$
n : $=\mathrm{n}$ - 1
n := n + 1
assign rn to (the root of) t
[Appel, 1998]

## Bibliography I

Appel, A. W. (1998).
Modern Compiler Implementation in C, Java, ML.
Cambridge University Press.
園 Briggs, P. (1992).
Register Allocation via Graph Coloring.
PhD thesis, Rice University, Houston, Texas.
( Matz, M. (2003).
Design and Implementation of a Graph Coloring Register Allocator for gcc.
pages 151-169.

